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2018

Preprint:

This is an accepted article published in Genetic and Evolutionary Computation Conference (GECCO). The final authenticated version is available online at: https://doi.org/10.1145/3205455.3205528

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Learning-Based Topology Variation in Evolutionary Level Set Topology Optimization

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ABSTRACT

The main goal in structural Topology Optimization is to find an optimal distribution of material within a defined design domain, under specified boundary conditions. This task is frequently solved with gradient-based methods, but for some problems, e.g. in the domain of crash Topology Optimization, analytical sensitivity information is not available. The recent Evolutionary Level Set Method (EA-LSM) uses Evolutionary Strategies and a representation based on geometric Level Set Functions to solve such problems. However, computational costs associated with Evolutionary Algorithms are relatively high and grow significantly with rising dimensionality of the optimization problem. In this paper, we propose an improved version of EA-LSM, exploiting an adaptive representation, where the number of structural components increases during the optimization. We employ a learning-based approach, where a pre-trained neural network model predicts favorable topological changes, based on the structural state of the design. The proposed algorithm converges quickly at the beginning, determining good designs in low-dimensional search spaces, and the representation is gradually extended by increasing structural complexity. The approach is evaluated on a standard minimum compliance design problem and its superiority with respect to a random adaptive method is demonstrated.

KEYWORDS

Mechanical engineering, Evolution Strategies, Big Data / Data mining, Neural networks / Deep learning, Adaptation / Self-adaptation

ACM Reference Format:
Mariusz Bujny, Nikola Aulig, Markus Olhofer, and Fabian Duddeck. 2018. Learning-Based Topology Variation in Evolutionary Level Set Topology Optimization. In *GECCO '18: Genetic and Evolutionary Computation Conference, July 15–19, 2018, Kyoto, Japan.* ACM, New York, NY, USA, 8 pages. https://doi.org/10.1145/3205455.3205528

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GECCO '18, July 15–19, 2018, Kyoto, Japan
© 2018 Association for Computing Machinery.
ACM ISBN 978-1-4503-5618-3/18/07…$15.00
https://doi.org/10.1145/3205455.3205528

1 INTRODUCTION

Thanks to the progress in development of simulation methods in engineering as well as growing computational power of contemporary computers, optimization methods are gaining more and more attention in real world engineering applications. In particular, structural Topology Optimization (TO) [3], based on simulations with use of the Finite Element Method (FEM), is a rapidly developing field. The main goal of the structural TO is to find an optimal material distribution within a specified design domain, under defined boundary conditions (supports, forces). Those methods are used usually in the initial phase of the product development process, in order to identify the best concept structures. A typical problem solved with TO is shown in Figure 1. A cantilever beam supported at the left side is loaded with a point force on the opposite side. The optimization algorithm aims to minimize the compliance of the structure, or equivalently - maximize the stiffness, under a constraint on the total volume of the structure.

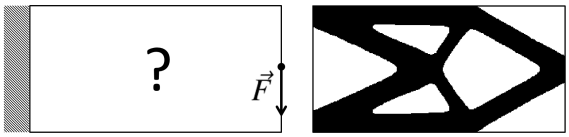

Figure 1: Typical problem in structural TO - compliance minimization of the cantilever beam. Design domain and boundary conditions (left) and the optimized design for 50% volume fraction [3] (right).

Most of the structural TO approaches rely on utilization of the analytical sensitivity information. In such a case, efficient gradient-based optimizers are applied to find the optimal material distribution. Among gradient-based approaches, density-based approaches, like SIMP and homogenization methods, and Level Set Methods (LSMs) can be distinguished [3]. Density-based approaches operate on a discrete mesh of cells, occupying the entire design domain. Each of the cells is parametrized (e.g. by defining density of the material), and the parameters are treated as design variables for the optimization algorithm. In LSMs, the material distribution is defined implicitly, by the Level Set Function (LSF), whose iso-contours define the boundaries between the material and void. As a result, unlike in the density-based approaches, a precise definition of the

material interfaces is possible (no intermediate densities) and numerical artifacts like checkerboard patterns or mesh dependency [3] can be avoided.

Although the gradient-based TO approaches offer very good numerical efficiency, there is a wide group of problems, for which analytical sensitivities are not available or very difficult to obtain. In particular, this applies to the problems in nonlinear crash dynamics, involving such phenomena as contact, failure or buckling, where correct gradient information cannot be obtained due to high nonlinearity and discontinuity of the objective functions as well as high levels of numerical noise [8]. In such problems, either heuristic approaches [1, 15, 19] or gradient-free methods [4, 7, 12, 16] are used. In particular, in this paper, we focus on the recently proposed Evolutionary Level Set Method (EA-LSM) [5, 7], which uses a level-set parametrization based on Moving Morphable Components (MMCs) [10] and has been successfully applied to problems in crash TO. Nevertheless, EA-LSM, which is based on Evolutionary Algorithms (EAs), suffers from high computational costs. This problem can be partially mitigated using parallelization of computations as well as hybridization of EAs with approximate gradient information [6], or for low-dimensional problems, by utilizing surrogate modeling techniques [16].

