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SIMILARITY CONTROL IN TOPOLOGY OPTIMISATION UNDER STATIC AND CRASH LOAD CASES

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ABSTRACT
Topology Optimisation (TO) redistributes the material within a design space to optimise certain objective functions under given constraints. The currently available TO methods do not consider the designer's preferences about the final material layout in the optimised design. Contrarily, an improved design similar to a reference design is required because of the economic, manufacturing, or assembly restrictions.

In this article, the proposed heuristic similarity control methods like Energy Scaling Method (ESM), weak passive material method, and an intuitive method of modified design domain are compared with the formal mathematical method of OC-based SIMP with similarity constraint. Initially, the methods are coupled with HCA and OC-based SIMP, for TO of a cantilever beam under static point load. ESM is found to be the most effective and further tested for similarity-based TO with HCA under crash load cases. The results show that ESM can be used to effectively control the TO process.

KEYWORDS
Similarity-based Topology Optimisation (TO); Optimality Criteria (OC)-based SIMP; Hybrid Cellular Automata (HCA); Crashworthiness; Energy Scaling Method (ESM)

1. Introduction

Topology Optimisation (TO) (Bendsøe and Sigmund 2004) is a computational tool used in the initial design phase of the product development to optimise structures under given load cases, boundary conditions, and constraints. Typical objectives involve compliance minimisation for TO under static loading conditions and maximisation of the absorbed energy for TO under crash/dynamic loading conditions.

The popular methods for TO include homogenisation method (Bendsøe and Kikuchi 1988) and density-based methods like Solid Isotropic Material with Penalisation (SIMP) (Bendsøe 1989; Bendsøe and Sigmund 1999). Another class of TO is based on Level Set Methods (LSMs) (van Dijk et al. 2013). To solve the actual TO problem, a very efficient gradient-based method is Optimality Criteria (OC)-based SIMP...
It requires the sensitivity information of the considered objective function and the constraints which is very difficult to obtain in highly non-linear crash scenarios. Since TO under crash load cases is extremely important to improve the crashworthiness of a vehicle, non-gradient TO methods, e.g. Bi-directional Evolutionary Structural Optimization (BESO) approaches (Yang et al. 1999; Huang, Xie, and Lu 2007), Hybrid Cellular Automata (HCA) techniques (Tovar 2004; Duddeck et al. 2016; Zeng and Duddeck 2017), State-Based Representation (SBR) approaches (Aulig and Olhofer 2016; Liu, Detwiler, and Tovar 2018), as well as Evolutionary (EA-LSM) and Kriging-guided (KG-LSM) Level Set Methods (Bujny et al. 2016, 2018; Raponi et al. 2019), have been developed. In particular, very efficient HCA approach, using high-fidelity crash simulations, is frequently used in the automotive industry. HCA is based on the assumption that uniform distribution of internal energy in the design leads to the optimal topology. The research is also being conducted to use Isogeometric Analysis (IGA) instead of Finite Element Method (FEM) in TO (Seo, Kim, and Youn 2010; Gao et al. 2019a, 2020a). Recently, Gao et al. (2019b, 2020b) has demonstrated Isogeometric Topology Optimisation (ITO) for the design of auxetic materials.

The currently available TO methods work very well in the industry to get an optimised structure without addressing the designer’s preferences about its layout. The final material layout is highly dependent on various factors like boundary conditions and the constraints specified at the start of the optimisation process. If any of these factors is changed, TO delivers a completely new design. On the other hand, an improved design with a certain degree of similarity to an already present reference design is usually required because of the economic reasons, limitations of the manufacturing and assembly process, or good attributes/aesthetics of the reference design. One practical implementation problem for TO of selected structural components which are part of a larger system, is how to re-integrate the new topology of the derived components back into the existing system. By utilizing the similarity control approach proposed in this paper, the designer can have confidence that the derived component topology will be based on their preferences for easier integration into the original system. It is also common in the automotive industry to share the platform part with slight modifications in different models of a vehicle. Sometimes, an optimised design under a certain high number of load cases is already known and re-optimisation is required to incorporate an additional load case. In this case, the re-optimised design would be similar to the already available design concept with some minor modifications. To save the computational cost, the designer should be able to re-run TO under only the new load case by controlling the similarity w.r.t. the previously optimised design. In other cases, the novelty or dissimilarity of the new design w.r.t. an already present design might be important to make the product stand out in the market. Therefore, a method or a framework is required which can intuitively control the current TO process and result in a structural layout consistent with the preferences of the designer.

