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Coordination of Home Appliances for Demand Response: An Improved Optimization Model and Approach

STEFFEN LIMMER[1], FERNANDO LEZAMA[2], (Member, IEEE), JOÃO SOARES[2], (Member, IEEE), AND ZITA VALE[2], (Senior Member, IEEE)

[1]Honda Research Institute Europe GmbH, 63073 Offenbach am Main, Germany
[2]Research Group on Intelligent Engineering and Computing for Advanced Innovation and Development (GECAD), Polytechnic Institute of Porto (ISEP/IPP), 4200-072 Porto, Portugal

Corresponding author: Steffen Limmer (steffen.limmer@honda-ri.de)

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ABSTRACT Home appliances constitute an interesting source of flexibility for demand response programs. However, their control and coordination are challenging, since typically a high number of such appliances has to be aggregated in order to provide a sufficient amount of flexibility. Thus, an efficient and scalable control approach is required. In a previous work, metaheuristic methods were evaluated for solving a control problem, which considers the regulation and time shifting of home appliances. The present paper presents two extensions of this problem, which consider a higher degree of flexibility, and which increase the practical relevance of the problem. Linear and quadratic formulations of the basic problem and its extensions are provided, which allow the use of mathematical programming for their solution. In experiments, the scalability of the proposed mathematical programming approach is evaluated and the impact of the problem extensions on the resulting schedules is investigated. The results show that problem instances with up to several thousand appliances can efficiently be solved and that aggregators can benefit from the additional flexibility considered in the problem extensions.

INDEX TERMS Demand response, energy management, home appliances, flexibility management, load shifting, mathematical programming, optimization.

NOMENCLATURE

PARAMETERS

Δt	Length of a time step.
c_k^A	Remuneration for shifting appliance $k \in A$.
c_k^B	Remuneration for regulating appliance $k \in B$.
c_k^C	Remuneration for shifting appliance $k \in C$.
c_k^D	Remuneration for regulating appliance $k \in D$.
$c_{DSO.}$	Penalty for deviation of load from load requested by DSO.
d_k^C	Remuneration for switching mode of appliance $k \in C$.
d_k^D	Remuneration for shifting appliance $k \in D$.
$I_{k,i.}^{B,max}$	Maximum intensity for i-th step in profile of appliance $k \in B$.
$I_{k,i.}^{B,min}$	Minimum intensity for i-th step in profile of appliance $k \in B$.
$I_{k,i.}^{B,pref}$	Preferred intensity for i-th step in profile of appliance $k \in B$.
$I_{k,i.}^{D,max}$	Maximum intensity for i-th step in profile of appliance $k \in D$.
$I_{k,i.}^{D,min}$	Minimum intensity for i-th step in profile of appliance $k \in D$.
$I_{k,i.}^{D,pref}$	Preferred intensity for i-th step in profile of appliance $k \in D$.
$M_k^{C,max}$	Number of modes of appliance $k \in C$.

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N_k^A	Length of load profile of appliance $k \in A$.
N_k^B	Length of load profile of appliance $k \in B$.
$N_{k,m.}^C$	Length of load profile of mode m of appliance $k \in C$.
N_k^D	Length of load profile of appliance $k \in D$.
$P_{k,i.}^A$	i-th value in load profile of appliance $k \in A$.
$P_{k.}^A$	Load profile of appliance $k \in A$.
$P_{k,i.}^B$	i-th value in load profile of appliance $k \in B$.
$P_{k.}^B$	Load profile of appliance $k \in B$.
$P_{k,m,i.}^C$	i-th value in load profile of mode m of appliance $k \in C$.
$P_{k,m.}^C$	Load profile of mode m of appliance $k \in C$.
$P_{k,i.}^D$	i-th value in load profile of appliance $k \in D$.
$P_{k.}^D$	Load profile of appliance $k \in D$.
P_t^{req}	Load requested by DSO for time step t.
T	Number of time steps.
$T_k^{A,min}$	Minimum start time of appliance $k \in A$.
$T_k^{A,max}$	Maximum start time of appliance $k \in A$.
$T_k^{A,pref}$	Preferred start time of appliance $k \in A$.
T_k^B	Start time of appliance $k \in B$.
$T_k^{C,max}$	Maximum start time of appliance $k \in C$.
$T_k^{C,min}$	Minimum start time of appliance $k \in C$.
$T_k^{C,pref}$	Preferred start time of appliance $k \in C$.
$T_k^{D,max}$	Maximum start time of appliance $k \in D$.
$T_k^{D,min}$	Minimum start time of appliance $k \in D$.
$T_k^{D,pref}$	Preferred start time of appliance $k \in D$.

SETS

A	Type-A appliances (shift).
B	Type-B appliances (regulate).
C	Type-C appliances (shift and operation mode).
D	Type-D appliances (shift and regulate).

VARIABLES

A_t	Load of type-A appliances in time step t.
B_t	Load of type-B appliances in time step t.
C_t	Load of type-C appliances in time step t.
D_t	Load of type-D appliances in time step t.
$F_{k,t.}^A$	Flag indicating start of appliance $k \in A$ in time step t.
$F_{k,m,t.}^C$	Flag indicating start of appliance $k \in C$ in mode m in time step t.
$F_{k,t.}^D$	Flag indicating start of appliance $k \in D$ in time step t.
$I_{k,i.}^{B,dev}$	Deviation from preferred intensity for i-th step in profile of appliance $k \in B$.
$I_{k,i.}^B$	Intensity for i-th step in profile of appliance $k \in B$.
$I_{k,i.}^{D,dev}$	Deviation from preferred intensity for i-th step in profile of appliance $k \in D$.
$I_{k,i.}^D$	Intensity for i-th step in profile of appliance $k \in D$.
M_k^C	Operation mode for appliance $k \in C$.
O_k^C	Flag indicating mode switch for appliance $k \in C$.
P_t^{dev}	Deviation from requested load in time step t.
P_t	Load in time step t.
Pen	Total penalty for deviation from requested load.
Rem^A	Remuneration for type-A appliances.
Rem^B	Remuneration for type-B appliances.
Rem^C	Remuneration for type-C appliances.
Rem^D	Remuneration for type-D appliances.
S_k^A	Flag indicating shift of appliance $k \in A$.
S_k^C	Flag indicating shift of appliance $k \in C$.
S_k^D	Flag indicating shift of appliance $k \in D$.
T_k^A	Start time of appliance $k \in A$.
T_k^C	Start time of appliance $k \in C$.
T_k^D	Start time of appliance $k \in D$.

I. INTRODUCTION

The proliferation of distributed energy resources based on renewable energy is quickly transforming the power grid at unprecedented rates [1]. This rapid penetration of uncontrollable variable energy generation is causing issues to grid balancing services since the grid inertia is dropping as large power plants are being displaced, consequently causing grid operator concerns [2], [3]. The reason lies in the fact that most of the renewable generation sources do not contribute to the system inertia due to the electrically decoupling of the generator from the grid [2], [4]. To achieve a sustainable grid based on – ideally close to 100% – renewable energy, substantial measures and changes to system operation are required [4]. Flexibility from the consumer side appears as a promising avenue to deal with increased levels of renewable energy in the grid [5], [6]. Beyond the necessary technology to enable demand-side flexibility, it is also important to engage consumers to get involved and interested in the grid events. A pilot study with 1232 participants conducted in Slovenia and Germany showed promising results within the Flex4Grid system [5]. The study suggests that the pilot's participants are engaged in the proposed system and reduced their demand by 10% on average during peak events. Previously, a study from the LINEAR (Local Intelligent Networks and Energy Active Regions) pilot project conducted in Belgium has quantified the potential flexibility of different residential appliances [7]. The analysis concludes that smart wet appliances can provide an average increase of 430 W per household at midnight and a maximum decrease of 65 W in the evening.

