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# Peak Load Reduction through Dynamic Pricing for Electric Vehicle Charging

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## Abstract

Typically, peak demand charges account for a considerable part of the operating costs of public electric vehicle charging stations. An intelligent control of the charging processes can help to reduce the peak load and the corresponding fees. This can be additionally supported by the use of a dynamic pricing scheme, which encourages customers to provide as much flexibility as possible. The present work proposes a framework for the setting of dynamic price offers for different charging deadlines and the scheduling of charging processes with the objectives of maximizing the charging station operator's daily profit and reducing the peak of the electrical load. In a simulation study based on a use case typical for Germany, it is shown that the proposed approach can increase the charging station operator's yearly profit by several thousand euros compared to a pricing and scheduling scheme without consideration of the peak load. Furthermore, an approach for increasing the scalability of the employed optimization of price offers is proposed and evaluated.

**Keywords:** electric vehicle, dynamic pricing, peak shaving, multi-objective optimization, evolutionary algorithm, linear programming

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## 1. Introduction

Nowadays, the majority of public charging stations for electric vehicles (EVs) are uncontrolled [1]. That means, when an EV is plugged in, it is immediately charged with the maximum possible power until its battery is fully charged, or the EV is unplugged. However, there is an increasing interest in controlled charging, which charges EVs according to a certain control plan [2]. This is on the one hand interesting for distribution grid operators, who can increase the grid stability by employing a smart charging scheme [3, 4], on the other hand, it is a way for operators of public charging stations to reduce their operating costs. In the literature, different approaches for the reduction of the operating costs based on an intelligent charging control can be found. Such approaches are, for example, the trading on the electricity market [5, 6], the provisioning of battery capacity to the frequency regulation market [7, 8] and the increase of photovoltaic self-consumption [9, 10].

However, in order to harness the full potential of controlled charging at public charging stations, a certain flexibility of customers is required [11]. If all customers want their EVs charged as fast as possible, the only option would be to charge them in the same way as with uncontrolled charging. A way to deal with this issue is the use of a dynamic pricing scheme, which incentivizes customers to provide flexibility for the charging of their EVs. A promising approach is to offer each customer different prices for charging her/his required amount of energy by different deadlines. The more time a customer provides for the

charging process, the lower the price. Bitar and Low [12] proposed such a pricing scheme in 2012 under the term *deadline differentiated pricing*. They describe an approach for scheduling the charging of EVs, which maximizes the charging stations operator's (CSO's) profit under the proposed pricing scheme. However, they do not provide a mechanism for setting prices. Salah and Flath [13] described an approach for stochastic optimization of price offers for different charging deadlines and of charging schedules. The approach is intended to maximize the CSO's profit by maximizing the self-consumption of local photovoltaic energy generation. In [14], Salah *et al.* evaluated the approach on a use case. Bitar and Xu [15] extended the work from [12] in 2017 and proposed an approach for setting the prices. They showed that under certain conditions, the approach maximizes the CSO's profit and the *social welfare* (the sum of the CSO's and the customers' profits). However, the approach requires customers to select their deadlines before they are informed about the corresponding prices.

The approaches described in [12, 13, 14, 15] are offline approaches, which require the knowledge or a good prediction of the charging demands over the complete planning horizon – e.g. the next day. Ghosh and Aggarwal [16, 17] proposed an online approach based on mixed-integer linear programming and a heuristic for the setting of price offers. They assume not only different price offers for different deadlines, but that customers can select from a menu of energy-deadline-pairs with corresponding prices. The approach is intended to yield a reasonable tradeoff between the CSO's profit and the social welfare. Limmer and Rodemann [18] described another online approach for the multi-objective optimization of price offers for different charging deadlines with the objectives of maximizing the CSO's profit, minimizing the number of customers who decline

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charging due to too high prices and minimizing the number of customers who have to be rejected because all charging stations are occupied. In [19], an analogous approach is employed for the optimization of price offers with the consideration of price fairness.

All the described dynamic pricing approaches consider the maximization of the CSO's profit. However, they neglect the so called *peak demand charges* or *maximum demand charges*, which can account for a high share of the CSO's operating costs. The peak demand charge is a fee per kW of the peak of the electrical load in a certain period (usually a month or a year). According to a recent study [20], peak demand charges can be responsible for over 90 % of electricity costs of public charging stations. Another study [21] states that in the USA peak demand charges often represent 30 % to 70 % of a commercial electric bill. Thus, reducing the peak load arising from charging, offers great potential for reducing the operating costs of public charging stations.

There are different approaches for peak load reduction in the context of EV charging. A natural approach is to control the charging of the EVs in a way that the peak load is minimized. Online and offline approaches for EV charging scheduling for peak load reduction are for example proposed by Chen *et al.* [22] and Zhang *et al.* [23]. Furthermore, different approaches for valley-filling over iterative distributed scheduling are described in the literature [24, 25, 26, 27]. These approaches are offline and they employ price signals to guide EVs in choosing their individual charging profiles. A special case of peak load reduction over an intelligent control is the scheduling of charging processes with the constraint of a capacity limit [28, 29]. This has the drawback that it is not guaranteed that the energy requirements of all customers are satisfied.

A further approach to reduce the peak load, is employing a stationary battery for shifting the load from peak periods to off-peak periods [30, 31, 32]. Vehicle-to-grid (V2G) technology allows to use the batteries of the EVs analogous to a stationary battery for peak load reduction. This is described in several works [33, 34, 35].

There are also approaches for peak load reduction based on dynamic pricing. The existing approaches address the customers' flexibility in energy requirements. The basic idea is to set different energy prices for different scheduling intervals and let the customer decide how much energy she/he wants to charge in each interval. It is assumed that customers charge less energy in periods of high prices. Thus, the total amount of energy a customer charges, depends on the offered prices. For example, Yan *et al.* [36] propose the setting of the energy prices for different intervals based on different zones of the total load. However, they do not state how to do this exactly. Wang *et al.* [37] propose a simple offline pricing policy based on the difference between a day-ahead forecast of the load and a target load. Customers indirectly decide on their charging schedule by specifying a maximum energy price they are willing to pay. The profit of the CSO is not considered in this approach. Soltani *et al.* [38] describe an online approach for setting energy prices for multiple households based on predictions, how much energy each household will charge for a given price. The

approach is not suited for public charging stations where continually new customers arrive. Flath *et al.* [39] propose the setting of dynamic energy prices for multiple locations, which allows the shifting of load from one location to another one. It requires customers to plan their charging for a long period ahead (Flath *et al.* assume one week in their simulations) since they have to switch between different charging locations. Furthermore, the CSO's profit is not considered. A general drawback of peak load reduction over setting of energy price profiles is that customers have to decide on their charging schedule. Some of the described approaches have the additional drawback that customers do not know in advance how much energy they will have charged and/or how much they will have to pay at departure time.

In the present work, we describe a framework for peak load reduction based on deadline differentiated pricing. The basic idea is that in periods of already high load the prices are set in a way that customers provide more time for charging, which allows to shift the charging to periods of lower load. With the proposed approach, customers have to decide only how much energy they want to charge by which deadline and it is guaranteed that they get the requested amount of energy by the requested deadline for the contracted price. This makes it more suitable for practical realization than existing approaches based on price profiles. The proposed approach is an online approach and it sets charging schedules and price offers considering the CSO's daily profit and the peak load. The price offers are optimized through evolutionary optimization taking into account uncertainties in customer preferences. The optimization of charging schedules with consideration of the peak load is formulated as a linear programming problem. However, the proposed price optimization approach can be also combined with other online control strategies for peak load reduction as well as with the employment of stationary batteries or V2G technology.

In a simulation study, the framework is evaluated on a use case intended to reflect typical operating conditions of public charging stations in Germany. The proposed approach employs robust optimization in order to deal with uncertainties in the customers' preferences. Since this can result in a high runtime, an approach for accelerating the robust optimization is proposed and evaluated.

The rest of the paper is organized as follows: Section 2 describes the assumed scenario and problem more in detail. In Section 3, the proposed framework for the setting of the price offers and the charging scheduling is described. Its evaluation over simulation experiments is described in Section 4. Section 5 describes the proposed approach for the speed-up of the robust optimization and its evaluation and finally, Section 6 provides a brief conclusion and outlook.

## 2. Problem Description

We consider the following scenario: Multiple public charging stations with a maximum power level of  $P_{max}$ , each, are operated by a charging station operator. The day is divided into  $T$  scheduling intervals  $i = 1, \dots, T$  of length  $\Delta t$ , and distributed throughout the day,  $N$  EVs arrive at the charging stations and

request charging. It is assumed that the number of charging stations is sufficient to serve all the charging requests.