Similarity control in TO is discussed very little in the literature. Recently, Oh et al. (Oh et al. 2019) has coupled generative design exploration (Krish 2011) with deep neural network generative models and TO to get designs with aesthetically appealing layout using previously present reference designs. In this framework, TO and deep generative models are used to generate a lot of designs in the iterative design exploration phase. Then, in the design evaluation phase, the designs are evaluated based on their novelty and other user-specified criteria. The framework was implemented in OC-based SIMP. Although the approach is very promising, the unavailability of gradient information and the computationally expensive nature of iterative design exploration
makes this approach unsuitable for similarity-based TO under highly non-linear crash loading conditions.

2. Methodology for Similarity Control

In this research, the target was to develop general similarity control methods that could work with the mostly used TO methods in the industry for different types of problems (both statics and crash). The ease of integration into commercial software was also one of the desired characteristics. In this article, the following four methods for similarity control w.r.t. a reference structure are proposed (Yousaf 2020):

1. OC-based SIMP with similarity constraint – gradient-based approach using analytical sensitivities of the dissimilarity metric, applicable to problems where gradients of the objectives and the constraints are available.

2. Energy scaling method (ESM) – heuristic similarity control technique applicable to both the gradient-based methods (e.g. OC-based SIMP) and heuristic approaches (e.g. HCA) which can address problems where analytical sensitivities are not available, e.g. crashworthiness.

3. Weak passive material method – based on the concept of guiding the optimisation towards local optima via definition of passive material regions. Since it does not involve any major modifications of the underlying TO method, it can be easily integrated into commercial software and work with different types of methods and problems, including structural crashworthiness.

4. Modified design domain method – the simplest approach, based on the idea of redefining the design space for TO to enforce the highest possible similarity.

The description of these methods is given in the following subsections.

2.1. OC-based SIMP with Similarity Constraint

In the 88-line Matlab code (Andreassen et al. 2011), volume fraction constraint is already implemented in the OC-based SIMP for compliance minimisation of 2D structures. In the current work, this code is modified to further incorporate extra similarity constraint w.r.t. a given reference design. After each iteration of TO, the obtained design is compared with the reference design. The mean squared (pixel/voxel) difference between the corresponding elements of the reference and the obtained design is used as the dissimilarity metric. We define dissimilarity metric \( s \) as follows:

\[
s = \frac{\sum_{e=1}^{N} (x_e - x_{ref}^e)^2}{N}.
\]

In Equation (1), \( N \) is the total number of elements, \( x_e \) is the relative density of element \( e \), and \( x_{ref}^e \) is the relative density of the corresponding element \( e \) in the given reference design. Moreover, all the elements are considered to be of the same size.

During similarity-based TO, the dissimilarity metric compares the element-by-element relative density values of the reference design with the design obtained after the current iteration of the TO. It shows the mean (squared) deviation in the material layout at different locations in the obtained design as compared to the reference
To achieve desired dissimilarity metric value of $\epsilon$ w.r.t. the reference design, the compliance minimisation problem with an additional similarity constraint is defined as below:

$$\min_x c(x) = U^T K U = \sum_{e=1}^{N} (x_e)^T P u^T_k u_e, \quad \text{subject to}$$

$$\frac{V(x)}{V_0} = f; \quad s(x, x^{ref}) = \epsilon; \quad K U = F; \quad 0 < x_{\text{min}} \leq x \leq 1.$$  \quad (2)