In the present paper, we present an extension of the work of Lezama *et al.* [8], who proposed an optimization model to solve the flexibility procurement problem of a distribution system operator (DSO). The DSO is looking to solve grid operation challenges while aggregators can match the procurement by accumulating the flexibility of their

customers' appliances and premises. Two types of appliances are considered: appliances whose operation time can be shifted and appliances whose energy consumption can be regulated. Lezama *et al.* formulated the problem in the form of a mixed-integer nonlinear programming (MINLP) model and solved it using metaheuristics. In the present paper, we propose and evaluate the use of an exact mathematical programming approach. Available exact solvers for MINLP models are prone to convergence issues. Hence, we convert the original MINLP model into a mixed-integer linear programming (MILP) model and solve it with an efficient commercial solver. In numerical experiments, we evaluate the approach on problem instances with up to 10,000 devices (compared to 140 devices as originally reported in [8]). The results show substantial improvements compared to the previously reported results achieved via metaheuristics. Therefore, we make two major extensions to the original optimization model to make the problem even more challenging and realistic:

- Extension 1: The original problem does not consider different operation modes, e.g., eco or express mode, of appliances. We extend the problem by adding a further class of appliances, whose operation modes can be switched and whose operation times can be shifted.
- Extension 2: A new class of appliances is added, which allows both, the shift of the operation time and the regulation of the energy consumption.

These two extensions lead to an increased number of binary variables (extension 1) and to quadratic terms (extension 2) in the optimization model, resulting in an increased computational burden and complexity compared to the original problem formulation. Hence, in the numerical experiments, we also evaluate the scalability of the proposed extensions. Furthermore, we investigate the impact of the extensions on the optimization results.

The key contributions of this paper can be summarized as follows:

- Consideration of (up to 10,000) appliances with DR capabilities (shifting, mode switching, and real-time reduction/increase capability) managed by an aggregator in response to DSO flexibility requests.
- Consideration of detailed consumption profiles (i.e., baseline profiles) of devices and operation times of each appliance in the home energy management system (HEMS).
- An efficient and scalable MILP/mixed integer quadratic problem (MIQP) optimization model solved using Gurobi solver, which can handle thousands of appliances.
- A systematic scalability analysis of the proposed method, investigating the monetary compensation for shifting devices using activation schemes (instead of the volume transacted) and optimizing the operation of appliances within a day-ahead horizon.

The rest of the paper is organized as follows: Section II provides an overview to related work. In Section III, the problem and its extensions are described and corresponding MILP/MIQP formulations are provided. Section IV describes the experiments and discusses their results. Finally, Section V provides a summary and conclusion.

II. STATE-OF-THE-ART ON FLEXIBILITY MANAGEMENT MODELS

Smart grid technologies are pushing the transformation of the energy grid towards a more sustainable network, in which energy flexibility plays a key role to achieve sustainable energy goals [9]. In this context, end-users equipped with modern information and communication technologies have been identified as a source of flexibility, taking advantage of the variable consumption patterns of some of their resources (e.g., loads with demand response (DR) capabilities) that can modify their profiles according to activation signals from an energy manager. Despite the low flexibility that single resources can offer to upper levels of the energy chain, different studies have pointed out that the aggregation of a considerable number of devices can be sufficient to alleviate some power grid issues [10]. Under these circumstances, the role of the aggregator seems to be the solution to gather overall flexibility volumes coming from small resources, enabling in this way the participation of end-users in local market activities and unlocking the value of their flexibility [11].

Demand response, as a solution to take advantage of the flexibility of resources, is a well-studied topic usually targeting resources connected to the medium and high voltage levels (i.e., big players having industrial loads) [12]–[16]. However, at the lower level, the aggregation and management of small resources is still a topic of interest in which researchers and facilities search for the most efficient way to take advantage of the high variability and low capacity of flexibility from end-users. Prieto-Castrillo *et al.* [17] consider the optimization of social welfare and flexibility using linear programming from the perspective of an aggregator that responds to the DSO, but they do not provide details about the scalability of the approach. Another work considering the interaction between DSO and aggregators is the one proposed by Lipari *et al.* [18], in which the flexibility management of batteries, and controllable and adjustable heat pumps (HPs) is analyzed in simulation. This work, however, does not provide actual coordination of flexible resources inside the house, and the study is limited to 173 customers that are part of the aggregator's portfolio. Similarly, Henríquez *et al.* [19] proposed a bi-level formulation for the profit maximization of DR aggregators and solved it as a MILP problem. The authors reported optimization times of around 1400 seconds for the base case, reducing such times down to 90 seconds when a single contract and 10 imbalance scenarios are considered. However, the actual number of devices controlled by the aggregators is not explicitly provided, so the scalability of the approach is difficult to assess. The authors conclude that the number of binary variables used to model DR contracts scales with the number of contracts, generators,