In this paper, we propose an adaptive representation as a step in this direction. In this approach, the number of design variables, proportional to the number of structural components, is increased gradually during the optimization, allowing the algorithm to find quickly optima in low-dimensional search spaces, which become good starting points for subsequent optimizations in search spaces of increased dimensionality. Since it is not clear how to extend the representation, a machine-learning-based method for prediction of promising topology variations is proposed. Once the topology variation predictor is trained on the data generated for a given type of problems (e.g. compliance minimization), it can be used without additional costs in optimizations of different cases.

Although the concept of adaptive representation has been already used in topology optimization of truss and thin-walled structures [11, 15], our method has a much more general character. First of all, the topology variations that are considered are much less restricted. Any connections between nodes and edges of the graph describing the structure that lead to different topologies can be introduced. Secondly, an evolutionary optimization approach utilizing the idea of speciation of different topologies in niches [18] is used. As a result, the method is able to deal with various problems in structural mechanics and lets different topologies to coexist and compete during the whole optimization process. Finally, the main contribution of this paper is the learning-based topology variation method, allowing for an automatic generation of rules for topology modifications based exclusively on finite element simulations. This is a novel, generic approach, which can be used instead of hand-crafted heuristic rules [15].

Beside already mentioned benefits in efficiency resulting from starting with a low-dimensional representation, adaptive representation does not require defining the complexity of the structure a priori, and can grow up to the complexity offering a required performance. Finally, the idea of learning-based topology variation predictors can be extended and serve as an artificial intelligence guidance system for designers, proposing promising topological changes based on the structural state of the design.

The paper is structured as follows. In Section 2 the optimization problem solved in this paper is defined. Section 3 introduces the Adaptive Evolutionary Level Set Method (A-EA-LSM), while Section 4 describes the learning-based topology variation approach. Finally, in Section 5 the experimental setup and the obtained results are described. Section 6 concludes the paper.

2 OPTIMIZATION PROBLEM

The evaluation of the method is based on the standard cantilever beam benchmark case [3] (Figure 1). Although in this case the analytical gradient information can be obtained, it serves as a suitable scenario for validation of any new TO method, which can be applied afterwards to more complex problems.

After discretization with FEM, the compliance minimization problem can be formulated as follows:

$$\begin{aligned} \min_{\mathbf{z}} \; & C(\mathbf{z}) = \mathbf{f}^T \mathbf{u} \\ \text{s.t.} : \; & V(\mathbf{z}) < V_{max} \\ & \left(\sum_{e=1}^{N} \rho_e(\mathbf{z}) \mathbf{K}_e \right) \mathbf{u} = \mathbf{f} \\ & 0 < \rho_{min} \leq \rho_e \leq 1, \quad e = 1, ..., N, \end{aligned} \tag{1}$$

where C is compliance and $\mathbf{z}$ is the vector of design variables. V is the total volume of the structure and V_{max} is the maximal allowed volume. The symbols $\mathbf{f}$ and $\mathbf{u}$ are used to denote the load and displacement vectors, respectively. $\mathbf{K}_e$ is the element's stiffness matrix, ρ_e is the element's density and ρ_{min} is a small value, e.g. 0.01, corresponding to the density of the void.

Finally, the cost function f minimized by the optimization algorithm takes the following form:

$$f(\mathbf{z}) = C(\mathbf{z}) + P \max\left(0, V(\mathbf{z}) - V_{max}\right), \tag{2}$$

where $P \max\left(0, V(\mathbf{z}) - V_{max}\right)$ is the penalty term used to enforce the volume constraint and P is a large constant.

3 ADAPTIVE EVOLUTIONARY LEVEL SET METHOD

3.1 Parametrization

In this section, first, the standard MMC parametrization used in EA-LSM is presented. Next, we propose a new graph-based parametrization with a mapping to MMCs. Finally, the optimization approach is described.

3.1.1 Moving Morphable Components. Let us formally introduce the parametrization used in EA-LSM. The global level set function, Φ, is defined in the following way:

$$\begin{cases} \Phi(\mathbf{x}) > 0, \mathbf{x} \in \Omega, \\ \Phi(\mathbf{x}) = 0, \mathbf{x} \in \partial\Omega, \\ \Phi(\mathbf{x}) < 0, \mathbf{x} \in D \backslash \Omega, \end{cases} \tag{3}$$

where Ω is a region, inside the design domain D, occupied by material. As a consequence, $D \backslash \Omega$ refers to the part of the design domain occupied by void, while $\partial\Omega$ is the interface between material and void.