Here, $c$ is the overall compliance of the structure. $U$, $K$, and $F$ are global displacement vector, stiffness matrix, and force vector, respectively. Similarly, $u_e$ is the elemental displacement vector, while $k_e$ is the elemental stiffness matrix. $V$, $V_0$ are the volume occupied by material and total design domain, respectively, and $f$ is the prescribed volume fraction value. $x = [x_1, x_2, ..., x_N]^T$ is the vector containing relative density of each element w.r.t. the base material and $P$ is the penalisation factor. Similarly, $x^{ref} = [x^{ref}_1, x^{ref}_2, ..., x^{ref}_N]^T$ is the vector of relative densities in the reference design. A non-zero $x_{\text{min}}$ is specified to avoid singularity. In the article by Zuo and Saitou (2017), an additional cost constraint along with the conventional volume fraction constraint has been implemented in OC. The same approach is followed here to handle the extra similarity constraint. The Lagrange function with active constraints of the optimisation problem (2) takes the form:

$$L = c + \lambda_f (V - fV_0) + \lambda_s (s - \epsilon) + \lambda_u (K U - F), \quad \text{(3)}$$

where $\lambda_f$ and $\lambda_s$ are Lagrange multipliers for the constraints of volume fraction and similarity, respectively. Similarly, $\lambda_u$ is a vector of Lagrange multipliers for static equilibrium. The optimality condition w.r.t. the design variables $x$ (i.e. $\partial L/\partial x_e = 0$) considering only the active constraints, becomes (Zuo and Saitou 2017):

$$- \frac{\partial c}{\partial x_e} \frac{\partial V}{\partial x_e} + \lambda_f = 1 = B_e.$$  \quad (4)

The sensitivities of compliance and dissimilarity metric are given by:

$$\frac{\partial c}{\partial x_e} = -P(x_e)^{P-1} u^T_k u_e.$$  \quad (5)

$$\frac{\partial s}{\partial x_e} = \frac{2}{N} \left( x_e - x^{ref}_e \right). \quad (6)$$
Finally, based on (Zuo and Saitou 2017), the update rule for the relative densities takes the form:

\[
x_{e}^{\text{new}} = \begin{cases} 
OC(B_e, x_e) & \text{if } B_e \geq 0, \\
\min(1, x_{e \text{old}} + m) & \text{if } B_e < 0,
\end{cases}
\]

(7)

where \( m \) is the move limit for update and \( OC(B_e, x_e) \) is defined by Sigmund (2001) as below:

\[
OC(B_e, x_e) = \begin{cases} 
\max(x_{\text{min}}, x_e - m) & \text{if } x_e B_e^\eta \leq \max(x_{\text{min}}, x_e - m), \\
x_e B_e^\eta & \text{if } \max(x_{\text{min}}, x_e - m) < x_e B_e^\eta < \min(1, x_e + m), \\
\min(1, x_e + m) & \text{if } \min(1, x_e + m) \leq x_e B_e^\eta,
\end{cases}
\]

(8)

where \( \eta \) (usually = 1/2) represents a numerical damping coefficient.

In a nutshell, OC-based SIMP with similarity constraint is a gradient-based method based on rigorously derived sensitivities of the dissimilarity metric, which can be used for compliance minimisation problems. Along with 2D problems, the algorithm described above can also be used for 3D problems without any further modification. For the problems where the sensitivity information is not available, e.g. crash, alternative, non-gradient similarity control techniques have to be developed. This is discussed in the following subsection.

2.2. Energy Scaling Method (ESM)

As discussed before, non-gradient heuristic methods like HCA are used for TO under crash load cases because of the unavailability of the gradient information. Because of their non-gradient nature, a similarity constraint cannot be incorporated directly. By analysing Equation (4), a non-gradient heuristic method based on energy scaling can be developed. In Equation (4), the denominator term \( \lambda_s (\partial s/\partial x_e) \) is responsible for controlling the material distribution in different parts of the design domain to achieve a given similarity level. This term, in fact, scales up or down the compliance sensitivity term \(-\partial c/\partial x_e\) to get the desired material distribution. Equation (5) shows that the elemental compliance sensitivities are proportional to the corresponding elemental strain energy values \( \frac{1}{2} u_e^T k_e u_e \).