buses, lines, and scenarios and recommend scenario reduction techniques to achieve acceptable computational times. Müller and Jansen [20] also consider the optimization of HPs by an aggregator, but from a large-scale demonstrator perspective. They provide results of experimental simulations considering 5 minutes granularity; however, despite calling it large-scale, only 300 devices with DR capabilities are considered in this study. A work that considers a more diverse set of devices (i.e., combined heat and power (CHP), auxiliary boilers (AB), PV, absorption chiller (AChil), HPs, batteries, and thermal energy storage) is presented by Di Somma *et al.* [21]. The maximization of aggregators' profits is achieved using clusters of local energy and the optimization times are kept within 25 minutes using a stochastic MILP approach. A more recent study presented by Olivella-Rosell *et al.* [22] proposed a model for the optimization of PV-battery systems using MILP. The problem is also solved using a decomposition approach (based on the alternating direction method of multipliers (ADMM)) that can provide solutions 5 to 12 times faster than the centralized approach. However, only 100 prosumers were considered in this study, so the scalability of the approach managing a larger number of devices was not assessed. A similar study for the optimization of PV-battery systems was introduced by Lezama *et al.* [23]. This study presents a scalability analysis, increasing the number of customers equipped with PV-battery systems to up to 10,000. The problem is solved using MILP/CPLEX and optimization times varied from 3.5 to 127 minutes depending on the devices considered in the optimization. Some other works have also explored the use of metaheuristics to leverage the computational burden that these types of large-scale optimization problems present [8], [24]–[26]. Recently, advanced metaheuristics such as the quadratic particle swarm optimization [27], or the enhanced leader particle swarm optimization [28] were proposed to tackle the optimal scheduling of appliances in smart homes, managing shiftable devices. Despite promising results, the case studies consider only up to 264 variables and 10 appliances, respectively, which cannot be considered a large-scale complex problem. Moreover, there is no discussion about running times or optimality guarantees, which undermines the applicability of such approaches under more realistic scenarios. In [29], a new bi-trajectory metaheuristic is proposed to solve exactly the same problem as the one proposed in [8]. While an improvement of up 24% compared with the results published in [8] are achieved, again, no scalability analysis or optimality guarantees are provided. It is worth saying that metaheuristics have the potential to become an alternative optimization method when dealing with complex mathematical problems in the new paradigm of power systems. However, such methods still present as a main drawback an inability of guaranteeing optimality, which hampers their acceptance and application in real-world scenarios. Moreover, metaheuristic methods are usually tailored for a specific application (taking advantage of expert knowledge and tailored techniques), and despite their effectiveness in solving particular complex problems, their application to general mathematical models is limited. As these limitations are overcome, metaheuristic optimization can find its true value as an alternative method to solve complex problems in the energy domain. As can be seen from the literature review, many works focus on the development of optimization models and solution methods that can handle a considerable number of devices in acceptable optimization times. Scalability is a critical aspect of such models since the aggregator needs to guarantee access to a considerable volume of flexibility coming from hundreds or thousand of devices to unlock their value. For instance, in [30], direct load control of appliances is used to solve a bilevel MILP formulation using Dantzig–Wolfe decomposition. Thanks to reducing the bilevel formulation to a single-level problem and using a distributed approach, the method can handle up to 10,000 users, compared to only 500 users without decomposition. In fact, some studies such as the one presented by Salgado *et al.* [31] are already in the phase of demonstration, implementing optimization models on, for instance, Raspberry processors, to manage the flexibility of resources. The work in [31] actually uses a two-stage MILP approach to minimize the demand deviations of devices such as thermal appliances, storage systems, and loads. Despite showcasing the practicality of MILP formulations to address such optimization problems, the demonstration only considers 4 different houses with 7 to 13 appliances, exposing the lack of more efficient optimization methods to handle a more considerable (and useful) number of devices. Table 1 summarizes the related works, highlighting their main characteristics and positioning our proposal. Considering the literature, our paper proposes a MILP/MIQP mathematical model for an aggregator that gathers the flexibility coming from residential end-users to match DSO/BRP flexibility requests considering a day-ahead optimization horizon. Through an efficient model, in contrast with [8] (that considers a MINLP model), we are able to handle thousands of devices in acceptable optimization times and with low optimization gaps.

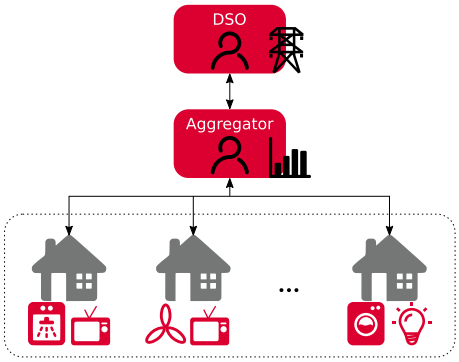

FIGURE 1. System overview.

III. PROBLEM DESCRIPTION

We assume a setting as illustrated in Figure 1. Different appliances of multiple households are under the control of an aggregator, who provides demand response capacities to a

TABLE 1. Summary of the state-of-the-art of flexibility management.

Year	Resources	Players	Objective	Planning horizon	Optimization method	Remarks
[17] (2018)	Shiftable loads in general (not detailed)	DSO, aggregators, bus load	Social welfare maximization	Day-ahead, 1 h interval	Linear programming	Optimizing flexible resources and social welfare, but there is no concrete information about scalability or number of handled resources.
[18] (2018)	Batteries, controllable heat pumps	DSO, aggregators, customers	Flexibility management	Day-ahead, 15 min interval	Planner optimization and simulation	No actual coordination of flexibility volume inside the house. Only up to 173 customers are considered in the aggregator's portfolio.
[19] (2018)	Shiftable and fixed loads (not detailed)	DR aggregators	Profit maximization	Day-ahead, 1h interval	Bi-level formulation solved as MILP	1400 s on average to solve the base case. Problems with a single contract and 10 scenarios of imbalance took 90 s and considering the transmission system took 3500 s.
[32] (2018)	Shiftable appliances (not detailed)	Aggregator, smart home	Energy cost minimization	Fine-scale (1 second monitoring)	Combinatorial optimization	Profile shift algorithm, simulation, no scalability analysis, only 5 appliances are considered.
[20] (2019)	Thermostatically controlled HP	Aggregator, buildings	Load reduction	5 min interval	Experimental simulation	Large-scale demonstration of DR only considered 300 houses with heat pumps (i.e., 300 devices).
[21] (2019)	CHP, AB, PV, AChil, HPs, storage	Aggregator, HEMS	Bidding maximizing aggregator's expected profit	Day-ahead, 1 h interval	Stochastic MILP, branch-and-cut	Clusters of local energy with interactions of various energy carriers (e.g., gas, electricity, heat, and cooling). DSO is not considered. Problem solved within 25 min with optimality gap lower than 0.01%.
[27] (2019)	Shiftable appliances	Smart home	Minimization of electricity bill and discomfort	Day-ahead 10 min interval	Multi-objective with binaries, metaheuristics	A novel quadratic PSO is proposed but only up to 264 variables equivalent to the sum of the length of operation ranges of all appliances are handled.
[28] (2020)	Shiftable appliances and EVs	Smart home	Minimization of electricity bill and discomfort	Day-ahead 30 min interval	Multi-objective with integers, metaheuristics	Enhanced leader PSO is used but only 10 appliances are considered. No discussion about optimality gaps or scalability is provided.
[30] (2020)	DLC appliances (not detailed)	Aggregator and customers	Cost minimization, profits maximization	Day-ahead 10 min interval	MILP and bilevel, DantzigâĂŞ-Wolfe decomposition	A distributed approach allows instances of up to 10,000 users. Decomposition is possible, but the modeling of appliances is missing.
[22] (2020)	PV, battery system	Aggregator, prosumers, BRP/DSO	Cost minimization	Day-ahead, 1 h interval	MILP, ADMM	A decomposition algorithm can provide solutions 5 to 12 times faster than centralized approaches. Only 100 prosumers were considered in the case study.
[8] (2020)	Real-time and shifting devices	Aggregator, DR consumers, BRP/DSO	Remuneration minimization	Day-ahead, 15 min interval	MINLP, metaheuristics	Consideration of real-time reduction and shifting load devices. Metaheuristics cannot guarantee optimality. Case study considering 20 houses (140 devices) is solved in around 145 min.
[25] (2020)	Shiftable, non-shiftable, and fixed appliances.	Aggregator, home appliances	Cost minimization, user comfort maximization	Day-ahead, 1 h interval	Multi-objective MILP, metaheuristics	Dimension of the problem or the number of devices is not clear. Runtimes around 10 s are reported, but no scalability analysis is provided. Optimality cannot be guaranteed.
[26] (2020)	PV, battery system, DR loads	Aggregator, HEMS	Minimize energy bill and DR curtailment	Day-ahead, 1 h interval	MILP, metaheuristics	A parallel approach is proposed to improve quality of solutions (although it is not used to improve optimization times). The optimization of 20 houses with PV-battery system and DR takes 5–10 min.
[23] (2020)	PV, battery system, DR loads	DSO, aggregator, HEMS	Minimize costs and maximize profits	Day-ahead, 1 h interval	MILP, CPLEX	The study considers up to 1000 households with PV-battery system. Optimization times span from 3.5 to 127 min, depending on the considered resources in the house.
[29] (2021)	Real-time devices and shifting devices	Aggregator, DR consumers, BRP/DSO	Remuneration minimization	Day-ahead 15 min interval	Metaheuristics	Bi-objective algorithm provides up to 24% of improvements. Only 140 devices are considered. No evidence of optimality guarantees or scalability analysis.
[31] (2021)	Thermal appliances, ESS, flexible loads.	Aggregator, HEMS	Minimization of demand deviations	Day-ahead, 4 min interval	Two-stage MILP	The demonstration was implemented in a Raspberry. Only 4 different houses with 7 to 13 appliances were considered.
This work	Real-time and shifting devices	Aggregator, DR consumers, BRP/DSO	Remuneration minimization	Day-ahead, 15 min interval	MINLP transformed to MILP	Consideration of thousands of devices. Scalability analysis. Optimality gap guarantees.