If the number  $N_i$  of EVs that arrive in interval  $i-1$  is non-zero, then these  $N_i$  EVs  $EV_{i,1}, \dots, EV_{i,N_i}$  are considered for starting the battery charging in interval  $i$ . Each EV  $EV_{i,n}$  of them has a certain energy requirement  $E_{i,n}$  and a minimum deadline  $D_{i,n}$ . The minimum deadline is the earliest interval by which the energy requirement of the EV can be satisfied and it can be computed as

$$D_{i,n} = i + \lfloor \frac{E_{i,n}}{P_{max} \cdot \Delta t} \rfloor. \quad (1)$$

At the end of interval  $i-1$ , the operator submits  $K+1$  prices  $\mathbf{p}_{i,n} = (p_{i,n}^0, \dots, p_{i,n}^K)$  to each  $EV_{i,n}$ ,  $n = 1, \dots, N_i$ , where  $p_{i,n}^k$  is the price for charging  $E_{i,n}$  energy by interval  $D_{i,n} + k$ . Thus, customers can decide to charge their required energy by their minimum deadline or to extend the charging time for up to  $K$  intervals. For the sake of simplicity, we assume  $D_{i,n} + K \leq T$  for all  $i = 1, \dots, T$  and  $n = 1, \dots, N_i$ . That means, all EVs can be charged within the day they arrive. The decisions of the customers are made based on the offered prices and the customers' utilities. The driver of  $EV_{i,n}$  gets a certain utility  $U_{i,n}^k$  for charging the required energy by deadline  $D_{i,n} + k$ ,  $k = 0, \dots, K$ . It can be assumed that  $U_{i,n}^{k_1} \geq U_{i,n}^{k_2}$  holds for  $k_1 < k_2$ , since customers prefer earlier deadlines over later ones. The customer makes a "profit" of  $U_{i,n}^k - p_{i,n}^k$  by selecting deadline extension  $k$ . If there is a  $k$  that results in a positive customer's profit, the customer selects the deadline extension  $K_{i,n}$  that maximizes her/his profit:

$$K_{i,n} = \arg \max_{k \in \{0, \dots, K\}} (U_{i,n}^k - p_{i,n}^k). \quad (2)$$

Otherwise – if no price offer resulting in a positive customer's profit exists – the customer declines charging and  $K_{i,n}$  is set to -1. Note, since customers can decline, the operator cannot make an arbitrarily high profit by setting the price offers arbitrarily high. Let  $\mathbf{K}_i = (K_{i,1}, \dots, K_{i,N_i})$  denote the vector of all customer decisions in interval  $i$ . Furthermore, let  $\mathcal{A}^{\mathbf{K}_i} = \{n \in \{1, \dots, N_i\} | K_{i,n} \neq -1\}$  be the set of indices of non-declining customers and  $\mathcal{D}^{\mathbf{K}_i} = \{1, \dots, N_i\} \setminus \mathcal{A}^{\mathbf{K}_i}$  be the set of indices of declining customers.

The drivers submit their decisions to the CSO and the EVs of drivers who did not decline charging are charged with their required amounts of energy by their selected deadlines. It is assumed that the power  $P_{i,n}^j$  with which  $EV_{i,n}$  is charged in an interval  $j \geq i$ , can be modulated between zero and  $P_{max}$ . Based on the decisions of the customers, the CSO determines a schedule  $\mathbf{P}_{i,n} = (P_{i,n}^i, \dots, P_{i,n}^T)$  of charging powers for each  $EV_{i,n}$ ,  $n = 1, \dots, N_i$ .

It is assumed that the CSO can purchase energy for certain real-time prices. Thus, the charging of an  $EV_{i,n}$  with a deadline extension  $k$  results in charging costs

$$C_{i,n}^k = \sum_{j=i}^{D_{i,n}+k} c_j \cdot P_{i,n}^j \cdot \Delta t \quad (3)$$

for the CSO, where  $c_j$  is the real-time electricity price per kWh in interval  $j$ . The profit the operator achieves with respect to the

EVs that are considered for starting battery charging in interval  $i$ , is

$$profit_i = \sum_{n \in \mathcal{A}^{\mathbf{K}_i}} (p_{i,n}^{K_{i,n}} - C_{i,n}^{K_{i,n}}). \quad (4)$$

We assume that the utilities of the customers are realized as random variables and that the CSO does not know the exact utility values of arriving customers. However, the probability distribution of the utilities is known by the operator. Furthermore, it is assumed that in an interval  $i$ , the operator has no knowledge about the number and energy requirements of EVs arriving in future intervals  $j > i$ . The real-time electricity prices of future intervals are assumed to be known to the CSO.

A natural goal of the CSO is to maximize his/her profit. Hence, the CSO wants to maximize  $profit_i$  for each interval  $i$ . However, the definition of  $profit_i$  according Eq. (4) only considers the electricity costs. Another important part of the CSO's operating costs is missing: the peak demand charges. Typically, commercial and industrial energy consumers have to pay a charge based on their peak load during a billing period – e.g. a year. Thus, when choosing the price offers and charging schedules in an interval  $i$ , the peak load should be taken into account in order to maximize the total profit over the complete billing period. A trivial way to reduce the peak load would be to set the prices so high, that customers decline charging in periods of already high load. However, it can be assumed that a high number of declining and thus unsatisfied customers has a negative impact on the reputation of the CSO. Thus, it is assumed that the CSO is interested in keeping the number of declining customers low.

In the following section, an approach for setting the price offers and for scheduling the charging processes under the described scenario is proposed. It is an online approach, which does not use knowledge or predictions of future charging demands. In order to deal with uncertainties in the customers' utilities, robust optimization is employed.

### 3. Framework for Determination of Charging Schedules and Price Offers

As explained in the previous section, it is assumed that the CSO's primary goal is to maximize the total profit over a complete billing period taking into account the demand charges. However, without knowledge of the charging demands over the complete period, it is hardly possible to optimize the price offers and charging schedules in each interval of a day directly with respect to the total profit. Hence, we propose an indirect approach, which optimizes the prices and charging schedules in each interval  $i$  w.r.t. the following two objectives:

- maximize the profit  $profit_i$  and
- minimize the amount  $V_i$  of load by which a certain limit  $P^{Lim}$  is exceeded.

More precisely, we define the load limit violation  $V_i$  in an interval  $i$  as

$$V_i = \max_{j=i, \dots, T} V_i^j, \quad (5)$$

where  $V_i^j$  is defined as

$$V_i^j = \begin{cases} 0 & , \text{ if } \sum_{n \in \mathcal{A}^{K_i}} P_{i,n}^j = 0 \\ \max \left( 0, \sum_{n \in \mathcal{A}^{K_i}} P_{i,n}^j + L_j^i - P^{Lim} \right) & , \text{ otherwise} \end{cases}, \quad (6)$$

with  $L_j^i$  being the load in interval  $j$  resulting from charging sessions started before interval  $i$ :

$$L_j^i = \sum_{l=1}^{i-1} \sum_{n \in \mathcal{A}^{K_i}} P_{l,n}^j. \quad (7)$$

Thus,  $V_i$  is the maximum violation of the load limit  $P^{Lim}$  in intervals  $j \geq i$  to which the charging of EVs starting in interval  $i$  contributes (without taking into account charging sessions that start after interval  $i$ ).

The two objectives are typically conflicting. Thus, a reasonable tradeoff solution has to be chosen. Additionally, the load limit  $P^{Lim}$  has to be properly set.

For the optimization of the prices and charging schedules in an interval  $i$ , we propose the framework outlined in Figure 1.

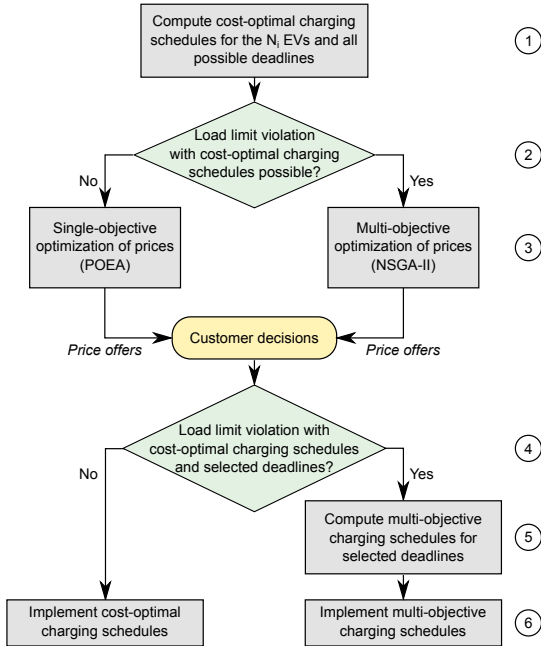


Figure 1: Framework for the setting of price offers and charging schedules for  $V_i > 0$  customers in an interval  $i$ .

