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Design Domain Dependent Preferences for Multi-disciplinary Body-in-White Concept Optimization

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Abstract

Recently methods for topology optimization are increasingly established in the virtual vehicle design process in the automobile industry. In particular a heuristic topology optimization process based on the assumption of uniform energy distribution throughout the structure combined with a scaled energy weighting approach was demonstrated to successfully to provide concepts for vehicle structures subject to static and crash loads concurrently. However, topology optimization for problems with multiple load cases is conventionally based on the assumption of all loads requirements being relevant throughout the complete design domain. This neglects potential design targets such as the restriction of certain load paths to specialized subdomains. For instance, typically, the energy absorption of a front crash of a vehicle is expected to be limited to components in the front of the vehicle. In this work we propose to address this issue for topology optimization of LS-DYNA models subject to multiple load cases by subdomains with design domain dependent preferences. This enables a specialization of subdomains to the designer's requirements. We show systematic evaluation results on a cantilever optimization problem and a possible application to the vehicle concept design.

1. Introduction

The virtual product design process uses computer aided design (CAD) and engineering (CAE) methods to design products and analyze their performance. A central goal of the design process focuses on the efficiency of the structural design. The task of designing a new structure is often supported by computational algorithms such as topology optimization methods to develop optimized designs which meet predefined performance targets with minimal material and cost.

Topology optimization methods are mathematical approaches manifested as numerical algorithms. Starting from devised initial geometry definition and boundary conditions, these algorithms provide the engineers with optimized design concepts in an automatized fashion [3, 4]. The designs obtained from such a topology optimization process are efficient solutions with respect to one or more defined objectives and are obtained more quickly compared to traditional ways of designing structures.

The known methods for topology optimization provide a means of optimizing the design within a single design domain or design space. All loading conditions are applied to this one design domain. The approach of topology optimization algorithms [4] is to find the distribution of materials and voids throughout the design domain. For this purpose material models can be used as for instance in density-based approaches [1, 2, 3]. Other approaches are based on level set [5] or discrete representations [6, 7]. The optimization variables define the design by determining how much and which material is present throughout the design domain and if material is present at all. Suitable optimization steps are performed based on metrics on the optimization variables such as mathematically computed sensitivities [1, 2, 3, 4, 5, 6, 7], specialized heuristics [8, 9, 10] or evolutionary selection schemes [11] that are using the available data from the analysis. Such iterative steps optimize the distribution of material and void throughout the design domain until a stopping criterion is fulfilled.

Frequently, topology optimization is applied to structural design problems subject to a number of different requirements. Users of this method must account for different loading conditions that are applied to the structure. Design problems for which the overall quality of the structure is defined by several objective measures are called multi-load case scenarios. For problems with several load cases, several analysis results are obtained. These various results are combined to enable an update of the optimization variables that considers all load cases. Usually, a weighting scheme is implemented so that the importance and magnitude of each individual load case is accounted for properly in the optimization scheme [3, 8]. As an example consider Figure 1(a). The figure shows a design domain of a minimum compliance cantilever problem with two load cases. For a regular single load case optimization of load case 1 in the complete design domain the topology optimization yields Fig. 1(b). The consideration of both load cases in the complete design domain, when applying a regular multi-load case optimization, yields the structure in Fig. 1(c). (For all optimizations we use the optimization algorithm described in Sec 2.1.)