DSO. The owners of the appliances have certain preferences regarding their operation (start times, load intensities, etc.). The DSO sends DR signals to the aggregator, requesting certain electrical loads for multiple time periods in the future. If the actual load curve resulting from the operation of the appliances does not match the DSO request, the aggregator

has to pay a penalty fee. On the other hand, the aggregator has to remunerate the owners of the appliances if they are not operated according to the owners' preferences. Thus, the aggregator is interested in coordinating the operation of the appliances to minimize the total cost.

In the following, the resulting scheduling problem is described more in detail. We first provide a description and a MILP formulation of the basic problem as considered in [8]. Then, the two extensions of the problem are described and corresponding MILP/MIQP formulations are provided.

A. BASIC PROBLEM

We assume a time horizon of T time steps of length Δt, each. An aggregator has access to two sets of appliances: A set A of appliances of type A, which can be shifted in time and a set B of appliances of type B, which can be down and up regulated. Each of these appliances has to be operated exactly once during the considered time horizon. Each appliance $k \in A$ is associated with a certain load profile $P_k^A = (P_{k,1}^A, \ldots, P_{k,N_k^A}^A)$ of length N_k^A, where $P_{k,i}^A$ is the load in kW of the appliance in the i-th time step of its operation. Furthermore, there is a time step $T_k^{A,pref}$ for each $k \in A$ in which the owner of the appliance prefers the start of its operation. Analogously, there is a load profile $P_k^B = (P_{k,1}^B, \ldots, P_{k,N_k^B}^B)$ and a start time step T_k^B for each appliance $k \in B$. Additionally, there is a preferred intensity $I_{k,i}^{B,pref} \in [0, 1]$ for each step i in the load profile of an appliance $k \in B$.

Operating all appliances according to the preferences of their owners would result in the following load in time step t:

$$P_t^{pref} = \sum_{k \in A} A_{k,t}^{pref} + \sum_{k \in B} B_{k,t}^{pref}, \tag{1}$$

where

$$A_{k,t}^{pref} = \begin{cases} P^A_{k,t-T_k^{A,pref}+1}, \text{ if } 0 \leq t - T_k^{A,pref} < N_k^A \\ 0, \text{ otherwise} \end{cases} \tag{2}$$

is the contribution of appliance $k \in A$, and

$$B_{k,t}^{pref} = \begin{cases} I^{B,pref}_{k,t-T_k^B+1} \cdot P^B_{k,t-T_k^B+1}, \text{ if } 0 \leq t - T_k^B < N_k^B \\ 0, \text{ otherwise} \end{cases} \tag{3}$$

is the contribution of appliance $k \in B$.

It is assumed that the aggregator participates in a demand response program and that for each time step t, the DSO requests a certain load P_t^{req}. If the actual load P_t in time step t deviates from the requested load, the aggregator has to pay a penalty of c_{DSO} Euro per kWh of deviation. Thus, the total penalty fee over the considered time horizon in Euros can be computed as:

$$Pen = \sum_{t=1}^{T} c_{DSO} \cdot |P_t - P_t^{req}| \cdot \Delta t. \tag{4}$$

The aggregator can reduce the deviation from the requested load, and thus the penalty, by shifting the start times T_k^A of each appliance $k \in A$ within certain limits $T_k^{A,min}$ and $T_k^{A,max}$. Appliances, which are flexible in their start times, could be, for example, washing machines or tumble dryers. If the start time of appliance $k \in A$ is set different from the preferred start time $T_k^{A,pref}$, the aggregator has to remunerate the owner of the appliance by c_k^A Euro and thus, the total remuneration for appliances of type A can be computed as

$$Rem^A = \sum_{k \in A} c_k^A \cdot S_k^A, \tag{5}$$

with

$$S_k^A = \begin{cases} 1, \text{ if } T_k^A \neq T_k^{A,pref} \\ 0, \text{ otherwise} \end{cases}. \tag{6}$$

In addition to the start times of type-A appliances, the aggregator can regulate the intensities $I_{k,i}^B$, $i = 1, \ldots, N_k^B$, for appliances $k \in B$ within certain limits $I_k^{B,min}$ and $I_k^{B,max}$. An appliance, which is flexible in the intensity, could be, for example, a lighting system. A deviation in the energy consumption from that resulting from the preferred intensities has to be remunerated by c_k^B Euro per kWh of deviation and the total remuneration for type-B appliances can be computed as:

$$Rem^B = \sum_{k \in B} \sum_{i=1}^{N_k^B} c_k^B \cdot |I_{k,i}^B - I_{k,i}^{B,pref}| \cdot P_{k,i}^B \cdot \Delta t. \tag{7}$$

The target of the aggregator is to set the start times of type-A appliances and the intensities of type-B appliances in order to minimize the total cost, resulting in the following optimization problem:

$$\min_{T_k^A, I_{k,i}^B} \; Pen + Rem^A + Rem^B \tag{8}$$

subject to

$$(4),\ (5),\ (6),\ (7), \tag{9}$$

$$P_t = A_t + B_t \quad \forall t, \tag{10}$$

$$T_k^{A,min} \leq T_k^A \leq T_k^{A,max} \quad \forall k \in A, \tag{11}$$

$$A_t = \sum_{k \in A | T_k^A \leq t < T_k^A + N_k^A} P^A_{k,t-T_k^A+1} \quad \forall t, \tag{12}$$

$$I_k^{B,min} \leq I_{k,i}^B \leq I_k^{B,max} \quad \forall k \in B,\ i = 1, \ldots, N_k^B, \tag{13}$$

$$B_t = \sum_{k \in B | T_k^B \leq t < T_k^B + N_k^B} I^B_{k,t-T_k^B+1} \cdot P^B_{k,t-T_k^B+1} \quad \forall t. \tag{14}$$