In the first step, cost-optimal charging schedules are computed for all possible combinations of deadlines that can be selected by the  $N_i$  customers. The costs arising from charging an EV do not depend on the charging schedule of other EVs. Hence, cost-optimal schedules can be computed for all EVs individually. For each  $EV_{i,n}$ ,  $n = 1, \dots, N_i$ , and each possible deadline extension  $k = 0, \dots, K$ , a schedule  $\hat{\mathbf{P}}_{i,n}^k = (P_{i,n}^{k,i}, \dots, P_{i,n}^{k,T})$  of charging powers is computed as

$$\hat{\mathbf{P}}_{i,n}^k = \arg \min_{(P^i, \dots, P^T)} \sum_{j=i}^T c_j \cdot P^j \cdot \Delta t \quad (8)$$

subject to the following constraints, denoted as  $\mathcal{X}_{i,n}^k$ :

$$\sum_{j=i}^{D_{i,n}+k} P^j \cdot \Delta t = E_{i,n}, \quad (9)$$

$$P^j = 0 \quad \forall j = D_{i,n} + k + 1, \dots, T, \quad (10)$$

$$0 \leq P^j \leq P_{max} \quad \forall j = i, \dots, D_{i,n} + k. \quad (11)$$

The constraints (9) and (10) ensure that the amount of energy required by  $EV_{i,n}$  is charged by the deadline resulting from the deadline extension  $k$  and that no energy is charged after this deadline. Constraint (11) ensures that the maximum charging power  $P_{max}$  is not exceeded. The optimization problem given by (8)–(11) can be efficiently solved over linear programming.

After the cost-optimal schedules are computed, it is evaluated in step ② if there is a combination  $\tilde{\mathbf{K}} \in \{0, \dots, K\}^{N_i}$  of deadline extensions for which the corresponding cost-optimal schedule  $(\hat{\mathbf{P}}_{i,n}^{\tilde{K}_1}, \dots, \hat{\mathbf{P}}_{i,n}^{\tilde{K}_{N_i}})$  of charging all EVs would result in a load limit violation  $V_i > 0$ . If there is no such combination of deadline extensions, the problem of selecting the price offers reduces to the single-objective optimization of the prices w.r.t. the (expected) profit. If, however, the cost-optimal schedules can result in a load limit violation, the prices are optimized in a multi-objective scheme. The single- and multi-objective price optimizations are described more in detail in Section 3.1 and 3.2, respectively.

The prices resulting from the optimization are then offered to the customers who make their decisions  $\mathbf{K}_i$  according Eq. (2). For these decisions, a cost-optimal charging schedule  $\bar{\mathbf{P}}_i^{\mathbf{K}_i} = (\bar{\mathbf{P}}_{i,1}^{\mathbf{K}_i}, \dots, \bar{\mathbf{P}}_{i,N_i}^{\mathbf{K}_i})$  for all EVs is created in step ④ by setting

$$\bar{\mathbf{P}}_{i,n}^{\mathbf{K}_i} = \begin{cases} \mathbf{0} & , \text{ if } K_{i,n} = -1 \\ \hat{\mathbf{P}}_{i,n}^{K_{i,n}} & , \text{ otherwise} \end{cases}. \quad (12)$$

If this schedule does not result in a load limit violation  $V_i > 0$ , it is implemented in step ⑥. Otherwise, a schedule  $\tilde{\mathbf{P}}^{\mathbf{K}_i}$  is implemented, which is the result of a multi-objective optimization (step ⑤) of the charging powers w.r.t. to the charging costs and the load limit violation. The two objectives are combined to a weighted sum:

$$\tilde{\mathbf{P}}^{\mathbf{K}_i} = \arg \min_{((P_1^i, \dots, P_1^T), \dots, (P_{N_i}^i, \dots, P_{N_i}^T))} \left( \sum_{n=1}^{N_i} \sum_{j=i}^T c_j \cdot P_n^j \cdot \Delta t + 1000 \cdot V^+ \right) \quad (13)$$

subject to

$$(P_n^i, \dots, P_n^T) \in \mathcal{X}_{i,n}^{K_{i,n}} \quad \forall n \in \mathcal{A}^{K_i}, \quad (14)$$

$$(P_n^i, \dots, P_n^T) = \mathbf{0} \quad \forall n \in \mathcal{D}^{K_i}, \quad (15)$$

$$P^{Peak} \geq \sum_{n=1}^{N_i} P_n^j + L_j^i \quad \forall j = i, \dots, T^{max}, \quad (16)$$

$$P^{Peak} - P^{Lim} = V^+ - V^-, \quad (17)$$

$$V^+ \geq 0, V^- \geq 0, \quad (18)$$

with  $T^{max} = \max_{n \in \mathcal{A}^{K_i}} (D_{i,n} + K_{i,n})$  being the latest deadline selected by one of the  $N_i$  customers. The constraints (14) and (15) ensure that non-declining customers are charged according their requirements and that declining customers are not charged. The constraints (16)–(18) ensure that  $V^+$  has to be set to the load limit violation  $V_i$  as defined in (5)–(7)<sup>1</sup>, in order to minimize the weighted sum in (13). Minimizing the load limit violation is the main objective of the multi-objective optimization of the charging schedules, since the weight for  $V^+$  is set very high in (13). Again, the problem defined by (13)–(18) can be efficiently solved over linear programming. Note, that unlike the cost-optimal scheduling, the multi-objective scheduling cannot be done for each EV individually since the load limit violation depends on the charging schedule for all EVs together.

### 3.1. Single-objective Optimization of Price Offers

As already stated, if independently of the customers' decisions the cost-optimal schedules cannot result in a load limit violation  $V_i > 0$ , then the problem of selecting optimal prices reduces to the single-objective problem of optimizing the prices w.r.t. the profit  $profit_i$  assuming cost-optimal charging schedules.

The exact utilities of the customers are unknown to the CSO and thus, the exact profit resulting from a certain choice of prices is also unknown at the time of optimization. However, since it is assumed that the probability distribution of the utilities is known, it is possible to optimize the prices w.r.t. the expected value  $\mathbb{E}(profit_i)$  of the profit.

Since it is often not possible to compute the expected profit analytically, we approximate it over Monte Carlo Simulation: In order to approximate the expected profit with given prices  $\mathbf{p}_i = (\mathbf{p}_{i,1}, \dots, \mathbf{p}_{i,N_i})$ ,  $S$  samples of customer utilities are drawn from the distribution of utilities. For each sample  $\mathbf{U}(s) = (\mathbf{U}(s)_{i,1}, \dots, \mathbf{U}(s)_{i,N_i})$ ,  $s = 1, \dots, S$ , the customer decisions are computed. More precisely, the deadline extension selected by the  $n$ -th customer with the  $s$ -th sample of utilities is computed according Eq. (2) as

$$K(s)_{i,n} = \arg \max_{k \in \{0, \dots, K\}} (U(s)_{i,n}^k - p_{i,n}^k), \quad (19)$$

if  $\max (U(s)_{i,n}^k - p_{i,n}^k) \geq 0$  and is set to -1, otherwise. Based on the customer decisions, the profit with the  $s$ -th sample of utilities is computed according Eq. (4):

$$profit_i(s) = \sum_{n \in \mathcal{A}^{K(s)_i}} (p_{i,n}^{K(s)_{i,n}} - C_{i,n}^{K(s)_{i,n}}), \quad (20)$$

where the costs  $C_{i,n}^{K(s)_{i,n}}$  for charging the  $n$ -th EV with deadline extension  $K(s)_{i,n}$  correspond to the costs with the cost-optimal charging schedule  $\bar{\mathbf{P}}_i^{K(s)_i}$  (Eq. (12)). For the determination of the cost-optimal schedules, the results of step ① can be reused. The expected profit with given prices  $\mathbf{p}_i$  is approximated as

$\hat{\mathbb{E}}(profit_i|\mathbf{p}_i) = \frac{\sum_{s=1}^S profit_i(s)}{S}$ . The goal of the optimization is to find prices  $\mathbf{p}_i^*$ , which maximize the (approximated) expected profit:

$$\mathbf{p}_i^* = \arg \max_{\mathbf{p}_i} \hat{\mathbb{E}}(profit_i|\mathbf{p}_i). \quad (21)$$

It is possible to formulate the described optimization problem as a stochastic mixed-integer linear programming (MILP) problem. However, the linear formulation of the problem requires several integer helper variables. Furthermore, a sufficiently large number  $S$  of Monte Carlo samples has to be considered in order to ensure a good approximation of the expected profit. This results in a large problem with many integer variables, which cannot be solved in reasonable time with conventional MILP techniques.

Thus, we propose the use of an evolutionary algorithm (EA) [40] for the price optimization. EAs are optimization algorithms inspired by biological evolution. They iteratively adapt a set (or *population*) of solution candidates (or *individuals*). An iteration is also termed *generation*. In each generation, the following steps are performed: parent individuals are selected from the population and are combined via a *crossover* operator in order to form offspring individuals. The offspring individuals are randomly altered over a *mutation* operator and are then evaluated with the objective function. The objective value of an individual is also called *fitness* of the individual. The last step in a generation is to select from the combination of the old population and the offspring individuals the survivors, which will form the population in the next generation. EAs have various advantages: They are global optimization approaches, which are able to escape from local optima, they can be applied to linear, non-linear, convex and non-convex problems and they require no problem knowledge like, for example, gradients.

For the single-objective optimization, we use an EA, which we term *Price Optimizing EA (POEA)* in the following. An individual of POEA consists of  $(K + 1) \cdot N_i$  real values reflecting the  $K + 1$  price offers for each of the  $N_i$  customers. The mutation operator from the Breeder Genetic Algorithm [41] and intermediate recombination<sup>2</sup> [41] are used as variation operators in POEA. Mating selection is done over tournament selection with two candidate individuals and a  $(\mu + 2)$  replacement strategy is used. The fitness of an individual  $\mathfrak{S}$  is the expected profit given the prices encoded in  $\mathfrak{S}$ . It is computed over Monte Carlo simulation as described above.