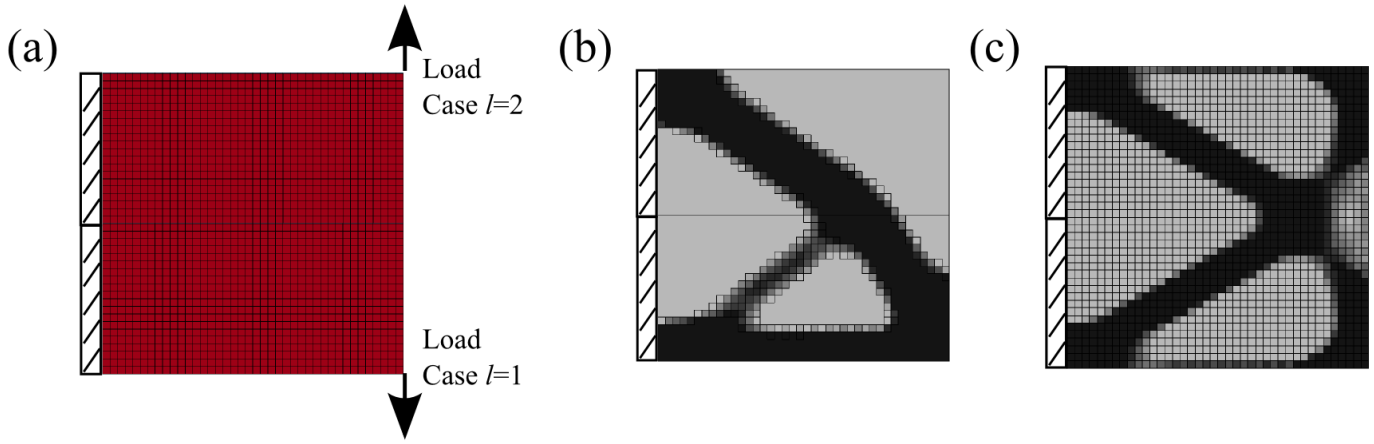

Figure 1: Example for a cantilever topology optimization problem with two loads and one design domain (a) and optimization results with single and (b) multi-load case (c) topology optimizations.

In a conventional multi-load case topology optimization, the complete design domain is subjected to the optimization of all load cases as in Fig. 1. The usage of the complete design domain for all loadings provides the best approach to realize an optimization of the design according to the defined objectives. A more general, although subjective optimization target, is to satisfy the user's expectations on the *load paths* for all intended use cases of the structure. We consider a load path as a path throughout the structure where material is present and contributes to satisfy the expectations towards at least one of the load cases.

These expectations might include which parts of the design domain are desired to be optimized for which loading requirements. A vehicle front crash is an example since a vehicle designer typically expects that the energy of the crash is absorbed within the front part of the vehicle while the cabin is not deforming and hence protecting the passengers; therefore, it is not desirable for the cabin to absorb energy for this load case. In a topology optimization of a concept design of a vehicle body structure where the vehicle body-in-white is the global design domain, current methods cannot explicitly limit the load paths of the front crash to the desired front region and energy absorption may occur in undesired parts of the cabin. A reduction of the design domain to just the front area is also not satisfactory since there are usually other load cases that are relevant globally. These load cases need to be considered throughout the complete design domain by the optimization process, including the front structure.

Hence, this work addresses the definition and achievement of specialization of structural subdomains by affording the user the ability to define preferences on load paths but still relying on one concurrent optimization. Concretely, the target is to realize topology optimization subject to multiple loadings where load paths of the obtained optimized result are controlled in preferred subdomains by incorporating user preferences on the load paths. However, conventional approaches to topology optimization consider one global design domain and do

not enable a specialization of certain parts of the design. The typical load case weighting is only applied to balance various load cases throughout the complete global design domain. Considering all load cases throughout the complete domain may not be satisfactory in cases where the preference of the user desires some of the load paths of the resulting optimized design to be limited or controlled within subdomains.

In this work, we assume that different design domains typically represent different conceptual entities, for instance different components of a larger assembly. In cases, in which the designer has a specified requirement such as loads and functionality for several components, he/she may prefer a separate concept solution for each component. Then, subdomains may be considered as global design domain for a subset of the load cases; however, this consideration requires defining boundary conditions and replacement loads that may insufficiently represent the optimization of the global design domain for all the load cases and the obtained solutions will be strongly influenced by the assumptions on the interface of the substructures. Consequently the results of the optimization of one subdomain may depend and/or lead to the boundary conditions for a neighboring component. This leads to a sequence of optimizations where the found optima might veer away more and more from the ideal (globally optimal) performance.
As an example consider the split of the cantilever design domain in two subdomains in Fig. 2(a). Results of optimizing load case 1 in the lower design domain for different boundary conditions are shown in Fig. 2(b) to (e). Figure 2(b) shows the result for assuming a weak material for which $E_{\text{weak}} = 0.25E_{\text{full}}$, where E_{full} is the Young's modulus of the original material. Figure 2(c) shows the result for assuming full material stiffness E_{full} in the subdomain 1. In Figure 2(d) the top edge nodal constraints support the top edge. Figure 2(e) shows the solution where top edge is not supported and design domain 1 is void.