The problem formulation (8)–(14) contains several nonlinear terms. However, it is possible to linearize the problem in order to solve it with a conventional MILP solver, as follows:

$$\min_{F_{k,t}^A, I_{k,i}^B} \; Pen + Rem^A + Rem^B \tag{15}$$

subject to

$$Pen = \sum_{t=1}^{T} c_{DSO} \cdot P_t^{dev} \cdot \Delta t, \tag{16}$$

$$P_t = A_t + B_t \quad \forall t, \tag{17}$$
$$P_t^{dev} \geq P_t - P_t^{req} \quad \forall t, \tag{18}$$
$$P_t^{dev} \geq P_t^{req} - P_t \quad \forall t, \tag{19}$$
$$F_{k,t}^A \in \{0, 1\} \quad \forall k \in A, \forall t, \tag{20}$$
$$F_{k,t}^A = 0 \quad \forall k \in A, \forall t \notin \{T_k^{A,min}, \ldots, T_k^{A,max}\}, \tag{21}$$
$$\sum_{t=1}^{T} F_{k,t}^A = 1 \quad \forall k \in A, \tag{22}$$
$$S_k^A = 1 - F_{k,T_k^{A,pref}}^A \quad \forall k \in A, \tag{23}$$
$$Rem^A = \sum_{k \in A} c_k^A \cdot S_k^A, \tag{24}$$
$$A_t = \sum_{k \in A} \sum_{l=\max\{1,t-N_k^A+1\}}^{t} F_{k,l}^A \cdot P_{k,t-l+1}^A \quad \forall t, \tag{25}$$
$$I_k^{B,min} \leq I_{k,i}^B \leq I_k^{B,max} \quad \forall k \in B, i = 1, \ldots, N_k^B, \tag{26}$$
$$I_{k,i}^{B,dev} \geq I_{k,i}^B - I_{k,i}^{B,pref} \quad \forall k \in B, i = 1, \ldots, N_k^B, \tag{27}$$
$$I_{k,i}^{B,dev} \geq I_{k,i}^{B,pref} - I_{k,i}^B \quad \forall k \in B, i = 1, \ldots, N_k^B, \tag{28}$$
$$Rem^B = \sum_{k \in B} \sum_{i=1}^{N_k^B} c_k^B \cdot I_{k,i}^{B,dev} P_{k,i}^B \cdot \Delta t, \tag{29}$$
$$B_t = \sum_{k \in B | T_k^B \leq t < T_k^B + N_k^B} I_{k,t-T_k^B+1}^B \cdot P_{k,t-T_k^B+1}^B \quad \forall t. \tag{30}$$

Binary flags $F_{k,t}^A$ are introduced, representing the start times of type-A appliances: If $F_{k,t}^A$ equals one, the operation of appliance $k \in A$ starts in time step t. Constraint (22) ensures that exactly one start time is set for each appliance and (21) ensures that the start times are within their corresponding limits. Helper variables P_t^{dev} and $I_{k,i}^{B,dev}$ are used in order to linearize the absolute terms in (4) and (7). The MILP formulation of the basic problem contains $|A| \cdot T$ binary variables.

B. PROBLEM EXTENSION 1

Appliances like dishwashers or washing machines, often provide the option to select between different operation modes, like eco, normal and express mode, resulting in different load profiles. Thus, in order to make the problem more realistic, we extend it by appliances of type C, whose start times can be shifted and whose operation modes can be selected from different alternatives. Let C denote the set of type-C appliances. We assume that each appliance $k \in C$ provides $M_k^{C,max}$ operation modes with corresponding load profiles $P_{k,1}^C, \ldots, P_{k,M_k^{C,max}}^C$ of different lengths $N_{k,1}^C, \ldots, N_{k,M_k^{C,max}}^C$. Thus, $P_{k,m}^C = (P_{k,m,1}^C, \ldots, P_{k,m,N_{k,m}^C}^C)$ for each mode $m = 1, \ldots, M_k^{C,max}$. The aggregator can select the operation mode M_k^C for each appliance $k \in C$ and if it differs from the operation mode $M_k^{C,pref}$, which is preferred by the owner of the appliance, a remuneration of d_k^C Euro has to be paid. Similar to type-A appliances, the aggregator can also set the start time T_k^C of appliance $k \in C$ within limits $T_k^{C,min}$ and $T_k^{C,max}$ and has to remunerate the owner by c_k^C Euro if it differs from the preferred start time $T_k^{C,pref}$.

In order to consider the described extension in the problem formulation, we extend the objective function (15) to:

$$\min_{F_{k,t}^A, I_{k,i}^B, F_{k,m,t}^C} \quad Pen + Rem^A + Rem^B + Rem^C, \tag{31}$$

change constraint (17) to:

$$P_t = A_t + B_t + C_t \quad \forall t, \tag{32}$$

and add the following constraints:

$$F_{k,m,t}^C \in \{0, 1\} \quad \forall k \in C, \forall t, m = 1, \ldots, M_k^{C,max}, \tag{33}$$
$$F_{k,m,t}^C = 0$$
$$\forall k \in C, m = 1, \ldots, M_k^{C,max}, \forall t \notin \{T_k^{min}, \ldots, T_k^{max}\}, \tag{34}$$
$$\sum_{m=1}^{M_k^{C,max}} \sum_{t=1}^{T} F_{k,m,t}^C = 1 \quad \forall k \in C \tag{35}$$
$$S_k^C = 1 - \sum_{m=1}^{M_k^{C,max}} F_{k,m,T_k^{C,pref}}^C \quad \forall k \in C, \tag{36}$$
$$O_k^C = 1 - \sum_{t=1}^{T} F_{k,M_k^{C,pref},t}^C \quad \forall k \in C, \tag{37}$$
$$Rem^C = c_k^C \cdot S_k^C + d_k^C \cdot O_k^C, \tag{38}$$
$$C_t = \sum_{k \in C} \sum_{m=1}^{M_k^{C,max}} \sum_{l=\max\{1,t-N_{k,m}^C+1\}}^{t} F_{k,m,l}^C \cdot P_{k,m,t-l+1}^C \quad \forall t. \tag{39}$$

Additional flags $F_{k,m,t}^C$ are introduced. If a flag $F_{k,m,t}^C$ equals one, the appliance $k \in C$ starts operation in time step t and in mode m. The variable O_k^C indicates whether the operation mode of appliance $k \in C$ is set differently from the preferred mode $M_k^{C,pref}$. Each appliance $k \in C$ adds $T \cdot M_k^{C,max}$ binary variables to the MILP problem.

C. PROBLEM EXTENSION 2

As a further extension, we consider appliances, like, for example, ventilation systems, which can be shifted and regulated at the same time. Let D denote the set of such appliances of type D and let $P_k^D = (P_{k,1}^D, \ldots, P_{k,N_k^D}^D)$ be the load profile of $k \in D$. Each appliance $k \in D$ allows to set the start time T_k^D within limits $T_k^{D,min}$ and $T^{D,max}$ as well as to set the intensity $I_{k,i}^D$ for each step i of the load profile within limits $I_k^{D,min}$ and $I_k^{D,max}$. Remunerations of c_k^D Euro per kWh and d_k^D Euro have to be paid, if the intensities differ from the preferred intensities $I_{k,i}^{D,pref}$ and the start time differs from the preferred start time $T_k^{D,pref}$, respectively.