In a previous study [19], we evaluated the performance of POEA and it turned out that it outperforms the popular *Covariance Matrix Adaptation Evolution Strategy (CMA-ES)* [42] for the described single-objective optimization of price offers. A possible explanation for this might be that POEA is more explorative than CMA-ES and thus better able to escape local optima.

### 3.2. Multi-objective Optimization of Price Offers

If the cost-optimal schedules computed in step ① can result in a load limit violation, then the price offers are optimized

<sup>1</sup>Strictly speaking,  $V^+$  might differ from  $V_i$  in the sense, that the first case in equation (6) is not considered in the problem formulation (13)–(18).

<sup>2</sup>The values of  $\alpha_i$  in the intermediate recombination are drawn uniform randomly from  $[-1.0, 1.0]$ .

multi-objective w.r.t. the two objectives of maximizing the profit  $profit_i$  and minimizing the load limit violation  $V_i$ . We propose the use of Pareto optimization, which does not compute a single solution, but a complete set of non-dominated solutions. EAs are well suited for Pareto optimization since they always work on a complete set of solution candidates. In this work, we use the popular *Non-dominated Sorting Genetic Algorithm II (NSGA-II)* [43] for the multi-objective optimization of the prices.

NSGA-II is an EA designed for multi-objective optimization. The non-dominated individuals in the final population of an optimization run form the resulting approximation of the Pareto set. The main difference between NSGA-II and a single-objective EA is how individuals are compared in order to select parent individuals and survivors in a generation. In the single-objective case, the fitness values are directly used for comparison: An individual is better than another one, if it has a higher fitness value (or a lower fitness value in the case of a minimization problem). In the multi-objective case, an individual has multiple fitness values, which correspond to the multiple objectives. This makes a comparison difficult. In NSGA-II, individuals are compared over a combination of so called *non-dominated sorting* and a *crowding distance*. Non-dominated sorting separates the population into multiple levels, so that an individual in level  $l$  dominates all individuals in level  $l + 1$ . The dominance level is used as first criterion for comparison: An individual with a lower dominance level is better than an individual with a higher one. Individuals with the same dominance level are compared over their crowding distance. The crowding distance is a measure for the distance of an individual in the solution space to its nearest neighbors in the same dominance level. Individuals with a higher crowding distance are preferred over individuals with a lower one, in order to preserve diversity in the Pareto front approximation.

Like in the single-objective optimization, an individual encodes the  $(K + 1) \cdot N_i$  price offers. The evaluation of an individual  $\mathfrak{S}$  is also done analogous to the single-objective optimization. Expected values of the profit and of the load limit violation are approximated through Monte Carlo simulation with  $S$  samples of customer utilities. Like in the single-objective price-optimization, the profit  $profit_i(s)$  with the  $s$ -th sample is computed according Eq. (20). However, there is one difference compared to the single-objective optimization: In the single-objective price optimization, the costs  $C_{i,n}^{K(s),i,n}$  in Eq. (20) are always set to the costs resulting from the cost-optimal charging schedule. In the multi-objective price optimization, this is only the case if the cost-optimal schedule does not result in a load limit violation. If there is a load limit violation with the cost-optimal schedule, the multi-objective charging schedule (Eq. (13)–(18)) is assumed and  $C_{i,n}^{K(s),i,n}$  is set to the corresponding costs. The load limit violation  $V_i(s)$  with the  $s$ -th sample is computed analogously. If the cost-optimal schedule does not result in a load limit violation,  $V_i(s)$  is set to 0. Otherwise,  $V_i(s)$  is set according Eq. (5)–(7) to the load limit violation resulting from the multi-objective charging schedule.

The evaluation of an individual in the multi-objective price

optimization might require multiple solutions of the multi-objective scheduling problem (13)–(18). In order to save computational time, the results of these internal optimizations are stored and reused whenever possible. That means, if during an optimization the multi-objective charging schedule is required multiple times for the same deadlines, it has to be computed only once.

With the setting of the multi-objective optimization described so far, an approximation of the Pareto set of solutions yielding different tradeoffs between the expected profit and the expected load violation can be computed. However, initial experiments showed that the resulting solutions with a low expected load violation are very likely to result in a high number of declining customers. As stated in Section 2, it can be assumed that the CSO is not interested in these trivial solutions. Thus, the minimization of the expected number  $\mathbb{E}(decl_i)$  of declining customers is used as a third objective of the optimization and the expected value is approximated analogously to  $\mathbb{E}(profit_i)$  and  $\mathbb{E}(V_i)$  in the evaluation of an individual.

For the selection of a solution from the resulting Pareto set  $PS$ , the following approach is used: Let  $\bar{\mathfrak{S}}$  be the individual in  $PS$  resulting in the highest expected profit  $\mathbb{E}(profit_i|\bar{\mathfrak{S}})$  and let  $\mathbb{E}(decl_i|\bar{\mathfrak{S}})$  be the corresponding expected number of declining customers. Then the individual  $\hat{\mathfrak{S}}$  with the lowest expected load limit violation  $\mathbb{E}(V_i|\hat{\mathfrak{S}})$  is selected from the set  $\{\mathfrak{S} \in PS | \mathbb{E}(decl_i|\mathfrak{S}) \leq \mathbb{E}(decl_i|\bar{\mathfrak{S}})\}$ . Thus, the expected number of declining customers with the prices selected from the Pareto set is not higher than with the prices yielding the maximum expected profit. This shall ensure that reducing the peak load is not at the expense of an increasing number of declining customers.

### 3.3. Summary of the Proposed Framework

In this section, we summarize the proposed framework and its working principle. Four optimizations are involved:

- Single-objective (or cost-optimal) charging scheduling,
- Multi-objective charging scheduling with consideration of the peak load,
- Single-objective price optimization and
- Multi-objective price optimization with consideration of the peak load.

The charging scheduling is done over linear programming and the optimization of prices is done over evolutionary algorithms. The framework consists of six steps. The first three steps are intended to optimize the prices offered to the customers. The optimization of the prices is actually multi-objective w.r.t. the profit and the load limit violation. However, in periods of low load, a load limit violation is not possible. In this case, the price optimization is done single-objective only w.r.t. the profit with help of POEA. Otherwise, NSGA-II is employed for multi-objective optimization.

The optimization of the prices is a bi-level optimization. That means, the evaluation of a given set of prices requires the optimization of a charging schedule in order to compute the corresponding charging costs. Analogous to the price optimization, the charging optimization is multi-objective w.r.t. costs and load limit violation, but in times of low load, it can be reduced to single-objective optimization only w.r.t. the costs. The cost-optimal schedules can be easily precomputed for all possible combinations of deadlines, which can be selected by the customers. This is done in the first step of the framework and the results are then used in the price optimization. However, the multi-objective optimization over NSGA-II also requires solutions of the multi-objective scheduling problem, which are computed during the optimization. In order to deal with uncertainties in the customers' utilities, Monte Carlo simulation is used for the evaluation of an individual in the price optimization. This requires the solution of the scheduling problem for multiple samples or scenarios.

In the steps four to six of the framework, it is determined which charging schedule is implemented, after the customers made their decisions and selected their charging deadlines.

#### 4. Experimental Evaluation of the Pricing Framework

##### 4.1. Experimental Setting

In simulation experiments, the pricing approach is evaluated assuming different scenarios. In the default scenario, the following settings are assumed: The length  $\Delta t$  of a scheduling interval is set to 15 min. The arrival times of the EVs are normally distributed with a mean of 12 p.m. and a standard deviation of two hours (eight intervals). Each arriving EV has a battery capacity of 25 kWh and the maximum charging rate  $P_{max}$  per EV is 11 kW. The initial states of charge (SoCs) of the EVs are chosen randomly from a uniform distribution between 10% and 80%, and it is assumed that the target SoC of all EVs is 100%. The maximum allowed deadline extension  $K$  is set to five intervals and the following utility function is used for  $EV_{i,n}$ :

$$U_{i,n}^k = E_{i,n} \cdot (\alpha + \delta_{i,n}) - \beta \cdot k - \gamma_{i,n,k}, \quad (22)$$

with  $\alpha = 0.3$  and  $\beta = 0.4$ . The value of  $\delta_{i,n}$  is normally distributed with a mean of 0 and a standard deviation of 0.01. The values  $\gamma_{i,n,k}$  are drawn from a truncated normal distribution with  $\mu = 0$  and  $\sigma = 0.1$ , truncated left at  $-\frac{\beta}{2}$  and right at  $\frac{\beta}{2}$ . The utility is stated in euro<sup>3</sup>, meaning that on average an EV driver is willing to pay up to 0.3 euros per charged kWh if the EV is charged by the minimum deadline and for each interval the charging takes longer, he/she expects a discount of 0.4 euros on average.