The proposed heuristic Energy Scaling Method (ESM) is based on the observation that if the elemental energies of a part of design domain are scaled up as compared to its surrounding, the optimisation algorithm (whether OC or HCA) will start depositing more material to that area by removing from the surrounding locations. To define different energy scaling values, the design domain in ESM is divided into \textit{preferred} and \textit{non-preferred} subdomains (Fig. 1) depending on the material distribution in the given reference design. Hence, we propose the following definitions:

\textbf{Preferred subdomain:} The part of the design domain which should have maximum amount of material to achieve maximum similarity.

\textbf{Non-preferred subdomain:} The part of the design domain which should have minimum amount of material to achieve maximum similarity.

ESM is implemented in OC-based SIMP and HCA by applying the scaling factors to
the compliance sensitivities and field variable strain energy density (SED), respectively (Table 1). The detailed procedure is also explained in the flowchart of Fig. 2.

For very high values of the scaling factor, e.g. \( p = 0.98 \), most of the material goes to the preferred region and a design which is almost the same as the reference design, is obtained. On the other hand, for very low values of the scaling factor, e.g. \( p = 0.02 \), a design completely dissimilar to the reference design is obtained. Furthermore, for \( p = 0.5 \), both the preferred and non-preferred regions get the same scaling factors and it is equivalent to running the TO without any similarity control (conventional TO). In short, ESM allows the user to enforce the similarity constraint indirectly, by choosing a suitable value of the scaling parameter \( p \). In practice, this requires carrying out multiple optimisations for different values of the parameter \( p \) to achieve the desired similarity level.

In this work, the proposed ESM has been implemented in the 88-line Matlab code for OC-based SIMP for TO of 2D linear elastic static test cases. To use ESM within HCA, a Python implementation of Honda Research Institute Europe (HRI-EU), based on the work of Patel et al. (2009), was modified accordingly. In this code, LS-Dyna is used as the FEM solver and this is achieved with the help of a Python-based interface for LS-Dyna (Aulig and Lepenies 2012).

ESM as discussed in this article is applicable to both gradient-based TO methods like OC-based SIMP and non-gradient methods like HCA. Moreover, the algorithm can be used for similarity-based TO problems subjected to multiple load cases as well as different types of loading conditions (static, dynamic or both) in both 2D or 3D.

### 2.3. Weak Passive Material Method

In conventional TO, passive material regions\(^1\) can be defined by setting the relative densities of the elements to a fixed value of 1. This concept is utilised to develop a method based on weak passive material to control the similarity. Here, weak passive material elements are those for which the minimum relative density value limit is specified by the user. During TO, the optimiser can arbitrarily change the relative densities of these elements within the range \((x_{\text{wp}, \text{min}}, 1)\), where \(x_{\text{wp}, \text{min}}\) is the minimum relative density value specified by the user. After certain number of initial TO iterations, or when the maximum change in the updated relative densities is lower than a certain limit, the user-defined threshold is removed and the conventional minimum limit for the relative densities (\(x_{\text{min}}\) as used in Equation (2)) is used in the whole design domain. In this way, the structure is pushed to a local optimum which has a certain degree of similarity w.r.t. the reference.

This method is implemented in the 88-line Matlab code for similarity control in compliance minimisation TO of 2D linear elastic static cases. A heuristic approach is followed where the preferred region (Fig. 1) has a higher minimum relative density limit \(d\) and a lower minimum value (e.g. \(1 - d\)) is specified for the non-preferred region. The algorithm is summarised in a form of a flowchart in Fig. 3. Just like ESM, this method also implements the similarity constraint indirectly by allowing the user to specify the minimum relative density limit value \(d\).

The major merit of this method is its generality and simplicity. This characteristic makes it very favourable for implementation in commercial tools without modifying the basic algorithm of TO. The heuristic nature of the method and the fact that it

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\(^1\)Regions in the design space with full material density, which are not modified by the optimisation algorithm.
does not require any sensitivity information, makes it applicable to TO with both static and dynamic loads. Moreover, the method can be used for similarity-based TO for both 2D and 3D problems.