The type-D appliances are considered in the problem description by further extending the objective function (31) to

$$\min_{F^A_{k,t}, I^B_{k,i}, F^C_{k,m,t}, F^D_{k,t}, I^D_{k,i}} Pen + Rem^A + Rem^B + Rem^C + Rem^D, \tag{40}$$

extending constraint (32) to

$$P_t = A_t + B_t + C_t + D_t \forall t, \tag{41}$$

and adding the following constraints to the problem:

$$F^D_{k,t} \in \{0, 1\} \quad \forall k \in D, \forall t, \tag{42}$$

$$F^D_{k,t} = 0 \quad \forall k \in D, \forall t \notin \{T^{D,min}_k, \ldots, T^{D,max}_k\}, \tag{43}$$

$$\sum_{t=1}^{T} F^D_{k,t} = 1 \quad \forall k \in D, \tag{44}$$

$$S^D_k = 1 - F^D_{k,T^{D,pref}_k} \quad \forall k \in D, \tag{45}$$

$$I^{D,min}_k \leq I^D_{k,i} \leq I^{D,max}_k \quad \forall k \in D, \ i = 1, \ldots, N^D_k, \tag{46}$$

$$I^{D,dev}_{k,i} \geq I^D_{k,i} - I^{D,pref}_{k,i} \quad \forall k \in D, \ i = 1, \ldots, N^D_k, \tag{47}$$

$$I^{D,dev}_{k,i} \geq I^{D,pref}_{k,i} - I^D_{k,i} \quad \forall k \in D, \ i = 1, \ldots, N^D_k, \tag{48}$$

$$Rem^D = \sum_{k \in D} c^D_k \cdot I^{D,dev}_{k,i} P^D_{k,i} \cdot \Delta t + d^D_k \cdot S^D_k, \tag{49}$$

$$D_t = \sum_{k \in D} \sum_{l=\max\{1, t-N^D_k+1\}}^{t} F^D_{k,l} \cdot I^D_{k,t-l+1} \cdot P^D_{k,t-l+1} \quad \forall t. \tag{50}$$

The constraint (50) contains a product of two decision variables. Thus, the problem becomes quadratic. It is possible to linearize the problem, but this requires a lot of additional binary variables, and initial experiments have shown that with the used solver (Gurobi), the linearization has a negative impact on the performance. Hence, we solve the MIQP directly without linearization.

IV. EXPERIMENTS

A. USE CASE

We generated problem instances for the experiments in a similar way as described in [8] in order to make the experimental results comparable to those reported in [8]: We consider a time horizon of $T = 96$ time steps with a length of $\Delta t = 0.25$ h (15 min), each. As it can be seen from Table 1, the choice of 15 min intervals is in line with related work, in which typically intervals of 15 min or 1 h are assumed. Furthermore, 15 min intervals appear reasonable since bids on the secondary reserve market typically apply for one or more blocks of 15 min [33]. The load profiles of the appliances are generated based on different base profiles, which are shown in Figure 2. These profiles are oriented on load curves reported by Stamminger *et al.* [34]. Base profile $A1\_1$ is assigned to the first third of type-A appliances, base profile $A2\_1$ is assigned to the second third of type-A appliances and $A3\_1$ is assigned to the last third of type-A appliances.

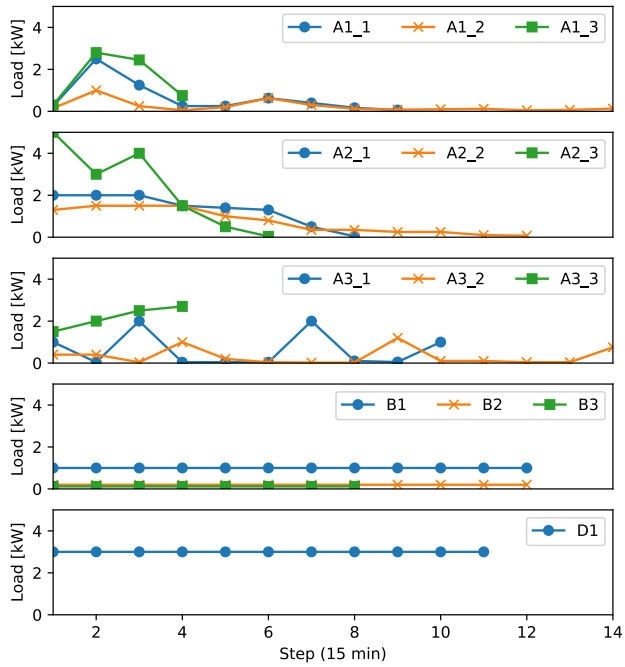

FIGURE 2. Base profiles used in the generation of load profiles of appliances.

Analogously, base profiles $B1$, $B2$, and $B3$ are assigned to type-B appliances and operation modes ($A1\_1$, $A1\_2$, $A1\_3$), ($A2\_1$, $A2\_2$, $A2\_3$), and ($A3\_1$, $A3\_2$, $A3\_3$) are assigned to type-C appliances. The base profile $D1$ is assigned to all type-D appliances. After assigning a base profile to each appliance, the values in the profiles are varied by adding a random value between -5% and +5%. For type-A and type-C appliances, one random values is chosen for each individual step of the profile and for type-B and type-C appliances, one random values is chosen for the complete profile.

The (preferred) start times of the appliances are randomly chosen as follows: With a probability of 10%, a time step between 1 and 40 is chosen, with a probability of 30%, a time step between 41 and 56 is chosen, with a probability of 20%, a time step between 57 and 76 is chosen, and with a probability of 50%, a time step between 77 and 96 is chosen. Minimum and maximum start times for type-A, type-C, and type-D appliances are set by substracting/adding a random integer value between 0 and 32 from/to the preferred start times.

The maximum intensities of all type-B and type-D appliances are set to 1 and the minimum intensities are uniformly sampled from the interval [0.6, 1]. The preferred intensities are chosen uniformly distributed between the corresponding minimum and maximum.

The preferred operation modes of the type-C appliances are randomly selected from their (three) available modes.

The load requested by the DSO is computed by first computing the base load resulting from operating all appliances with their preferred settings and then adding a flexibility in the form of percentage values as shown in Figure 3 to the

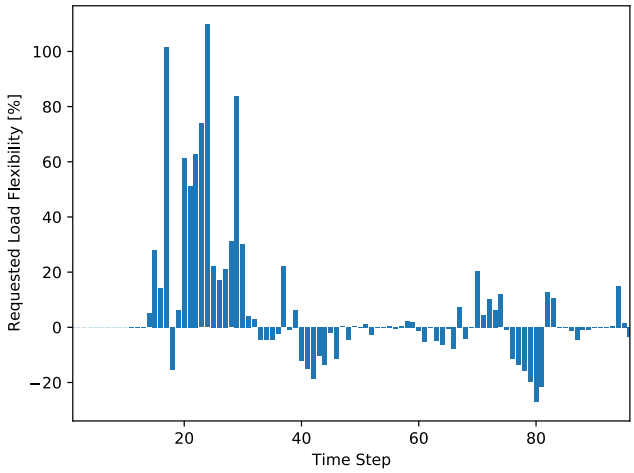

FIGURE 3. Flexibility requested by the DSO in each time step as percentage of the base load.