It is assumed that the real-time electricity prices are composed of a fixed fee of 10 euro cent per kWh and a time varying amount. For the latter one, we use the quarter-hourly prices from the German intraday market EPEX SPOT SE [44] averaged over all days in June 2017. The fixed fee reflects the fact that in Germany several taxes and levies have to be paid for electricity. The used electricity prices are shown in Figure 2.

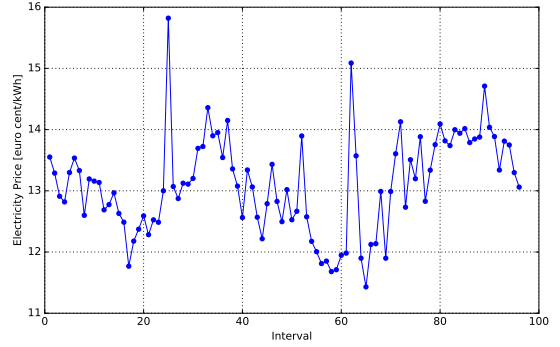


Figure 2: Electricity prices used in the simulations.

It is assumed that the CSO has to pay 76 euros demand charge per kW of the yearly peak of his/her 15-minute-average electricity load. This corresponds to the demand charges arising to our institute. A study of demand charges in the USA [21] comes to the conclusion, that the demand charges vary widely across the country and that they are very high in some states. For example, in California, the peak demand charges are on average 11.45 USD per kW of the monthly peak. Assuming similar peaks in the different months of a year, this would correspond to around 120 euros per kW of the yearly peak. Hence, the peak demand charges assumed in the experiments can be regarded as moderate.

In the experiments, we want to evaluate to what extent the CSO's yearly profit can be increased by using the approach described in Section 3 compared to choosing the prices and charging schedules without consideration of the peak load. However, the simulation of multiple years with different settings would be too time consuming. Thus, in the experiments 20 yearly peaks and total profits are approximated as outlined in Algorithm 1.

---

**Algorithm 1** Approximation of 20 yearly peaks and profits in the experiments.

---

```

for  $y = 1, \dots, 20$  do
  for  $d = 1, \dots, 30$  do
     $profit_d, peak_d \leftarrow$  simulate day  $d$ 
   $peak_y = \max_{d=1, \dots, 30} peak_d$ 
   $profit_y \leftarrow \sum_{d=1}^{30} profit_d \cdot \frac{365}{30} - 76 \cdot peak_y$ 

```

---

For each "year", 30 days with different charging demands (arrival times, energy requirements and utilities) are simulated and the resulting daily profits and peak loads are computed. The yearly peak is assumed to be the maximum of the 30 daily peaks and this together with the average daily profit is used to approximate the yearly profit.

For the computation of charging schedules as described in Section 3, version 5.0.0. of the (mixed-integer) linear programming solver SCIP (Solving Constraint Integer Programs) [45] is used in the experiments. In the single-objective price optimizations, the recombination rate of POEA is set to 0.6, a gene is mutated with a probability of  $\frac{1}{dim}$ , where  $dim$  is the number

<sup>3</sup>One euro is about 1.14 USD at the time of writing (November 2018).



of parameters to optimize ( $6 \cdot N_i$  in our case), and the population size is set to 100. For the optimizations with NSGA-II, version 1.1.6 of the C implementation provided by Kanpur Genetic Algorithms Laboratory [46] is employed. A population size of 60, a mutation rate of  $\frac{1}{dim}$  and a crossover rate of 0.9 is used in NSGA-II. The parameters  $\eta_c$  and  $\eta_m$  of the real-valued crossover and mutation operators of NSGA-II are set to 5 and 20, respectively.

For both POEA and NSGA-II the budget of fitness function evaluations in an optimization for an interval  $i$  is set to  $30,000 \cdot N_i$ , and  $S = 1000$  Monte Carlo samples are used in an evaluation of an individual. The lower and upper bounds of the search variables are set as follows in the price optimizations: For each price offer  $p_{i,n}^k$ , the lower bound is set to the corresponding charging costs  $C_{i,n}^k$  resulting from the cost-optimal charging schedule and the upper bound is set to  $5 \cdot C_{i,n}^k$ .

#### 4.2. Results

In a first experiment, the approach for setting the price offers and charging schedules is evaluated on the default scenario described in the previous section. The approach is compared to two baseline approaches: The first baseline approach sets the prices and charging schedules without consideration of a load limit. For its computation the described framework (Figure 1) is used but with a very high load limit  $P^{Lim}$ . This results in an optimization of the prices and charging schedules solely w.r.t. the profit  $profit_i$ . The second baseline approach does not employ the multi-objective optimization in step ③ of Figure 1. It always uses the single-objective optimization. Hence, the load limit is not considered in the optimization of the price offers, but only in the optimization of the charging schedules (steps ④–⑥). In the following, the approach with multi-objective optimization is denoted as `multi`, the baseline approach with consideration of a load limit and without multi-objective optimization is denoted as `single` and the approach without load limit is denoted as `no_limit`.

Figure 3 and Table 1 show the average of the yearly profit and peak load (approximated as outlined in Algorithm 1) resulting from the `no_limit` approach for different numbers  $N$  of EVs per day.

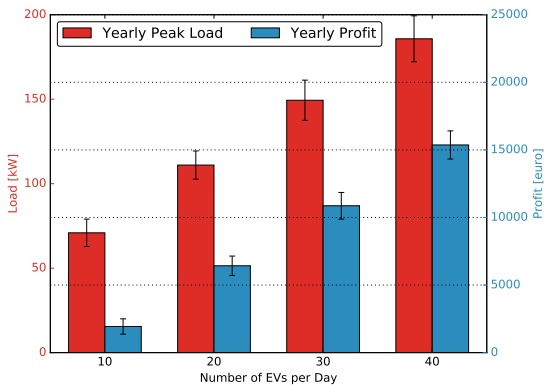


Figure 3: Average and standard deviation of the yearly profit and peak load with the `no_limit` approach for different numbers  $N$  of EVs per day.

Table 1: Average and standard deviation of the yearly profit and peak load with the `no_limit` approach for different numbers  $N$  of EVs per day.

$N$	Peak Load [kW]	Yearly Profit [euro]
10	71 +/- 8	1936 +/- 566
20	111 +/- 8	6426 +/- 719
30	149 +/- 12	10867 +/- 986
40	186 +/- 14	15365 +/- 1043

The average yearly peak loads shown in Figure 3 are used as basis to set the load limits for the simulations with consideration of the peak load. Let  $P_N^{no\_lim}$  denote the average yearly peak load resulting from the simulations with the `no_limit` approach for  $N$  EVs per day. For the simulations with the `multi` and `single` approaches with  $N$  EVs per day, the load limit is set to

$$P^{Lim} = P_N^{no\_lim} - \Delta P, \quad (23)$$

for different values of  $\Delta P$ . In the simulations with the `multi` approach, the load limit is probably adjusted during the simulation of a year. At the beginning of each simulation of a year, the load limit is set according (23), but if in a time step of a day the load exceeds the limit, the limit is set to the current peak. In the simulations with the `single` approach, the load limit is not adjusted since initial experiments showed that this has a negative impact on the results with the `single` approach.

Figure 4 shows the average increase in the yearly profit compared to the `no_limit` approach resulting from the `single` and the `multi` approach for different values of  $\Delta P$ . The detailed numbers for  $\Delta P = 50$  can be seen in Table 2. With both

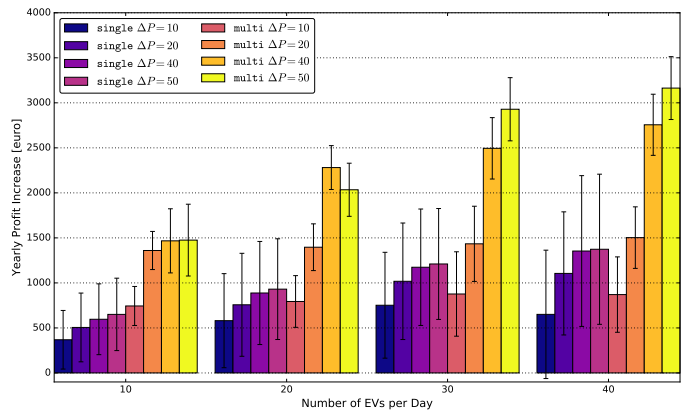


Figure 4: Average and standard deviation of the increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for different values of  $\Delta P$  and different numbers  $N$  of EVs per day.

approaches and with all regarded values of  $\Delta P$ , an increase in the average yearly profit compared to the `no_limit` approach is achieved. However, the `multi` approach, which considers the peak load in the price and charging schedule optimization, yields notable higher profits than the `single` approach, which considers the peak load only in the charging scheduling.

With an increasing number  $N$  of EVs per day, the increases in the profit tend to grow. The reason is that with an increasing  $N$ , the load can be better reduces. This can be seen in Figure

Table 2: Average and standard deviation of the increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for  $\Delta P = 50$  and different numbers  $N$  of EVs per day.