2.4. Modified Design Domain Method

Modified design domain method is an intuitive, and perhaps the simplest way to get a design similar to a given reference design. Here, the non-preferred region is specified as a non-design region and TO is performed only in the preferred region. It is important to realise that this method can give only one design, which will have the maximum possible similarity w.r.t. the reference design (lowest dissimilarity metric value), and it cannot control the similarity in an effective way. The algorithm is explained in the flowchart of Fig. 4. Here, the non-design region is specified by restricting the maximum relative density in the non-preferred region to a value \( q \), which is very close to zero.

Modified design domain method can only work if the target volume fraction is not greater than the volume fraction of the reference structure. Despite these drawbacks, the generality and simplicity of the method make it very easy for implementation in a commercial TO tool. Since the method just modifies the original design domain according to the reference structure, it can be used for 2D and 3D similarity-based TO subject to both static and dynamic loads.

Using the similarity-based TO methods proposed in this research, the designer has more control over the final obtained design by adjusting its degree of similarity or novelty (dissimilarity) w.r.t. an already present design. It is worth mentioning that the final design is neither fixed nor known before starting the similarity-based TO. The designer only has an idea about the affordable material layout deviation in the desired new design w.r.t. an already present reference design. This material layout deviation has been integrated as an additional similarity constraint. The goal of the research was to develop methods to find the material layout which optimises the objective function subject to the additional similarity constraint.

3. Test Cases and Results

The test cases discussed below are used to assess the capabilities of the methods proposed for similarity control w.r.t. the reference design. The first, 2D linear elastic static compliance minimisation, test case is considered for all the proposed methods to investigate their properties and select the best approach. Moreover, a test case with crash loading conditions is considered to further investigate the applicability of the selected method for highly non-linear problems. The corresponding results and important observations are also mentioned after each test case.

3.1. 2D Square Plate under Static Loading

A 2D square plate (100 \( \times \) 100 square finite elements of unit dimensions) fixed at the left end, with downward load on the midpoint of the right edge (Fig. 5(a)), is considered for compliance minimisation problem with a volume fraction constraint under linear elastic static scenario. The default TO results using conventional OC-based SIMP and HCA without any similarity control, for the simulation parameters given in Table 2, are shown in Fig. 5(b) and Fig. 5(c), respectively. The considered reference structure
having the same size as the test case, is shown in Fig. 1(b).

3.1.1. Experimental Setup

The procedure for evaluation of methods for similarity-based TO, based on the test case under consideration, is discussed below:

- 14 complete TO runs are performed using OC-based SIMP with similarity constraint (Section 2.1). In these optimisations, designs having dissimilarity metric values in the range between 0 and 1 w.r.t. the considered reference structure are obtained.
- To test ESM in OC-based SIMP and also in HCA, 25 sampling points for the scaling parameter $p$ are generated uniformly in the range from 0.02 to 0.98. For each value of the scaling parameter $p$, a complete TO run is performed for the considered reference structure.
- In order to investigate the weak passive material method in OC-based SIMP, 25 complete TO runs are performed by varying uniformly the weak passive relative density value $d$ for the preferred region in the range from 0.02 to 0.80. The corresponding value of relative density for the non-preferred region is found by $(0.82 - d)$. Furthermore, the weak passive material in both the preferred and non-preferred subdomains is removed when the maximum change in the design variables is less than or equal to 0.04, which is four times higher than the threshold used for termination of the optimisation process.
- To test the modified design domain method in OC-based SIMP, one complete TO run for the considered reference structure is performed by specifying the non-preferred region as the non-design region.

3.1.2. Results and Discussion

The comparison of ESM and OC-based SIMP with similarity constraint for similarity control w.r.t. the reference structure (Fig. 1) is shown in Fig. 6.