Similarly, a local level set function of the ith elementary structural component (MMC) is given by:

$$\begin{cases} \phi_i(\mathbf{x}) > 0, \mathbf{x} \in \Omega_i, \\ \phi_i(\mathbf{x}) = 0, \mathbf{x} \in \partial\Omega_i, \\ \phi_i(\mathbf{x}) < 0, \mathbf{x} \in D \backslash \Omega_i, \end{cases} \tag{4}$$

where Ω_i is a part of the design domain D, where the ith elementary component is situated. Hence, the part of the design occupied by the material can be defined as:

$$\Omega = \bigcup_{i=1}^{E} \Omega_i, \tag{5}$$

where E is the number of elementary components. In this work, a local level set function after Guo et al. [10] was used. For $D = \mathbb{R}^2$, $\mathbf{x} = (x, y)^T$, the level set function of a component can be defined as:

$$\begin{aligned} \phi_i(\mathbf{x}) = -\Bigg(& \left(\frac{\cos\theta_i \left(x - x_{0i}\right) + \sin\theta_i \left(y - y_{0i}\right)}{l_i/2} \right)^q \\ & + \left(\frac{-\sin\theta_i \left(x - x_{0i}\right) + \cos\theta_i \left(y - y_{0i}\right)}{t_i/2} \right)^q - 1 \Bigg), \end{aligned} \tag{6}$$

where (x_0, y_0) denotes the position of the center of a component of length l and thickness t. Component rotation angle is denoted by θ. As in the work by Guo et al. [10], the exponent value $q = 6$ is used. Figure 2 shows the resulting parametrization with the level set function defined by Equation (6).

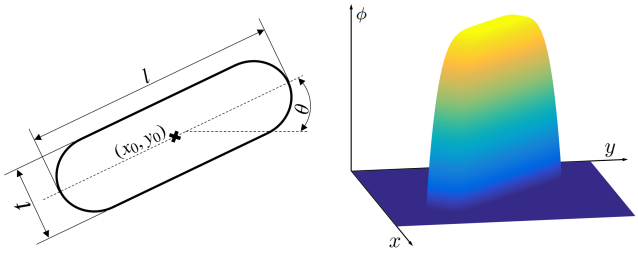

Figure 2: Parametrization and a plot of the level-set function given by (6). For readability, values $\phi < 0$ were set to 0 [6].

As in the other TO approaches [3], a standard density-based geometry mapping is used. In this case, the relation between the global level set function and the material density $\rho(\mathbf{x})$ at position $\mathbf{x} \in D$ is given by:

$$\rho(\mathbf{x}) = \max\left(\rho_{min}, H\left(\Phi(\mathbf{x})\right)\right), \tag{7}$$

where:

$$\Phi(\mathbf{x}) = \max\left(\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \ldots, \phi_E(\mathbf{x})\right), \tag{8}$$

and $H(x)$ denotes the Heaviside function:

$$H(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0. \end{cases} \tag{9}$$

Figure 3 presents an exemplary layout of structural components for the compliance minimization of the standard cantilever beam benchmark case.

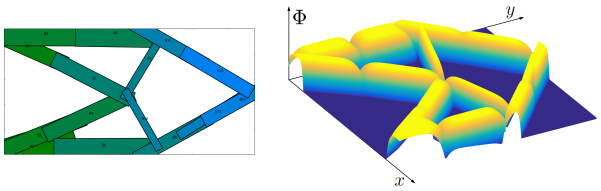

Figure 3: Possible layout of the elementary beam components for the cantilever beam problem (left) and the corresponding level-set field (right) [6].

3.1.2 Graph-Based Parametrization. In the standard EA-LSM [6, 7] the parameters defining each of the local level set functions $(x_{0i}, y_{0i}, \theta_i, l_i, t_i)$ are treated as design variables for the optimization problem. However, this type of representation is not suitable for an adaptive evolutionary approach, where the number of components is very low at the beginning and increases gradually during the optimization. The main problem that occurs in such a situation is illustrated in Figure 4. Suppose that the initial topology for the cantilever beam problem consists of 2 beams connected at the right end. This structure can be represented as a graph consisting of 2 edges and 1 node (design A). In this low-dimensional representation, this design is very close to the global optimum and corresponds to a local optimum in high-dimensional search spaces. After introduction of an additional connection in the graph (design B), the structure would consist of 4 beams and would no longer correspond to the global optimum in the extended search space (design C). In order to move from the local to the global optimum using the standard parametrization, a correlated change of 12 design variables is necessary. The probability of such an evolution of the design via the mutation operator of Evolution Strategies (ESs) [2], used in this paper, is very low and in most cases, the optimization would lead to the design A by decreasing the thickness of the newly inserted connection. Alternatively, one could parametrize the structure by defining the positions of the nodes and thicknesses of the edges of the corresponding graph. In such a case, the conversion from the design B to C would require a correlated change of just 2 design variables, defining the position of the newly added node.