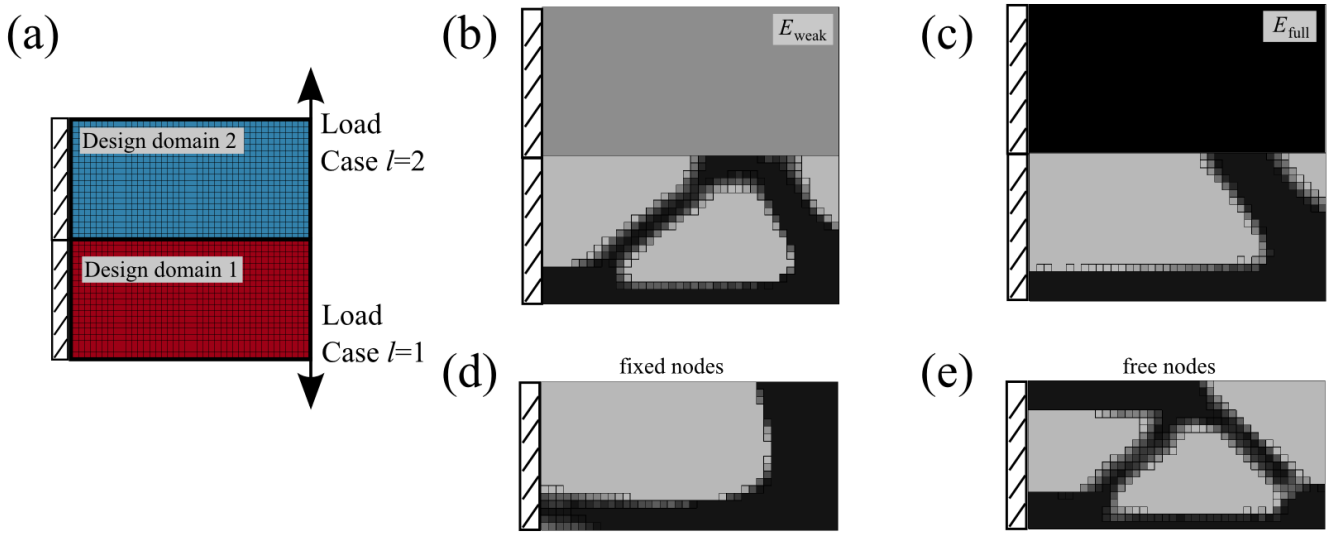

Figure 2: The cantilever problem from Fig. 1 split in two design domains (a) and optimization results for optimizing load case 1 only in design domain 1 subject to different boundary conditions: weak material in top design domain (b), regular (full) material in top design domain (c), void for design domain 2 and nodal constraints along top edge (d), void for design domain 2 and no constraints along top edge (e).

The examples highlight that the assumption on the boundary conditions greatly influences the concept we obtain. One can imagine that a sequential optimization of design space 2 would lead to very different result depending on the results in Fig. 2 (a)-(d). Although the solution obtained from a multi-load case optimization that considers both loads concurrently in one optimization as in Fig. 1(c) is generally preferable, this concurrent process does not have the capability to influence *where* the loads are considered, hence the designer cannot express preferences on the load paths. In the case where a vehicle structure is subjected to a crash load case,

typically there is the intention to restrain the absorption of the crash energy to certain dedicated zones. We believe that there are more such rules based on design experience that make it desirable to account for load path preferences that may lead to a favorable bias of the design.

2. Optimization Method Subject to Load Path Preferences

This work is based on the Scaled Energy Weighting extension of the Hybrid Cellular Automata topology optimization algorithm (SEW-HCA) [8]. In the first subsection the SEW-HCA method is introduced briefly. The next subsection extends the algorithm for multiple design domains that enable the expression of load path preferences.