$N$	Yearly Profit Increase [euro]	
	<code>single</code>	<code>multi</code>
10	650 +/- 402	1475 +/- 399
20	931 +/- 559	2034 +/- 295
30	1210 +/- 616	2929 +/- 351
40	1374 +/- 834	3164 +/- 349

5, which shows the average reduction of the yearly peak load compared to the `no_limit` approach. With 10 EVs per day, the

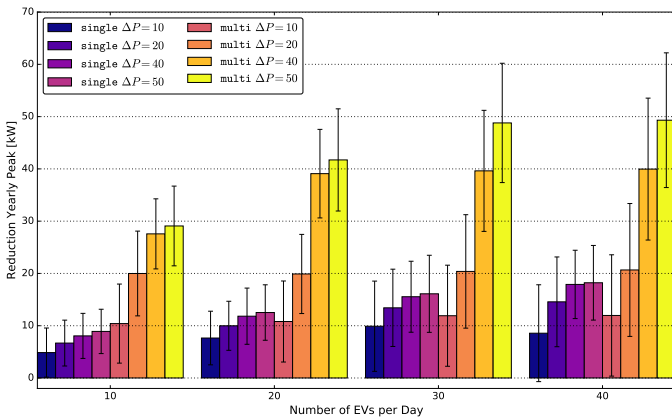


Figure 5: Average and standard deviation of the reduction in the yearly peak load with the `single` and `multi` approach compared to the `no_limit` approach for different values of  $\Delta P$  and different numbers  $N$  of EVs per day.

`multi` approach with  $\Delta P = 50$  can reduce the peak load only by about 30 kW on average. However, with 40 EVs per day, the peak can be reduced by nearly 50 kW on average. Note, that with the `multi` approach the average peak load reduction can be even higher than  $\Delta P$  because of the uncertainties in the customers' utilities. The percentage increase of the yearly profit is shown in Figure 6. It can be seen that the percentage increase

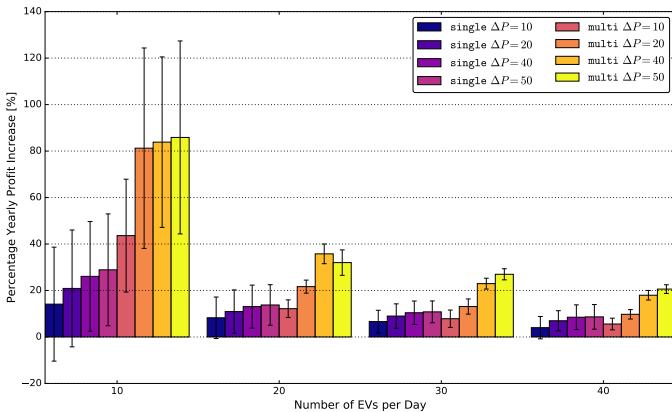


Figure 6: Average and standard deviation of the percentage increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for different values of  $\Delta P$  and different numbers  $N$  of EVs per day.

reduces with an increasing number of EVs per day. However, with 40 EVs per day, there is still an average percentage increase of the yearly profit of about 20%.

Table 3 shows the average number of declining customers per day and the average selected deadline extension with the `no_limit` approach and the `multi` approach with  $\Delta P = 50$ . One can see that the optimization of the price offers with con-

Table 3: Average number of declining customers per day and average selected deadline extension per charging session with the `no_limit` approach and the `multi` approach with  $\Delta P = 50$  for different numbers  $N$  of EVs per day.

Approach	$N$	Avg. Declines per Day	Avg. Deadline Extension
<code>multi</code> , $\Delta P = 50$	10	0.28	0.48
	20	0.56	0.35
	30	1.04	0.18
	40	1.49	0.11
<code>no_limit</code>	10	0.37	0.028
	20	0.67	0.024
	30	1.13	0.024
	40	1.62	0.020

sideration of the peak load does not increase the number of declining customers. On the contrary, with the `multi` approach even less customers decline than with the `no_limit` approach. Furthermore, it can be seen that with the `no_limit` approach nearly no customer extends her/his minimum deadline. The reason is that the electricity prices are in a range that allows the CSO (or the price optimization, respectively) to grant only small discounts for deadline extensions. However, the `multi` approach grants higher discounts in some intervals in order to reduce the peak load, resulting in a higher deadline extension selected on average. Interestingly, the higher the number of EVs per day, the lower the average deadline extension, although the reduction of the peak load increases with an increasing number of EVs (Figure 5).

The results so far show that the proposed approach is able to reduce the peak load and to increase the CSO's yearly profit in the assumed default scenario. In order to investigate, if the approach is robust regarding changes in the default scenario, additional experiments with variations in parameters of the scenario were conducted. In one of these experiments the variance in the electricity prices is varied. More precisely, random numbers drawn from  $\mathcal{N}(0, \sigma_p^2)$  are added to the prices shown in Figure 2 and the standard deviation  $\sigma_p$  is varied between zero and three (euro cent per kWh). The resulting prices are shown in Figure 7.

Figure 8 shows the results with the `no_limit` approach for 20 EVs per day with the different electricity prices. With an increasing noise on the prices, the profit slightly increases and the peak load slightly decreases. The reason for this is that with an increasing variance in the electricity prices, it gets more profitable for the CSO if customers extend their charging deadline and thus, higher discounts for deadline extensions are offered to the customers. Due to the increase in the deadline extensions, the charging is more distributed over the day, resulting in

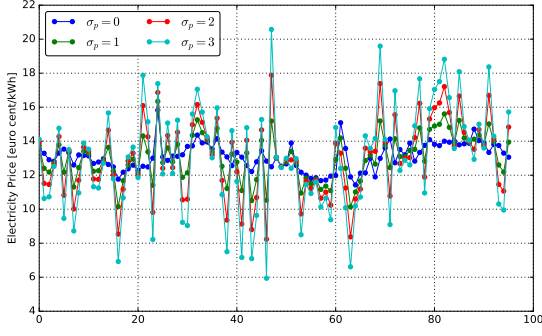


Figure 7: Electricity prices with noise with different standard deviations  $\sigma_p$ .

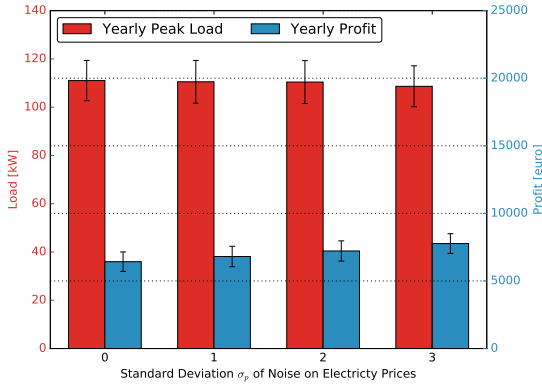


Figure 8: Average and standard deviation of the yearly profit and peak load with the `no_limit` approach for 20 EVs per day and different noise on the electricity prices.

a lower peak load.

Figure 9 shows the corresponding increases in the yearly profit with the `single` and `multi` approach. The increase in

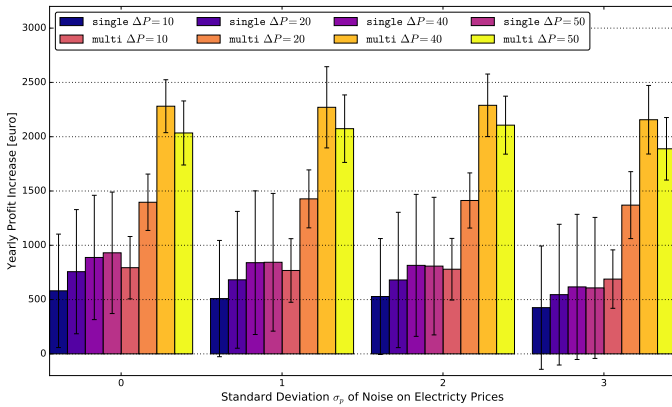


Figure 9: Average and standard deviation of the increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for 20 EVs per day with different values of  $\Delta P$  and with different noise on the electricity prices.

the profit tends to reduce with a higher variance in the electricity prices. The reason is that the difference in the costs resulting from the cost-optimal charging scheduling and from the multi-objective charging scheduling increases with an increasing variance in the prices. However, it can be seen that also with a high

variance in the electricity prices a notable increase in the profit can be achieved by considering the peak load in the setting of the price offers and in the charging scheduling.