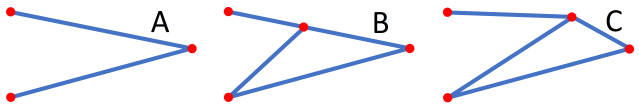

Figure 4: Initial topology (A), corresponding to the global optimum in low-dimensional search space, topology extended by introducing a topology variation (B) and a design corresponding to the global optimum in the extended search space (C).

The problem described above is a standard situation one has to deal with when using adaptive representations and speaks in favor of the graph-based parametrization. Additionally, the graph-based representation reduces the dimensionality and the size of the search space, assuming that the beams are connected at the

ends in a prescribed fashion, which can be only changed by means of adaptation. This is very beneficial when using EAs and can lead to a significant increase of the convergence speed. However, the graph-based representation reduces the flexibility of the optimization and the possibility to introduce topological changes with use of a simple mutation operator. Nevertheless, in case of the adaptive topology optimization, it seems to be the most suitable parametrization. Therefore, in this paper, we use the graph-based parametrization, which is subsequently mapped to the level-set formulation and corresponding finite element models are created to evaluate the fitness of the designs. The mapping between the graph-based parametrization and the standard parametrization of the EA-LSM with MMCs is defined as follows:

$$\begin{aligned}\left(x_{0i}, y_{0i}, \theta_i, l_i, t_i\right) = \Bigg(&\frac{x_{1i}+x_{2i}}{2}, \frac{y_{1i}+y_{2i}}{2},\\ &\arctan\left(\frac{y_{2i}-y_{1i}}{x_{2i}-x_{1i}}\right), \sqrt{\left(x_{1i}-x_{2i}\right)^2+\left(y_{1i}-y_{2i}\right)^2}, t_{1,2i}\Bigg),\end{aligned} \tag{10}$$

where (x_{1i}, y_{1i}) and (x_{2i}, y_{2i}) are the coordinates of the first and the second node of the ith graph edge of thickness $t_{1,2i}$. Parameters $(x_{1i}, y_{1i}, x_{2i}, y_{2i}, t_{1,2i})$ describing each edge of the graph $(i = 1, ..., E)$ are collected in a vector of design variables, used by the optimizer. Figure 5 shows exemplary graphs and their mapping to the material distribution.

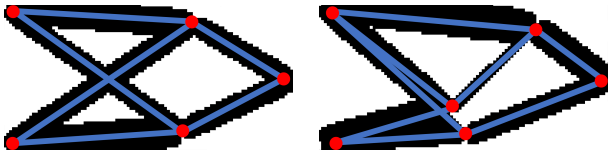

Figure 5: Material distribution with a superimposed graph representation for a structure corresponding to the global (left) and a local (right) optimum for the cantilever beam design problem.

3.2 Optimization Approach

In this section, we propose modifications of a standard Evolution Strategy (ES) [2], allowing for an optimization with the adaptive parametrization described above. Those changes include modification of the recombination operator, introduction of a strategy for topology variations, utilization of the concept of speciation and derandomization of the step size adaptation. The exact description of those modifications is given below.

3.2.1 Recombination. EA-LSM, in its simplest variant, uses a standard ES, which assumes that the dimensionality of the search space remains unchanged. However, that is not true for the adaptive parametrization used in this paper. The easiest solution of this problem is based on a modification of the recombination operator. The recombination operator plays a secondary role in ESs, since the most changes are done based on random mutations. As a result, recombination of design variables' vectors is neglected in this work and only intermediate recombination of strategy parameters, which is essential for ES [2], is carried out.

Once the recombination operator is modified, it is possible to carry out optimization in search spaces of different dimensionality. In order to exploit the accelerating effect of the adaptive representation, the optimization should start from the design of low structural complexity. When the optimization converges to the optimum in the low-dimensional space, the representation is extended. In order to check if the optimization converges, the step size σ of the best individual is monitored. Once it drops below a given threshold σ_{th}, the topology of the best individual is modified.

3.2.2 Topology Variation. We consider two methods for modification of the topology. The simplest idea is a random choice of a node-edge[1] pair and insertion of a new connection. Another approach is based on prediction of favorable topology variations with use of a pre-trained neural network and is described in Section 4.

Once the node-edge pair is selected, it is crucial to identify the best position of the newly inserted edge, in order to escape from the local optimum. This is accomplished by exploring different configurations in a 2-dimensional search space based on the Latin Hypercube Sampling (LHS) [9] and has been illustrated in Figure 6. The algorithm searches for the best (with respect to the objective function that is optimized) elongation of the newly inserted edge and position of the new node along the edge that it splits, while the rest of the structure remains unchanged. In order to respect the volume constraint, the thicknesses of the edges are scaled and the modified individual is inserted back to the parent population.