TABLE 2. Costs assumed in the experiments. Remuneration costs are selected randomly in a range of -/+ 30% around the shown values.

c_{DSO}	c_k^A	c_k^B	c_k^C	d_k^C	c_k^D	d_k^D
0.2	0.1	0.09	0.1	0.05	0.09	0.1

base load. The flexibility in the form of percentage load de-/increases was derived from the absolute values of the base load and the requested load assumed in [8]. An example of the base load and the corresponding requested load is illustrated in Figure 4. The example was generated considering 100 appliances of each type.

The penalty costs c^{DSO} for the deviation from the requested load is set to 0.2 Euro per kWh. The remuneration costs for the different appliances are chosen uniformly random from a range of -/+ 30% around the values shown in Table 2.

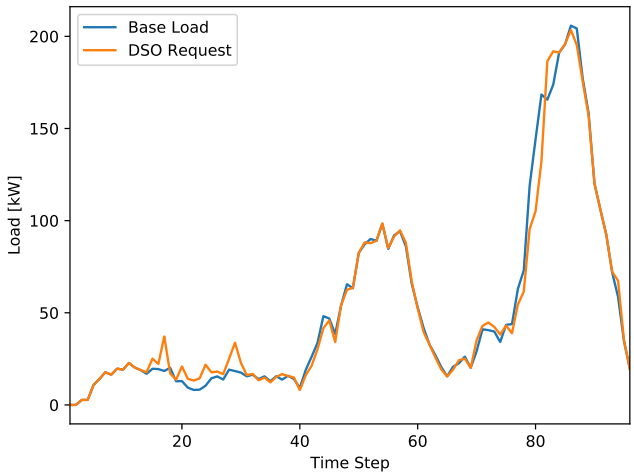

FIGURE 4. An example of the base load and the load requested by the DSO.

B. EXPERIMENTAL SETTINGS

All experiments are run on a 3.8 GHz Intel Core i5-7600K quad-core CPU with 15.6 GB RAM. Version 9.1.0 of the Gurobi solver is used for the optimizations. If not otherwise stated, a time limit of 1 h (3600 s) is set per optimization. All other settings of the solver are left to their default values. The binary variables of each problem instance are initialized according to the preferred start times and operating modes.

TABLE 3. Results on the basic problem considering only appliances of types A and B. For each problem size, the results on five problem instances are summarized.

$\|A\|/\|B\|/\|C\|/\|D\|$	Obj^{pref}	Obj	Gap [%]	Optimal
50/50/0/0	1.76	1.48	0.00	5/5
100/100/0/0	3.61	2.39	1.59	3/5
500/500/0/0	17.53	7.08	7.28	0/5
1000/1000/0/0	35.35	13.35	3.43	0/5
5000/5000/0/0	176.06	63.24	0.94	0/5

C. EXPERIMENTAL RESULTS

In a first experiment, we investigated the scalability of the MILP approach on the basic problem, considering only appliances of types A and B. We varied the total number of appliances between 100 and 10,000 assuming that half of the appliances is of type A and the other half is of type B. For each problem size, five problem instances were solved. The results are summarized in Table 3. The columns "Obj" and "Gap" show the average of the resulting objective value and of the (worst-case) optimality gap, respectively, over the five problem instances per problem size. The column "Optimal" lists, how many of the five instances were solved to proven optimality (within the time limit of 3600 s). Furthermore, in order to allow a better interpretation of the resulting objective values, the column "Obj^{pref}" shows the objective value resulting from keeping all integer variables (start times and operation modes) at their preferred values and optimizing only the continuous variables (intensities). Fixing the integer variables results in a purely continuous linear programming problem, which can be efficiently solved within a few seconds. The shown values of "Obj^{pref}" are again the average over the five problem instances per problem size.

All five problem instances with 100 appliances were solved to proven optimality. Out of the five problem instances with 200 appliances, three could be solved to optimality. No problem instance with 1000 or more appliances was solved to proven optimality within the time limit of one hour. However, there is a significant improvement of the objective value compared to Obj^{pref}. Furthermore, having a look at the optimality gaps and keeping in mind that the reported values are only upper bounds for the actual (unknown) gaps, one can assume that the quality of the solutions is still acceptable for the practical application. Thus, we can conclude that the MILP approach can handle the basic problem with up to several thousand appliances sufficiently well.

In a second experiment, we investigated the scalability of the MILP approach on the first extension of the problem considering only appliances of types B and C. Again, we varied the problem size from 100 to 10,000 appliances and

TABLE 4. Results on the first extension of the problem considering only appliances of types B and C. For each problem size, the results on five problem instances are summarized.

$\|A\|/\|B\|/\|C\|/\|D\|$	Obj^{pref}	Obj	Gap [%]	Optimal
0/50/50/0	2.26	1.51	0.00	5/5
0/100/100/0	4.52	2.30	9.23	1/5
0/500/500/0	22.00	6.59	10.96	0/5
0/1000/1000/0	43.10	11.09	6.86	0/5
0/5000/5000/0	209.63	49.03	2.02	0/5

solved five problem instances per problem size. The results are summarized in Table 4. The increase of the number of binary variables compared to the basic problem results in higher optimality gaps and fewer instances, which are solved to proven optimality. However, the performance degradation is limited and the quality of the solutions should be still tolerable for most practical applications. Thus, we can conclude that also for the first problem extension, the MILP approach scales sufficiently well up to several thousand appliances.

TABLE 5. Results on the second extension of the problem considering only appliances of types B and D. For each problem size, the results on five problem instances are summarized.

$\|A\|/\|B\|/\|C\|/\|D\|$	Obj^{pref}	Obj	Gap [%]	Optimal
0/50/0/50	3.41	2.90	5.47	0/5
0/100/0/100	7.28	5.25	7.44	0/5
0/500/0/500	36.38	26.09	24.55	0/5
0/1000/0/1000	73.10	56.25	35.57	0/5
0/5000/0/5000	362.33	362.33	83.11	0/5

In a third experiment, we investigated the scaling of the MIQP approach on the second problem extension. The results are summarized in Table 5. None of the problem instances could be solved to proven optimality. The optimality gaps for problem sizes of 100 and 200 appliances appear tolerable but for larger problem sizes they are in a range, which is probably no longer tolerable for practical application. For the problem instances with 10,000 appliances, the solver was not even able to improve the binary variables compared to the start solution since it could not perform the root relaxation within the time limit of one hour. Hence, the experiments show that the MIQP approach scales only up to a few hundred appliances of type D. Larger problems are too hard for a conventional mathematical programming approach due to the quadratic constraints.

In a last experiment, we investigated the impact of the problem extensions on the results. More precisely, we investigated the impact of allowing operation mode switches for type-C appliances and of allowing start time shifts for type-D appliances. For this, we solved a problem instance with 100 appliances of each type with four different settings **S1** to **S4**:

- **S1** - The original setting with full flexibility of the appliances.

TABLE 6. Detailed results of optimizations of problem instance with 100 appliances of each type with different settings S1 to S4.