Both of the methods are able to get the designs similar (left ends of the curves), dissimilar (right ends of the curves), and also the designs of intermediate similarity level w.r.t. the reference. The increased similarity comes at the cost of worse structural performance (increased compliance). The designs obtained with both methods for the same dissimilarity metric values are comparable in most cases, both when it comes to the structural performance and the types of topologies. For the dissimilar designs, i.e. for dissimilarity metric values $s \geq 0.7$, it can be observed that the heuristic ESM outperforms the formal mathematical approach based on rigorously derived sensitivities of dissimilarity metric to impose a similarity constraint in OC. In Fig. 6, the objective values for the designs with $s \geq 0.7$ obtained with ESM are better than the corresponding objective values of the designs obtained using OC-based SIMP with similarity constraint. This might be because of the fact that it was difficult to obtain the 0-1 designs of higher dissimilarity metric values for the method of OC-based SIMP with similarity constraint even for higher penalisation values. The problem of intermediate densities did not occur for ESM even for the standard penalisation value of 3.

One possible reason for this is that the similarity constraint method tries to achieve the specified dissimilarity metric value by scaling the energies of each element independently (Equation (4)). These independent energy scaling values come from the gradient information. The element-by-element energy scaling can hinder the convergence to 0-1 design. On the other hand, in the proposed heuristic ESM, the whole preferred region
is assigned a single energy scaling value $p$ and the whole non-preferred region is assigned a single energy scaling value $1 - p$, which gradually drives the structure to a 0-1 design of certain similarity level.

Fig. 7 compares the energy scaling, weak passive material, and modified design domain methods to control the similarity of the 2D square plate w.r.t. the reference structure. It can be seen that the modified design domain method results in the design of maximum similarity, but it cannot generate the designs of varying similarity w.r.t. the reference structure. Weak passive material method gives some flexibility, but still, it does not cover the full range of similarity. On the other hand, ESM gives the full range of designs, starting from the design of maximum similarity (left end of the curve) to the completely dissimilar design (right end of the curve). Each design obtained with the energy scaling or weak passive material method corresponds to a certain value of scaling parameter $p$ or weak passive relative density $d$ of the preferred region, respectively. The structures obtained for a given level of similarity with all the similarity control methods were qualitatively the same.

The results of ESM implemented in HCA for similarity control w.r.t. the reference structure are shown in Fig. 8. Visual inspection shows that these results represent topologically similar concepts as the ones obtained with ESM in OC-based SIMP.

Each point on the curves in Figures 6, 7, and 8 represents one separate design obtained after performing similarity-based TO. Each of these designs has a certain dissimilarity metric value w.r.t. the reference. The design which the designer should select depends on the similarity constraint value which they want to satisfy.

The testing of the proposed similarity control methods for TO of 2D square plate shows that ESM is the most promising approach, giving the most control over the similarity of the optimised structure to the reference design. Therefore, ESM is further tested for more complex TO problems, i.e. TO under highly non-linear load cases. However, the other methods like weak passive material and modified design domain method are still useful especially in the case if the designer can afford very little or no changes in the main TO algorithm.

### 3.2. 2D Beam with Fixed Edges under Impact Loading

To test ESM for TO with dynamic loading conditions, a 800 mm×200 mm 2D beam fixed on vertical edges and impacted by a pole of radius 70 mm on the top with an initial velocity of 60 m/s is considered. The pole impacts on the top edge with an offset from the midpoint to the left (Fig. 9(a)). Fig. 9(b) shows the LS-Dyna FEM mesh where the DOFs in the z-direction are assigned a value of zero to make the problem 2D. The TO for this test case is considered by using HCA. Fig. 9(c) shows the default HCA result without any similarity control. The reference structure considered here for similarity control is shown in Fig. 9(d). This reference structure was obtained by TO for a separate test case with the pole impacting the beam exactly at the midpoint of the top edge. For simulations, piecewise linear plasticity material model (*MAT\_PIECEWISE\_LINEAR\_PLASTICITY$^2$) is considered for the beam, while the pole is modelled as rigid (*MAT\_RIGID$^2$). The parameters used for HCA are the same as suggested by Patel et al. (2009). Other simulation parameters are given in Table 3.