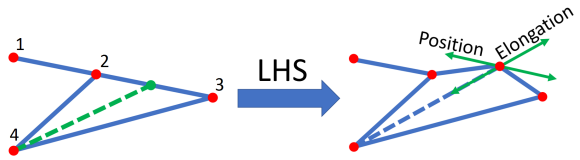

Figure 6: Insertion of a new connection (green dashed line) at the node-edge pair (4, (2,3)) and exploration of different configurations by changing the position of the node along the split edge and elongation of the new edge.

3.2.3 Speciation. Even though the topology variations are tentatively optimized by carrying out the steps described above, they are often still not able to compete with the simpler designs. After introduction of a topology variation, the design has to be optimized to fully exploit the potential of the extended representation. One have to take into account that the optimization in the extended search space is much slower, as well. As a result, it is often recommended to protect the topology variation for some time, to let the modified structure optimize and be able to compete with simpler designs [14, 18]. The simplest way is to evolve the extended individual for a couple of generations in a separate subpopulation and insert it back to the parent population [14]. Another method, which is used in this work, is to protect the individuals in niches. As a result, the individuals with varied topologies compete within the niche and have sufficient time to optimize. The niches are created

[1]This term refers to an already existing node and edge of the graph describing the topology. As shown in Figure 9, topology variations are always introduced by adding a graph edge from one of the existing nodes (red) to a new node (green) that splits one of the existing edges.

by modifying the cost function (f_k), which is minimized, in the following way:

$$f_k' = f_k s_k, \tag{11}$$

where s_k is the number of individuals of the same species as the kth individual, i.e. having the same number of edges, or equivalently, level set basis functions.

3.2.4 Step Size Adaptation. Finally, the standard self-adaptation mechanism of ES [17] is not suitable for the proposed approach. This is because the main idea of self-adaptation relies on the assumption that the good fitness values result from using an appropriate step sizes in the mutation operator [2]. In case of the adaptive approach, topology variations can result in large improvements of the fitness. Hence, the assumption that fitness improvements result from the mutation step is wrong and the step size is very unstable. Therefore, we use derandomization of the step size adaptation [13], which helps to improve the stability. As a result, the mutation step is realized according to the following formulas:

$$\sigma_k^{(g+1)} = \sigma_k^{(g)} \exp\left(\frac{||\mathbf{u_k}|| - E\left[||N(\mathbf{0},\mathbf{I})||\right]}{d\sqrt{n_k}}\right), \tag{12}$$

$$\mathbf{z_k}^{(g+1)} = \mathbf{z_k}^{(g)} + \sigma_k^{(g)}\mathbf{u_k}, \tag{13}$$

where $\sigma_k^{(g)}$ and $\sigma_k^{(g+1)}$ are step sizes of the kth offspring individual at generation g and $g+1$, respectively. $\mathbf{u_k} \sim N(\mathbf{0},\mathbf{I})$, n_k is the dimensionality of the problem for the kth individual and d is a damping parameter.

4 LEARNING-BASED TOPOLOGY VARIATION

The easiest way to introduce topology variations within the proposed adaptive TO framework is, as pointed out in the previous section, to simply choose any node-edge pair at random. As we demonstrate later, this approach can already yield good results thanks to the ability of the algorithm to discard inappropriate topology variations. However, introducing prior knowledge in form of heuristic rules for topology modifications proved to be a successful strategy [15]. Usually, such rules are defined based on the expert knowledge, which might be very difficult to obtain and be valid only for selected cases. Alternatively, we propose to find such rules based on computer experiments only.

In this paper, a method for automatic generation of rules for topology modification within the framework of the A-EA-LSM is proposed. The main idea is to build a neural network model able to predict favorable topological changes based on the state of the structure defined by such properties as stresses or displacements, which can be obtained from the finite element analysis. Below, the main steps of the proposed method are explained.

4.1 Sampling

First of all, training samples have to be generated. To assure the best generalization of the neural network model, this should be done for many different topologies and different boundary conditions. Therefore, in the proposed method, N_T different topologies are generated and for each topology, different boundary conditions are applied. Generation of topologies is based on the cases presented in Figure 7, while boundary conditions are varied by changing the position and orientation of the load as well as applying randomly one of the 3 different support types shown in Figure 8. For each of the N_T cases generated in this way, an underlying graph representation of the topology is identified and all feasible[2] node-edge pairs are listed. For each of the node-edge pairs, a new connection is introduced as illustrated in Figure 9. The exact position of the newly added node along the edge is optimized with respect to the

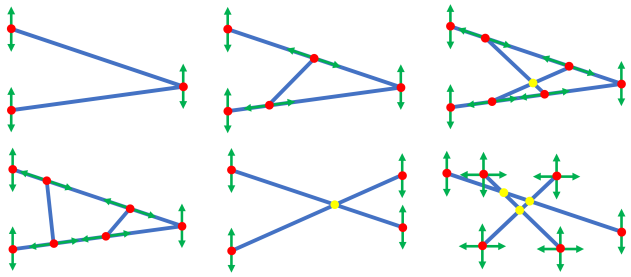

Figure 7: Base topologies used for generation of training samples. Green arrows correspond to the degrees of freedom that are randomly varied to create many more topologies. The yellow nodes result from intersecting the randomly positioned edges.