	S1	**S2**	**S3**	**S4**
$Shifts_A$	6	6	11	13
Rem^A	0.51	0.59	0.97	1.21
Reg_B	3.26	3.34	3.66	4.21
Rem^B	0.26	0.26	0.29	0.34
$Shifts_C$	12	15	19	18
$Switches_C$	8	0	11	0
Rem^C	1.42	1.44	2.31	1.79
$Shifts_D$	7	9	0	0
Reg_D	15.98	18.55	16.34	20.12
Rem^D	1.80	2.23	1.25	1.58
Rem^{tot}	4.09	4.52	4.82	4.91
Pen	0.93	0.81	0.95	1.11
Obj	5.02	5.33	5.77	6.02

- **S2** - The operation modes of type-C appliances are fixed to the preferences of the owners.
- **S3** - The start times of type-D appliances are fixed to the preferences of the owners.
- **S4** - The operation modes of type-C appliances and the start times of type-D appliances are fixed to the preferences of the owners.

With each setting, we ran an optimization with a time limit of five hours. The details of the results are shown in Table 6. Shown are the number of shifts of type-A, type-C, and type-D appliances ($Shifts_A$, $Shifts_C$, $Shifts_D$), the amount of up and down regulated energy for type-B and type-D appliances (Reg_B, Reg_D), and the number of mode switches of type-C appliances ($Switches_C$). Furthermore, the total remuneration, the remunerations per appliance type, the penalty, and the complete total costs are shown (Rem^{tot}, Rem^A, Rem^B, Rem^C, Rem^D, Pen, Obj).

One can see that with all settings, type-C appliances are shifted more than appliances of other types. The reason could be that the profiles of type-C appliances are comparatively short in certain operation modes, which allows a better reduction of the load deviation by shifting of the start time. Comparing the results with settings **S1** and **S2**, one can see that the missing option to switch the operation mode of type-C appliances in **S2** is mainly compensated by more shifting and regulation of type-D appliances. The missing option to shift type-D appliances in **S3** is however compensated by more shifting / switching / regulation of all appliance types. The increase of the total costs compared to setting **S1** is notably higher with setting **S3** than with setting **S2**. Fixing the operation modes of type-C appliances and the start times of type-D appliances in setting **S4** results in an increase of the total costs by about 20% compared to the setting **S1** with full flexibility. Summarizing, one can say that both the option to switch operation modes and the option to shift and regulate appliances at the same time, are beneficial for cost reduction, but that the latter has a higher impact.

V. CONCLUSION

We proposed and evaluated mathematical programming for solving the problem of optimally coordinating the operation of home appliances with shift and regulation capabilities for the provision of demand response, as introduced in [8]. We converted the mixed integer nonlinear programming (MINLP) formulation from [8] into a mixed integer linear programming (MILP) formulation. The MILP model contains a high number of binary variables. However, experiments have shown that it can still be efficiently solved. Problem instances with up to 200 appliances could be solved to proven optimality within one hour. This is a notable improvement, compared to the results reported in [8], where meta-heuristics failed to solve a problem instance with 145 appliances to optimality within around 140 minutes. It has been further shown that the proposed approach scales well to even higher numbers of appliances. Although the approach was not able to solve instances with 1000 to 10,000 appliances to proven optimality within an hour, it yielded acceptable optimality gaps in a range of around 1 to 7%. The ability to provide guarantees on the optimality gap is a further advantage of the proposed exact approach compared to metaheuristic approaches evaluated in [8]. We proposed two extensions of the problem formulation, which make the problem more challenging but also more practically relevant. The first extension adds appliances to the problem, which can be shifted in time and whose operation modes can be switched. The ability to consider this further level of flexibility in the problem comes at the price of an even higher number of binary variables in the MILP model. However, the experiments have shown that the negative impact on the performance is limited. The proposed approach still presents a good scalability on the extended problem and is able to yield solutions of acceptable quality for problem instances with up to 10,000 appliances. The second proposed problem extension considers appliances, which can be both shifted and regulated. This second extension induces quadratic terms, which cannot be efficiently linearized. Hence, a mixed integer quadratic programming (MIQP) formulation is provided for the second extension. The experiments have shown that the MIQP problem scales significantly worse than the MILP problems. However, problem instances with up to a few hundred appliances can still be efficiently solved. A deeper investigation of the impact of the problem extensions on the results has shown that both extensions can yield lower costs and that especially the second extension is beneficial. The problem extensions yielded 20% lower costs compared to the basic problem formulation on the considered use case. We can finally conclude that compared to the previous study in [8], we can solve more challenging and more practically relevant problems more efficiently but that the scalability on the second problem extension remains an open issue. The key transformation that allowed to achieve more efficiency was the linearization of nonlinear terms. This proved essential to enable scalability of the original model without loss of accuracy while benefiting from quality solutions of mathematical techniques when compared with heuristics. In practical applications, this would be highly appreciated.

A practically relevant question, which might be investigated as part of future research, is how uncertainties – for example, in the load requested by the DSO – can be efficiently considered in the optimization. A common approach to consider uncertainties in mathematical programming is stochastic programming. However, this would result in a notable increase of the runtime. Another topic of future work could be to investigate the interaction of the aggregator with the regulation market and to develop bidding strategies, which maximize the aggregator's revenue. This typically also requires to take uncertainties into account.

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STEFFEN LIMMER received the M.Sc. (Diploma) degree in computer science from the University of Jena, Germany, in 2009, and the Ph.D. degree in engineering from the University of Erlangen-Nürnberg, Germany, in 2016. From 2009 to 2016, he worked as a Scientific Assistant at the Chair of Computer Architecture, University of Erlangen-Nürnberg. Since December 2016, he has been a Senior Scientist with the Honda Research Institute Europe. His current research interests include optimization and data-driven modeling in the context of energy management systems.

FERNANDO LEZAMA (Member, IEEE) received the Ph.D. degree in ICT from ITESM, Mexico, in 2014. Since 2017, he has been a Researcher with GECAD, Polytechnic of Porto, where he contributes in the application of computational intelligence (CI) in the energy domain. He has also been a part of the National System of Researchers of Mexico, since 2016, the Chair of the IEEE CIS TF 3 on CI in the Energy Domain, and has been involved in the organization of special sessions, workshops, and competitions (at IEEE WCCI, IEEE CEC, and ACM GECCO), to promote the use of CI to solve complex problems in the energy domain.

JOÃO SOARES (Member, IEEE) received the B.Sc. degree in computer science and the master's degree in electrical engineering from the Polytechnic Institute of Porto, in 2008 and 2011, respectively, and the Ph.D. degree in electrical and computer engineering from UTAD University, in 2017. He is currently a Researcher with ISEP/GECAD. His research interests include optimization in power and energy systems, including heuristic, hybrid, and classical optimization. He is also the Vice-Chair of the IEEE CIS TF 3 on CI in the Energy Domain, and has been involved in the organization of special sessions, workshops, and competitions to promote the use of CI to solve complex problems in the energy domain.

ZITA VALE (Senior Member, IEEE) received the Ph.D. degree in electrical and computer engineering from the University of Porto, Porto, Portugal, in 1993. She is currently a Professor with the Polytechnic Institute of Porto, Porto. Her research interests include artificial intelligence applications, smart grids, electricity markets, demand response, electric vehicles, and renewable energy sources.