In a further experiment, the influence of the distribution of the customers' arrival times is evaluated. For that purpose, the standard deviation (eight intervals in the default setting) is varied between four and 12 intervals. Figure 10 shows the corresponding yearly profits and peak loads with the `no_limit` approach for 20 EVs per day. Not surprisingly, the lower the

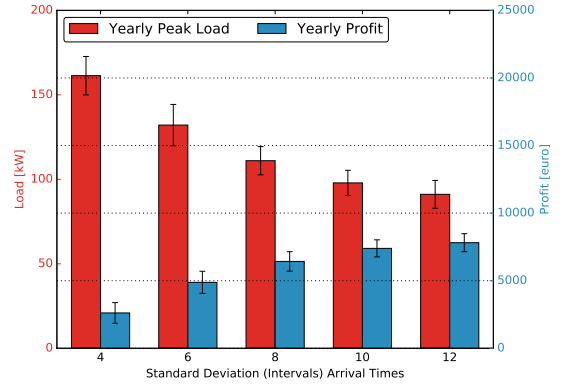


Figure 10: Average and standard deviation of the yearly profit and peak load with the `no_limit` approach for 20 EVs per day and different standard deviations of the distribution of the customers' arrival times.

standard deviation in the arrival times, the higher the peak load and thus, the lower the profit. The results with the `single` and `multi` approach are shown in Figure 11. The increase in the

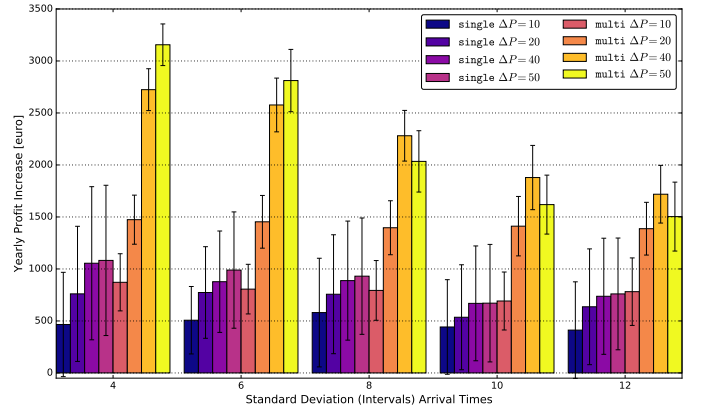


Figure 11: Average and standard deviation of the increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for 20 EVs per day with different values of  $\Delta P$  and with different standard deviations of the distribution of the customers' arrival times.

profit reduces with an increasing standard deviation of the arrival times. The same holds for the percentage increase as can be seen in Figure 12. With a standard deviation of four intervals (one hour), the `multi` approach with  $\Delta P = 50$  increases the yearly profit by around 130% compared to the `no_limit` approach. With 12 intervals (three hours) standard deviation, the percentage increase is significantly lower, but still about 20%.

In the experiments so far, the  $\beta$  in the utility function (Eq. (22)) is set to 0.4, meaning that on average a customer expects

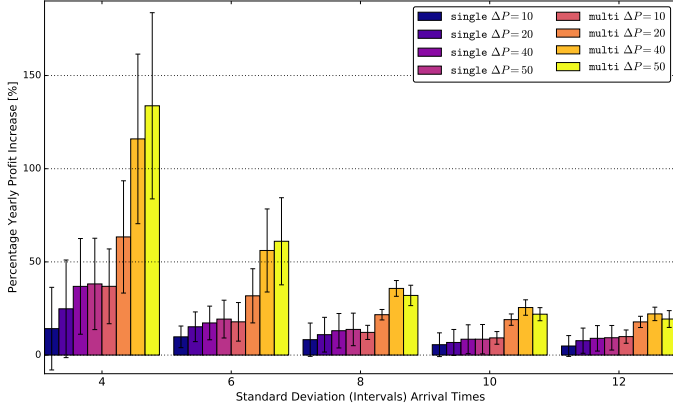


Figure 12: Average and standard deviation of the percentage increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for 20 EVs per day with different standard deviations of the distribution of the customers’ arrival times.

a discount of 40 euro cent for each interval he/she extends the charging deadline. In practice, a higher discount might be necessary in order to encourage customers to extend their deadlines. Thus, in a further experiment, the value of  $\beta$  is varied between 0.4 and 1.2. The results with the `single` and `multi` approach with 20 EVs per day are shown in Figure 13. As ex-

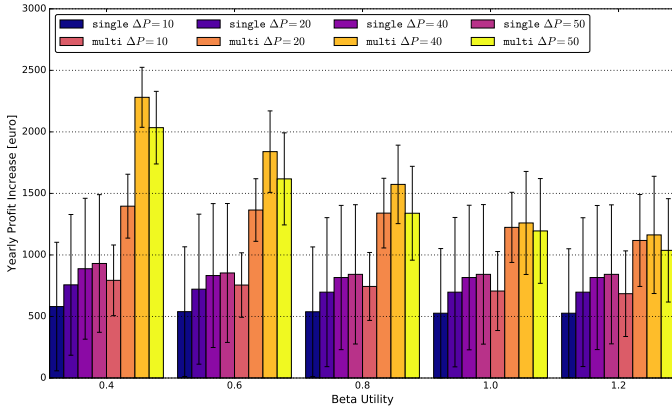


Figure 13: Average and standard deviation of the increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for 20 EVs per day with different values of  $\Delta P$  and with different values  $\beta$  in the utility function.

pected, the increase in the profit reduces with a higher  $\beta$ . However, with a  $\beta$  of 1.2, still a notable increase in the profit can be achieved.

It can be concluded that the proposed approach is reasonably robust regarding variations in the scenario. For all settings considered in the experiments, a positive increase in the average yearly profit compared to the `no_limit` approach is achieved.

## 5. Speed-up of the Robust Optimization

A drawback of the proposed approach is that the multi-objective optimization (in step ③ of Figure 1) can be very time consuming if the number  $N_i$  of considered customers in an interval  $i$  is high. In the optimization, Monte Carlo simulation is

employed in order to evaluate an individual. For many samples of the Monte Carlo simulation a solution of the multi-objective scheduling problem (Eq. (13)–(18)) is required. Solutions to the scheduling problem are stored and reused during an optimization run if possible. However, there are  $(K + 2)$  possible decisions one customer can choose (decline or select one of the  $K + 1$  deadlines) and thus, there are  $(K + 2)^{N_i}$  possible combinations of decisions,  $N_i$  customers can choose. Hence, in the worst-case the multi-objective scheduling problem has to be solved  $(K + 2)^{N_i}$ -times in a run of the multi-objective optimization in an interval  $i$ . Even for a moderate number  $N_i$ , of for example 10, this is too time consuming since the optimization has to be done online and thus, certain real-time requirements have to be fulfilled.

One way to speed-up the robust optimization can be to use only a small number  $S$  of samples in the Monte Carlo simulations. However, this can be expected to result in a low accuracy of the approximations of the expected profit and of the expected load limit violation. Another approach, we call *lazy approach* in the following, can be to compute over Monte-Carlo simulation only the most probable combination of customer decisions given the prices encoded in an individual and to approximate the expected profit and expected peak load violation by solving the multi-objective scheduling problem for these most probable customer decisions. This requires only one solution of the scheduling problem per evaluation of an individual. However, the resulting approximations of the expected values are typically also of low accuracy.

We propose the use of a coevolution-based approach, we already evaluated on general benchmark functions [47]. Applied to the robust optimization of the price offers, the approach works as follows: Coevolution as proposed by Potter and De Jong [48] is employed for the price optimization. This is outlined in Algorithm 2. First, the  $N_i$  customers are separated into  $G$  groups of a certain size  $g$ , each<sup>4</sup> and the corresponding price offers are initialized randomly. Then a certain number  $C$  of cycles is computed. In each cycle, the prices are updated group by group via optimization. In an optimization of a certain group of prices, the prices corresponding to other groups are fixed.

**Algorithm 2** Coevolutionary optimization of price offers for  $N_i$  customers in an interval  $i$ .

- 1: Group the  $N_i$  customers in  $G$  groups of size  $g$ , each, with corresponding price offers  $\mathbf{p}_1, \dots, \mathbf{p}_G$
- 2: Initialize  $\mathbf{p}_1, \dots, \mathbf{p}_G$  randomly
- 3: **for**  $i = 1, \dots, C$  **do**
- 4:   **for**  $j = 1, \dots, G$  **do**
- 5:      $\mathbf{p}_j \leftarrow$  optimize  $\mathbf{p}_j$ , treating  $\mathbf{p}_i$  for  $i = 1, \dots, G, i \neq j$  as fixed

The optimization of a price group is done analogous to the multi-objective optimization described in Section 3.2. However, in order to speed-up the optimization, the Monte Carlo simulations in the evaluations of individuals are done only over

<sup>4</sup>One group might be of size  $N_i \bmod g$  if  $N_i \bmod g \neq 0$ .

the utilities of customers belonging to the currently optimized group. For customers belonging to other groups, it is assumed that they make the most probable decisions given the current (and fixed) price offers of the other groups. Since the decisions of customers belonging to other groups are assumed to be fixed, there are only  $(K + 2)^g$  possible combinations of customer decisions that have to be regarded in the Monte Carlo simulations of one optimization. In total, the coevolution-based approach requires  $C \cdot G \cdot (K + 2)^g$  solutions of the multi-objective scheduling problem. By choosing a small value for the group size  $g$ , the runtime of the complete optimization can be kept low. The coevolution-based approach can be seen as a combination of the conventional approach with full Monte Carlo simulation and the lazy approach.