Figure 8: Three types of support conditions applied randomly in the sampling process. Design domain fixed at the left, right or both sides. Point load is applied at a random position inside the design domain and in the random direction.

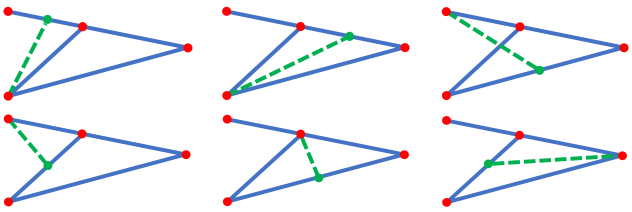

Figure 9: Exemplary reference topology (solid blue lines) and feasible topology variations (dashed green lines). Topology variations are always introduced by adding a graph edge from one of the existing nodes (red) to a new node (green) that splits one of the existing edges. The sampling procedure explores all feasible node-edge pairs.

[2]Feasibility in this case refers to the ability to create a topology variation by introducing a new connection between a node and an edge. Obviously, by introducing a new connection between a node and an edge the node belongs to, only shape variations can be introduced. Therefore, those node-edge pairs are not considered in the sampling procedure.

objective function (see Section 3 and Figure 6) to find the best configuration for the considered node-edge pair. Finally, performance of each of new topologies and the reference topology they have been derived from is evaluated using FEM. The improvements of the objective function (relative to the reference topology) resulting from introduction of a topology variation at a given node-edge pair are computed and used to create a ranking of node-edge pairs, separately for each of the N_T cases. The ranking will be used later to build a neural network model able to predict the most promising node-edge pairs for introducing topology variations.

4.2 Feature Extraction

The data generated by means of finite element simulations can be used now to extract the most suitable features for the model. The features used in this paper can be divided into two groups:

- Geometric features - defining relative position of a node and an edge as well as geometry of the edge.
- Structural state features - defining structural state of the design, based on such features as displacements, equivalent (von Mises) stress and strain energy.

Precise definition of the used geometric and structural state features is presented in Table 1 and Table 2, respectively. In total, 13 different features defining the structural state of the design are taken into consideration.

ID	Feature
1	Minimal distance between the node and the straight line the edge lies on.
2	Distance between the node and the middle of the edge.
3	Shortest path along the graph from the node to the middle of the edge.
4	Length of the edge.

Table 1: Geometric features. Geometric features are normalized with the characteristic length (e.g. diagonal of the design domain).

In order to train a model for prediction of the ranking of topology variations, a transformation of the previously defined features is necessary. For each of the N_{NEt} node-edge pairs within the tth generated reference topology ($t = 1, ..., N_T$), the following pairwise transform of the features and ranking positions is applied:

$$\left(\mathbf{X'_k}, \mathbf{Y'_k}\right) = \left(\mathbf{X_i} - \mathbf{X_j}, sgn\left(\mathbf{Y_i} - \mathbf{Y_j}\right)\right), \tag{14}$$

where $\mathbf{X'_k}$ and $\mathbf{Y'_k}$ are the kth ($k = 1, ..., (N_{NEt})^2$) transformed feature and target vectors, respectively. $\mathbf{X_i}$ and $\mathbf{X_j}$ are the vectors of features characterizing the ith ($i = 1, ..., N_{NEt}$) and jth ($j = 1, ..., N_{NEt}$) node-edge pair within a given reference topology. $\mathbf{Y_i}$ and $\mathbf{Y_j}$ are the positions of those pairs in the ranking (the higher the better).

Transformed samples from different N_T cases are subsequently collected, resulting in a total number of $N_S = \sum_{t=1}^{N_T} (N_{NEt})^2$ samples, which are used for training of a classifier. According to the initial evaluation of different classification models (i.a. logistic regression, SVM, k-NN), a multilayer perceptron neural network had the highest accuracy on the test data and was therefore used in this paper. The classifier can be used to perform comparisons of node-edge pairs (as the potential candidates for creating a new connection in the graph) within a given topology. Based on that, a ranking of potential topology variations (corresponding to node-edge pairs) is created and the best node-edge pair is used to introduce topology variation during the optimization with the A-EA-LSM.