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$^2$LS-Dyna card.
3.2.1. Experimental Procedure

Total 14 samples for different energy scaling values \( p \) of the preferred region are selected uniformly in the range of 0.05 to 0.95. For each sample, a complete TO run is performed. Each scaling value \( p \) results in a structure of certain similarity w.r.t. the reference structure and performance, measured by the maximum intrusion of the pole into the beam. The results are discussed in the following subsection.

3.2.2. Results and Discussion

Fig. 10 shows the performance indicator (maximum intrusion of the pole) of the structures with different similarity level w.r.t. the reference structure. As for the 2D linear elastic static case, ESM also works for the crash loading test case and is able to generate a wide range of designs with varying similarity w.r.t. the reference. Left end of the curve shown in Fig. 10 corresponds to the designs of the highest similarity w.r.t. the reference. The dissimilar designs are shown on the right end of the curve. Moreover, the designs in-between the similar and dissimilar are also obtained with ESM and are depicted in the middle of the plot. As we move away from the default design to the designs of lower dissimilarity metric values, the similarity w.r.t. the reference increases on the cost of performance. It is up to the designer to decide about the proper compromise between the performance and the similarity level.

It is worth mentioning that ESM does not increase computational cost and can be easily used for similarity-based TO of large-scale crash cases. Moreover, no adaptation is needed to apply ESM to 3D industrial problems with multiple static and dynamic loads.

4. Conclusion

By implementing the similarity constraint in OC-based SIMP after rigorously deriving the sensitivities of the dissimilarity metric and carefully observing this formal mathematical method, a heuristic Energy Scaling Method (ESM) is developed. The comparison of the similarity control results of ESM with the method of OC-based SIMP with similarity constraint for linear elastic static TO demonstrates the superiority of ESM. The main problem with OC-based SIMP with extra similarity constraint is that it does not converge to 0-1 design for certain dissimilarity metric values w.r.t. the reference. The problem persists even for the higher values of power law penalisation. On the other hand, ESM always delivers a 0-1 design even for the default penalisation value of 3.

The comparison of ESM with other investigated similarity control methods such as weak passive material method and modified design domain method for linear elastic static TO with OC-based SIMP shows that ESM is capable of delivering a wide variety of designs depending on the applied energy scaling value \( p \). The weak passive material method can generate a small subset of the designs possible to obtain with ESM, while the modified design domain method can give just one design which will be nearly the same as the reference design. Although ESM is found to be the best, the weak passive material method and modified design domain method are very simple and easy to be implemented in any commercial TO tool. Moreover, they give results which are qualitatively comparable to ESM in their range of applicability.

ESM does not require any sensitivity information to control the similarity and therefore, can be used in non-gradient TO approaches such as HCA. In this article,
we use a 2D beam fixed on the vertical edges and impacted on top with a pole to
demonstrate the capabilities of ESM implemented in HCA for similarity-based TO
under a crash scenario. Again, the obtained results show that ESM is able to generate
a wide range of designs of varying similarity w.r.t. the considered reference.
The simplicity of ESM makes it favourable to be used for controlled TO in the
industry. The fact that ESM allows designers to intuitively express their structural
layout preferences can greatly influence the way TO is used for large scale practical
problems.
The algorithmic formulation of the methods proposed in this research allows them
to be used for similarity-driven TO for both 2D and 3D problems. Since mathemati-
cally derived method of OC-based SIMP with similarity constraint requires sensitivity
information of the objective function and the constraints, it can only be used for TO
under static loads. On the other hand, the heuristic methods like weak passive material
method, modified design domain method, and ESM can be used for similarity-driven
TO under both static and dynamic (crash) loads. In short, the similarity-based mindset
gives the designer an ability to intuitively control the TO process in industry.

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Table 1. ESM for similarity control in TO with OC-based SIMP and HCA.