2.1 SEW-HCA

In topology optimization the target is to find the optimal material distribution within a two or three dimensional design space or design domain Ω. Each finite element of the discretized design domain is an optimization variable, i.e. the optimization assigns a density variable that controls the material properties within the element. The SEW-HCA applies a power law approach according to the well known SIMP approach:

$$E_i(\rho_i) = \rho_i^p E_0 \ , \tag{1}$$

where ρ_i is the density of element i, E_0 is the Young's modulus of the full material and p is a penalization exponent and there are $i = 1 \dots N$ elements.

In typical mathematical optimization approaches the densities are iteratively updated based on the gradients of the objective function. We utilize the heuristic Hybrid Cellular Automata approach due to its capability to address certain types of non-linear crashworthiness topology optimization problems. The assumption of the HCA optimizer is to target a uniform distribution of a field variable by iteratively performing a control-based update of the variables according to:

$$\rho_i^{\text{new}} = \rho_i + K_{\text{P}}(S_i - S^*) \ , \tag{2}$$

where K_P is a control parameter, S_i is the field variables of the element, and S^* is a set-point for the field variables, which is adapted in each iteration so that a desired volume constraint holds. For the kind of problem addressed by SEW-HCA the field variables are usually the strain or internal energy densities of the elements.

In case of multiple load cases, the field variables are combined before the update by a preference-based weighting as proposed in [8]:

$$S_i = \sum_{l=1}^{L} w_l S_{il} = \sum_{l=1}^{L} p_l \frac{1}{s_l} S_{il} \ . \tag{3}$$

with the number of load cases L, a weight w_l for each load case and the field variable S_{il} associated with element i for load case l. The SEW-HCA approach refactors the weight w_l in a user-defined preference factor p_l and a scaling factor s_l. This refactoring enables to separate the task of scaling each of the load cases to the same level from the task of expressing how important the load case is for the user, hence the preference factor.

Results of previous work, where typically the compliance subject to static loads or the energy absorption subject to crash loads are optimized by SEW-HCA, good results were obtained with choosing the scaling factor according to [8]:

$$s_l = \frac{W_l^{(\text{full})}}{W_{\min}^{(\text{full})}}\ , \tag{4}$$

where $W_l^{(\text{full})} = \sum_{i=1}^{N} S_{il}^{(\text{full})} v_i$ is the work of the structure obtained from the analysis of the load cases within the initial iteration of the topology optimization.

2.2 Design Domain Dependent Preferences

Our approach addresses the problem of incorporating load path preferences in the topology optimization by a decomposition of the global design domain Ω into a set of D subdomains Ω_i, hence $\Omega = \bigcup_{d=1}^{D} \Omega_i$. An example for this is the two design domains in Fig. 2(a). The subdomains each consist of a set of design variables i.e. elements that define the distribution of material within in the subdomain.

The split in subdomains can be used to express load path preferences of the user by using additional preference parameters. Concretely, we introduce additional preferences parameters by replacing (3):

$$S_i = \sum_{l=1}^{L} p_{ld(i)} \frac{1}{s_l} S_{il}\ , \tag{5}$$

where $p_{ld(i)}$ is the preference for the load case within subdomain *d =1...D*. The index of the design domain *d=d(i)* is a function of the element index, more precisely, *d* is equal to the index of the design domain that includes the element. Hence we have extended the weighting of the load cases by design domain *dependent* preferences. Note that the proposed method is independent on the specific update rule (2) and can be applied as well when the optimizer is replaced by more general gradient based method, where the weighting of the field variable becomes a weighting of sensitivities.

The preference-based update creates an accumulated field variable from which the optimizer (2) creates an updated design. The load path preferences $p_{ld(i)}$ enable the designer to consider each load case with an individual preference in each subdomain. A zero load path preference will remove the effects of the load case within the corresponding subdomains and consequently the load case will not be considered locally. This enables a specialization of subdomains towards the desired load case, or a removal of undesired load paths. Compared to the previous approaches, this novel approach increases the number of tunable preference parameters from *L* to *L*D*. For instance for two load cases and two subdomains (as in Fig. 2) we obtain four preference parameters.