ID	Feature
5	Maximal resultant displacement in the neighborhood of radius 1 around the graph node.
6	Maximal strain energy in the neighborhood of radius 1 around the graph node.
7	Maximal von Mises stress in the neighborhood of radius 1 around the graph node.
8	Maximal resultant displacement along the graph edge.
9	Maximal strain energy along the graph edge.
10	Maximal von Mises stress along the graph edge.
11	Maximal resultant displacement in the middle of the edge.
12	Maximal strain energy in the middle of the edge.
13	Maximal von Mises stress in the middle of the edge.

Table 2: Structural state features. Structural features are normalized with the maximal value of each quantity (e.g. displacement, von Mises stress, strain energy).

The method presented above has a generic character and can be applied to generate rules for topology variations for different objective functions and, once appropriate features are extracted, different optimization problems in structural mechanics (e.g. crash TO). Potentially, it could be also used with different parametrizations and utilized as an alternative to hand-crafted heuristic rules for topology modification [15]. Although the numerical effort to create the model is very high, once the model is trained on a set of samples that is diverse enough, it can be used as a support for solving different optimization tasks.

5 EXPERIMENTS

5.1 Setup

In this paper, the cantilever beam benchmark test, as in Figure 1, was considered. The beam is fixed at one end and a static, point, unit load is applied in the middle of the opposite end. The objective function minimized in this case is the compliance of the structure. Exact configuration of the case is shown in Table 3.

As an optimization algorithm, the optimization method from Section 3 was used. The exact specification of the algorithm is presented in Table 4.

Finally, the configuration for training of the topology variation predictor is specified in Table 5.

5.2 Results

In this section, the results obtained with the A-EA-LSM are described. Figure 10 illustrates the working principles of the method

Property	Symbol	Value	Unit
Design domain length	L	20	mm
Design domain height	H	10	mm
Design domain volume	V_d	200	mm^3
Maximal volume	V_{max}	100	mm^3
Young's modulus	E	$2.1 \cdot 10^5$	MPa
Poisson's ratio	ν	0.3	-
Load	F	1	N
Mesh resolution	-	100 x 50	-

Table 3: Configuration of the cantilever beam test case.

Property	Symbol	Value
Number of parent individuals	μ	20
Number of offspring individuals	λ	70
Initial step size	σ_{init}	0.02
Adaptation damping	d	2
Penalty constant for the volume constraint	P	1000
Step size threshold for topology variations	σ_{th}	0.01
Number of topology variations	N_{TV}	10
Number of edges in the initial topology	-	2
Number of LHS samples in variation step	N_{LHS}	8

Table 4: Configuration of the optimization algorithm.

Property	Symbol	Value
Number of hidden layers	N_{HL}	2
Number of neurons per hidden layer	N_N	100
Activation function	–	ReLU
Optimization algorithm	–	LBFGS
Loss function	–	Log-loss
Number of sampled topologies	N_T	317
Number of training samples	N_S	259025
Number of test samples	N_{TS}	28780

Table 5: Configuration of the neural network classifier (multilayer perceptron) for topology variation prediction.

based on one of the optimization runs. The algorithm used in this case takes advantage of the learning-based topology variation predictor. The neural network classifier reached 85% accuracy on the test data for the pairwise comparison task (see Section 4). The optimization starts with a simple structure consisting of only 2 components connected on one end. After ca. 40 generations, the step size of the best individual drops below 0.01 and the first topology variation takes place. The newly generated topology has over 50% lower compliance than the best existing design and is additionally protected in its niche[3]. As a result, adjusted cost for the newly generated design, consisting of 4 components, is equal to $f_4' = 0.11 \cdot 1 = 0.11$, since there is only one such individual. For the best design consisting of 2 components, adjusted cost is equal to $f_2' = 0.24 \cdot 69 = 16.61$. Thus, the design consisting of 4 components quickly dominates the whole population. Once the population of 2-component designs shrinks, its adjusted cost drops, which should prevent this species from dying out. However, the niche of that species also shrinks, resulting in smaller effective population size and lower convergence speed than in the niche of the new design. Around generation 65, the species consisting of 2 components becomes extinct. The third species (6 components), appearing around 60th generation, corresponds to the reference optimal design for the cantilever beam problem. It survives and evolves during the entire optimization run, eliminating more complex, suboptimal designs. The convergence plot shows very fast convergence of the algorithm in the low-dimensional search spaces in the initial phase of the optimization, which is a very important property in real world applications, where the budget of FEM evaluations is often restricted. The plot of the average step size shows the stabilizing effect of the derandomized step size adaptation scheme. Until the final phase of the optimization run, where just the species consisting of 6 components survives, the step size is kept above $\sigma_{th} = 0.01$, rising in the moments when the new species are introduced. After 200 generations, the average step size steadily drops, as the optimization converges.