### 5.1. Experimental Evaluation

In experiments, we compared the coevolution-based approach with the lazy approach and the usual Monte Carlo approach with only 10 samples. For the comparison, the optimizations are done for 30 days with 10 EVs per day, which all arrive in the same interval (the interval and the energy requirements of the EVs vary over the 30 days). Without consideration of the peak load, the average of the peak load over the 30 days is about 106 kW. In the experiments, the load limit  $p^{Lim}$  is set to 60 kW and is not adapted if it is exceeded. The number of allowed fitness function evaluations in an optimization of a group in a cycle of the coevolution-based approach is set to 4000 times the number of EVs in the group. The experiments are run with different settings for the number  $C$  of cycles and the group size  $g$  in the coevolution-based approach, resulting in different numbers  $lp_{max}$  of required solutions to the multi-objective scheduling problem. For a fair comparison with the lazy approach and the traditional Monte Carlo approach, we set the maximum number of allowed computations of solutions of the scheduling problem with these approaches accordingly. Thus, the optimization is terminated when the number of computed solutions of the multi-objective scheduling problem<sup>5</sup> exceeds the maximum number  $lp_{max}$ . For each day, the results yielded by the different approaches are evaluated by computing the expected profit and the expected load limit violation over Monte Carlo simulation with 1000 samples. In the experiments, the default scenario and the settings of NSGA-II as described in Section 4.1 are used.

The results of the experiments are shown in Table 4. With a group size of 1 and with up to 3 cycles, the corresponding values of  $lp_{max}$  are too low to allow the evaluation of the start population with the conventional Monte Carlo approach with 10 samples per fitness evaluation. One can see that for low values of  $lp_{max}$  under 1000, the coevolution-based approach yields the best results with the highest expected profit and a low expected load limit violation. For higher numbers of  $lp_{max}$ , the lazy approach and the usual Monte Carlo approach yield higher expected profits. However, especially with the lazy approach, the expected load limit violation is also high compared

Table 4: Average expected profit and expected load limit violation  $V$  over the 30 days considered in the experiments with different approaches for the robust optimization and different numbers  $lp_{max}$  of allowed solutions of the multi-objective linear programming problem. The traditional Monte Carlo simulation approach is done with 10 samples per fitness evaluation. A “-” means that the value of  $lp_{max}$  is too low to evaluate the start population.

$g$	$C$	$lp_{max}$	coevo		lazy		MC <sub>10</sub>	
			profit	$V$	profit	$V$	profit	$V$
1	1	70	<b>13.31</b>	0.80	6.73	<b>0.32</b>	-	-
1	2	140	<b>13.69</b>	<b>0.02</b>	8.17	0.55	-	-
1	3	210	<b>13.61</b>	<b>0.03</b>	8.55	0.56	-	-
1	4	280	<b>13.81</b>	<b>0.01</b>	8.51	0.64	6.60	0.24
2	1	245	<b>13.20</b>	0.75	8.45	0.84	6.40	<b>0.23</b>
2	2	490	<b>13.20</b>	<b>0.66</b>	9.20	0.86	6.40	0.23
2	3	735	<b>13.29</b>	<b>0.20</b>	10.11	1.33	8.16	0.57
2	4	980	<b>13.27</b>	<b>0.09</b>	10.01	1.26	8.83	0.29
4	1	4851	13.12	0.69	<b>15.61</b>	5.18	12.58	<b>0.18</b>
4	2	9702	13.48	0.47	<b>16.72</b>	6.13	12.58	<b>0.18</b>
4	3	14553	13.55	<b>0.28</b>	<b>16.77</b>	5.77	15.65	1.68
4	4	19404	13.48	<b>0.66</b>	<b>16.82</b>	5.80	15.57	1.53

to the coevolution-based. Additionally, the values of  $lp_{max}$  corresponding to a group size  $g$  of 4 are already high and result in a high runtime. It can be seen that an increase of the group size does not improve the results of the coevolution-based approach. Increasing the number of cycles is more beneficial. The conventional approach based on Monte Carlo simulation with 1000 samples and without an upper limit for the solutions of the multi-objective scheduling problem, as it is used in the experiments described in Section 4, yields an average expected profit of 15.11 euros and an average expected load limit violation of 0.90 kW. Hence, the results are not significantly better than with the coevolutionary-based approach, but the runtime is much higher.

In order to provide an impression of the compute intensity, we measured the time required for the simulation of a day as described above on a machine with a Core i5-4460-CPU (3.2 GHz) and 8 GB RAM and averaged the times over 10 simulations of different days. With the conventional approach with 1000 Monte Carlo samples, a simulation takes on average 138 minutes. The runtimes with the different approaches for speeding up the optimizations can be seen in Table 5 for different values of  $g$  and  $C$ .

It can be seen that for all settings of  $g$  and  $C$ , the runtimes are much lower than with the conventional approach. With group sizes  $g$  of 1 and 2, the runtimes are in a range that might be acceptable for a practical realization. However, the runtimes with a group size of 4 are probably too high for practice.

The speed-up of the robust optimizations allows to simulate the charging with higher numbers of EVs per day compared to the simulations described in Section 4. Thus, we repeated the simulations for the default scenario with 80, 100, 120 and 150 EVs per day. The multi-objective optimization of price offers for intervals  $i$  with more than two EVs is done over the coevolution-based approach with a group size  $g$  of one and

<sup>5</sup>The reuse of an already computed solution of the scheduling problem is not counted in.

Table 5: Average runtime of the simulation of a day (with 10 EVs arriving in the same interval) with the different approaches for the robust optimization and different numbers  $lp_{max}$  of allowed solutions of the multi-objective linear programming problem.

$g$	$C$	$lp_{max}$	Runtime [s]		
			coevo	lazy	MC <sub>10</sub>
1	1	70	18	6	-
1	2	140	30	9	-
1	3	210	43	13	-
1	4	280	56	16	17
2	1	245	27	14	16
2	2	490	51	25	24
2	3	735	74	36	38
2	4	980	98	47	49
4	1	4851	257	241	221
4	2	9702	503	523	435
4	3	14553	760	695	626
4	4	19404	1005	825	814

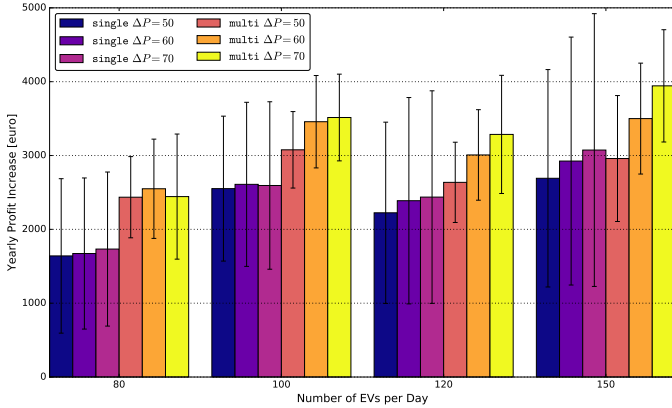


Figure 14: Average and standard deviation of the increase in the yearly profit with the `single` and `multi` approach compared to the `no_limit` approach for different values of  $\Delta P$  and different numbers  $N$  of EVs per day. The coevolution-based approach is employed in the multi-objective optimizations of price offers.

$C = 4$  cycles. For intervals with up to two EVs, the optimizations are done in the conventional way. The results can be seen in Figure 14. With 150 EVs per day, an average increase of the yearly profit by nearly 4000 euros (corresponding to a percentage increase by around 6%) is achieved.

## 6. Conclusion

In the present work, we proposed an approach for the reduction of the peak load at public electric vehicle charging stations, which is based on dynamic pricing and which takes uncertainties in customer preferences into account. Through deadline differentiated pricing, the flexibility provided by customers is increased, which allows a more efficient scheduling of the charging processes. Simulations have shown that considering the peak load in the scheduling of charging processes, can result in a notable increase of the charging station operator's

yearly profit. By considering the peak load in the setting of the dynamic prices, a further considerable increase of the yearly profit can be achieved. In simulations based on a use case typical for Germany, average increases of the yearly profit of nearly 1500 euros could be achieved through intelligent scheduling of the charging processes. The consideration of the peak load in the dynamic pricing resulted in a further increase of more than 1500 euros. In regions with higher peak demand charges, like California, even higher gains in the profit can be expected. It has been shown that the proposed approach is reasonably robust regarding changes in the scenario, like changes in the variance in the arrival times of customers. An advantage of the proposed approach compared to existing approaches for peak load reduction based on dynamic price profiles is that it is better suited for a practical realization since it requires less customer decisions and it provides customers certainty in the planning of the amount of charged energy and the price to pay.

Furthermore, we proposed an approach for accelerating the robust optimization and compared it in simulation experiments to two other approaches. Especially for small numbers of allowed solutions of the scheduling problem, the proposed approach outperforms the other two ones in terms of the quality of the solutions. It has been shown that for the optimization of prices and charging schedules of 10 electric vehicles, the approach can reduce the runtime from more than two hours to several seconds, while yielding still acceptable results.

However, the scalability is still an issue. Thus, a direction of future research might be to evaluate possibilities for further speed-ups, for example, the use of heuristics or of surrogate models. A further topic of future work can be to investigate if the results can be improved by allowing the rescheduling of charging processes or by even making new offers to already charging customers. However, this would result in a further increase of the compute intensity. A further issue besides the scalability is the lack of user studies on dynamic pricing for electric vehicle charging. A deeper investigation of customer preferences or utilities is required in order to evaluate the practicability of the proposed approach.

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