The approach enables the designer to impose geometric constraints on the global design domain that define which load cases, and as a consequence, which load paths are considered in the subdomain and to which importance. In contrast to previous methods, the proposed method can explore a larger space of solutions and enables designs which are more fitting according to the needs of the designer. Hence, this method is a step towards a control of load paths throughout the global design domain and could enable a specialization of certain subdomains to certain load cases affording the removal, reduction, or emphasis of load paths in selected subdomains.

3. Case Study on Cantilever Beam

3.1 Experimental Setup

In this section we evaluate the proposed approach from Sec. 2.2 for the minimum compliance cantilever problem as introduced in Fig. 2(a). The standard problem of minimum compliance can be formulated as:

$$\begin{aligned} &\min_{\vec{\rho}} c(\vec{\rho}) = \vec{u}\vec{f} \\ &s.t.: \vec{K}(\vec{\rho})\vec{u} = \vec{f} \\ &\quad V(\vec{\rho}) = V_t \\ &\quad 0 < \rho_{\min} \leq \rho_i \leq 1, i = 1, \dots, N \end{aligned} \qquad (6)$$

with the compliance *c*, the displacement vector $\vec{u}$ and the load vector $\vec{f}$. In the equilibrium equation, $\vec{K}$ denotes the stiffness matrix. A constraint V_t imposes a target on the volume of the structure. In order to avoid numerical instabilities a minimum density $\rho_{\min}$ is defined.
The cantilever beam design domain is split in two subdomains for this experiment as shown in Fig 2(a). There are two symmetric loads, represented as Load Case 1 (LC1) and Load Case 2 (LC2). Thus, the design domain dependent weighting (5) in our optimization process has four preference parameters: two for each load case in each of the two subdomains. Accordingly, p_{l1} is the preference for load case *l* in design subdomain 1 and p_{l2} is the preference for load case *l* in design subdomain 2. For all experiments we vary p_{22} i.e. the preference of LC2 in subdomain 2 from 0.1 to 1.0 with a step of 0.1, where $p_{12} = 1 - p_{22}$. This section describes the results of the four following different experiments:

- Experiment 1: This is a regular multi-load case optimization that serves as a baseline for comparison. In this experiment only it holds $p_{21} = p_{22}$ and $p_{11} = p_{12}$. This emulates a conventional multi-load case optimization with only two preferences over the complete design domain.
- Experiment 2: We introduce LC2 in the top design domain, while the bottom subdomain only considers LC 1: $p_{11} = 1,\ p_{21} = 0$.
- Experiment 3: We introduce LC2 in the top design domain, while the bottom subdomain considers both load cases with 50% preference: $p_{11} = p_{21} = 0.5$.
- Experiment 4: We introduce LC2 in the top design domain, while the bottom subdomain considers LC 1 only to 10% and LC2 with a preference of 90%. Hence we start from an assignment to the opposing design domain for each load case: $p_{11} = 0.1,\ p_{21} = 0.9$.

The different preferences are visualized in Fig. 3. For the experiments we utilize our own Python based implementation of the SEW-HCA which is extended by the functionality for multiple design domains with design domain dependent loading. The structure is modelled in LS-Dyna with a 40x40 shell mesh and analyzed with LS-Dyna implicit. No symmetry constraint is applied, but loads and domains are symmetric. The optimization is coupled to LS-Dyna by a specifically developed LS-Dyna interface [14]. The following section discusses the obtained results.

3.2 Cantilever with Design Domain Dependent Preferences Results

Figure 4 shows the compliance of the obtained structures from the optimization experiments subject to the two load cases. For all plots the compliance is normalized to the compliance of the single load case optimization result. As can be expected, the compliance of LC1 increases for all experiments, when its preference is lowered in favor of an increase of the preference for LC2. Vice versa the compliance of LC2 decreases, although not in all cases monotonously. The new freedom of parameters enables different structural concepts and trade-offs between the load cases. Figure 5 shows the structural results for the experiments. In the following part we discuss the experiments separately.