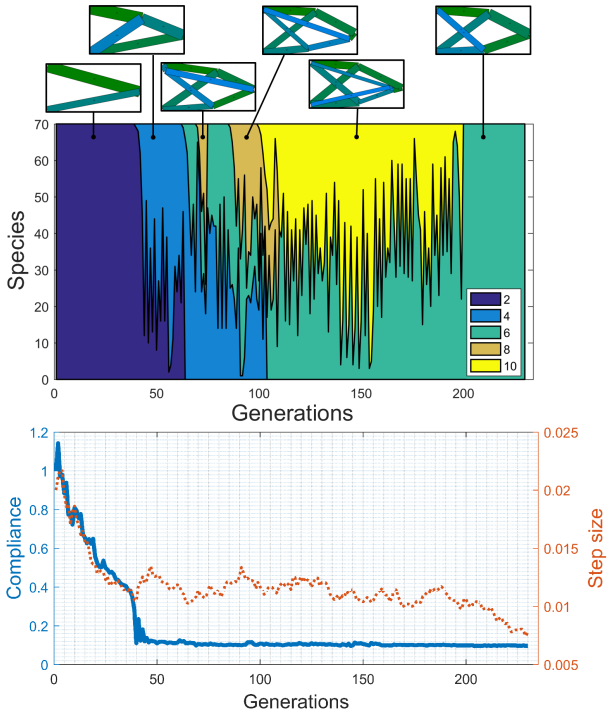

Figure 10: Exemplary run of the A-EA-LSM with the learning-based topology variations. Speciation during the optimization (top) based on the number of components (legend). Convergence of the compliance normalized with the initial value (solid blue line) and the step size averaged over the population of parent individuals (dotted red line).

[3]It should be noted that within one niche, more than one topology can exist, since the criterion for classification of topologies to one species is based on the number of components. Therefore, the final number of species is smaller than the specified number of topology variations N_{TV}. In Figure 10 only the best design within each species is depicted.

In order to compare topology variation methods, 25 A-EA-LSM runs for each of the approaches were performed. The optimizations started from a simple, 2-component topology and 10 topology variations were introduced. Figure 11 shows the box plots of the compliance values after 200 generations. The Wilcoxon rank sum test allows for rejection of the null hypothesis about equality of the medians at the 1% significance level, confirming the superiority of the learning-based approach visible in the box plots. Table 6 shows a comparison of the final topologies obtained with the random and learning-based approach. Again, the learning-based approach was able to converge much more frequently to the best design. Nevertheless, also the random approach is able to optimize the structure and often, thanks to the ability of the algorithm to eliminate bad topology variations, comes up with very good designs.

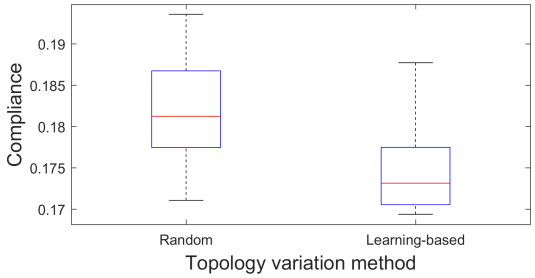

Figure 11: Box plots showing statistical evaluation of A-EA-LSM after 200 generations for 25 optimization runs with random and 25 runs with learning-based topology variations.

Topology type	Compliance	Random	Learning-based
	1.0	8%	72%
	1.013	8%	4%
	1.062	40%	4%
	1.126	16%	8%

Table 6: Main topology types obtained in 25 runs of the A-EA-LSM with random and learning-based topology variations. Compliance values normalized with the best value. Frequency of different topologies in the 2 last columns.

6 CONCLUSIONS

The main focus of this paper was a development of a Topology Optimization (TO) approach exploiting the concept of adaptive representation [14], where the favorable topological changes are predicted using a pre-trained neural network. The proposed method was embedded in the framework of the Evolutionary Level Set Method (EA-LSM), which has been successfully used to solve different problems in structural TO [6, 7]. Thanks to the reduction of the problem dimensionality, the method offers a great possibility to reduce high computational costs of evolutionary methods.

The results show that the Adaptive Evolutionary Level Set Method (A-EA-LSM) can be successfully used in structural TO. The method works both with random and learning-based topology variation approaches, taking advantage of its ability to discard inappropriate topological changes. However, the utilization of the learning-based approach leads to much better performance of the final designs and much better ability to find the global optimum.

The further development of the method seems to be very promising. A-EA-LSM offers great flexibility, allowing for incorporation of different objectives and constraints. The topology variation prediction models can be trained for different types of problems and used to generate promising topological changes. Finally, universality of Evolutionary Algorithms makes the approach applicable to a wide range of structural TO problems in many real world applications.

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