<table>
<thead>
<tr>
<th>Region</th>
<th>OC-based SIMP</th>
<th>HCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferred region</td>
<td>$\frac{\partial c}{\partial x} := p \times \frac{\partial c}{\partial x}$</td>
<td>$SED := p \times SED$</td>
</tr>
<tr>
<td>Non-preferred region</td>
<td>$\frac{\partial c}{\partial x} := (1 - p) \times \frac{\partial c}{\partial x}$</td>
<td>$SED := (1 - p) \times SED$</td>
</tr>
</tbody>
</table>

Table 2. Simulation and optimisation parameters for OC-based SIMP and HCA for 2D square plate test case.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>OC-based SIMP</th>
<th>HCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force magnitude</td>
<td>1 N</td>
<td>0.01 N</td>
</tr>
<tr>
<td>Number of elements</td>
<td>$100 \times 100$</td>
<td>$100 \times 100$</td>
</tr>
<tr>
<td>Element type</td>
<td>default 4-node shell</td>
<td>4-node shell</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Young’s modulus $E$</td>
<td>1 MPa</td>
<td>1 MPa</td>
</tr>
<tr>
<td>Power law penalisation</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Filter radius</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Considered volume fraction</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3. Simulation and optimisation parameters for 2D beam test case under crash loading.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole mesh</td>
<td>22 elements of 4-node shell</td>
<td></td>
</tr>
<tr>
<td>Pole density</td>
<td>$5.0 \times 10^{-6}$</td>
<td>ton/mm$^3$</td>
</tr>
<tr>
<td>Beam mesh</td>
<td>$160 \times 40 \times 1$ elements of type 8-node solid</td>
<td></td>
</tr>
<tr>
<td>Beam density $\rho$</td>
<td>$2.7 \times 10^{-9}$</td>
<td>ton/mm$^3$</td>
</tr>
<tr>
<td>Beam Young’s modulus $E$</td>
<td>$7.0 \times 10^4$</td>
<td>MPa</td>
</tr>
<tr>
<td>Beam Poisson’s ratio $\nu$</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Beam yield strength $\sigma_y$</td>
<td>241</td>
<td>MPa</td>
</tr>
<tr>
<td>Beam tangent modulus $E_{tan}$</td>
<td>70</td>
<td>MPa</td>
</tr>
<tr>
<td>LS-Dyna termination time</td>
<td>$6.0 \times 10^{-3}$</td>
<td>s</td>
</tr>
<tr>
<td>Considered volume fraction</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Filter radius</td>
<td>5.5</td>
<td>mm</td>
</tr>
</tbody>
</table>

Figure 1. Concept of preferred and non-preferred regions in ESM.
Figure 2. Flowchart for similarity control by ESM in OC-based SIMP and HCA.
Figure 3. Flowchart for similarity control by weak passive material method in OC-based SIMP.
Figure 4. Flowchart for similarity control by modified design domain method in OC-based SIMP.
(a) Design domain and boundary conditions. 
(b) Default TO result with conventional OC-based SIMP of 88-line Matlab code. 
(c) Default TO result with conventional HCA. 

Figure 5. 2D square plate test case.

Figure 6. Results for similarity-based TO by implementing ESM and similarity constraint in OC-based SIMP. The plot shows the final objective function (compliance) values for the structures of different dissimilarity metric values w.r.t. the reference structure.
Figure 7. Results for similarity-based TO of 2D square plate by implementing energy scaling, weak passive material, and modified design domain method in OC-based SIMP. The plot shows the final objective function (compliance) values for the structures of different dissimilarity metric values w.r.t. the reference structure.

Figure 8. Results for similarity-based TO by implementing ESM in HCA. The plot shows the final objective function (compliance) values for the structures of different dissimilarity metric values w.r.t. the reference structure.
(a) Design domain and boundary conditions. (b) LS-Dyna FEM mesh of the test case.

(c) Default HCA result for the test case without any similarity control. (d) Considered reference structure for similarity control.

Figure 9. Test case of 2D beam with fixed edges under impact loading.

Figure 10. Performance vs. dissimilarity metric value of the obtained structures w.r.t. the reference design for the 2D crash test case. The shown deformed structure corresponds to the moment when intrusion reaches its maximum value.