Experiment 1: In Fig. 4(a) we see the result of the conventional multi-load case optimization. As we increase the preference on LC2 (which is increased equally in both domains) the compliance increases for LC1 and drops for LC2 until it reaches the compliance of the single load case optimization. Since both load cases and design domains are symmetric, also the compliance in this plot for both load cases is symmetric (note however that the axis starts at 0.1 instead of 0.0). The structures in Figure 5(a) show a transition from the single load case

design of the load case, visiting the multi-load case design for $p_{22} = 0.5$ and again the single load case design for LC2. Note that although the design domain and load cases are symmetric, symmetry is broken by the unequal preferences.

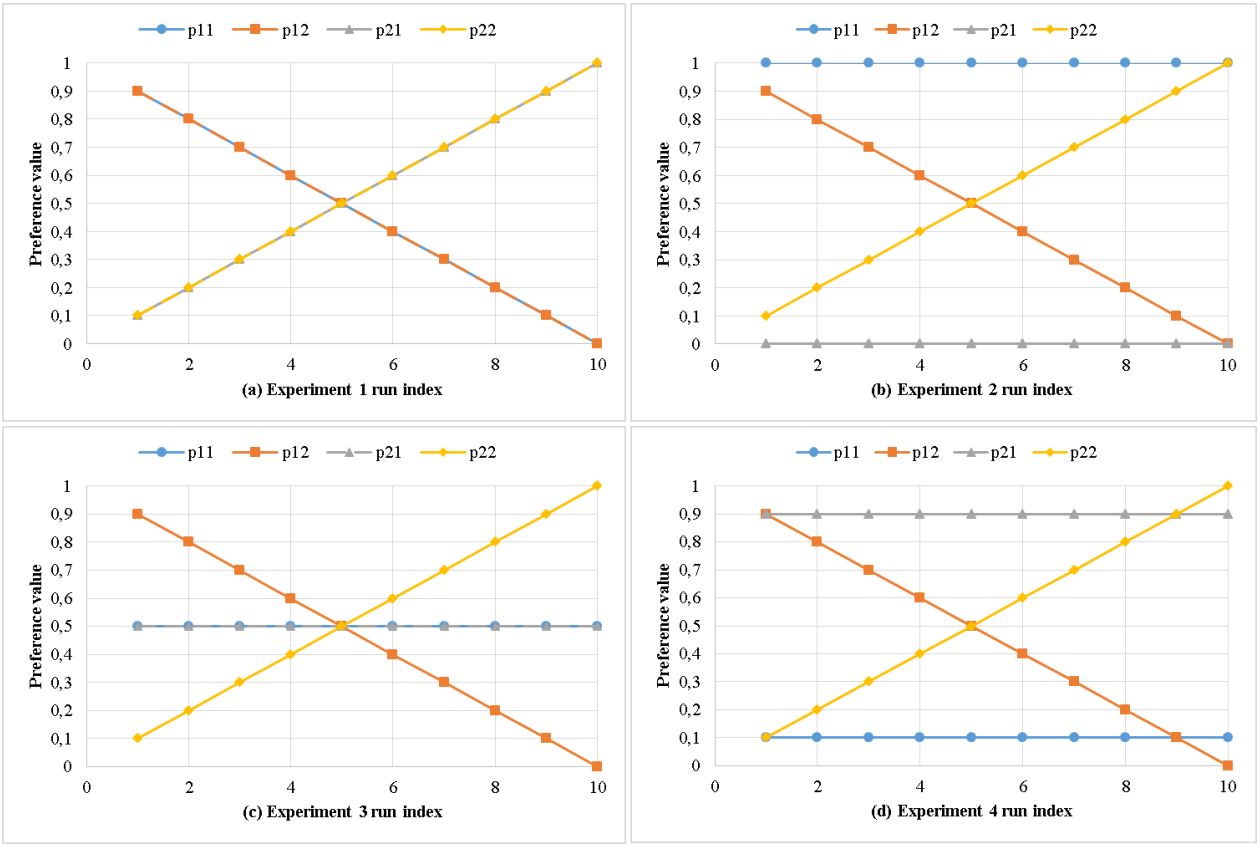

Figure 3: The picture visualizes the variation of the preference parameters for each of the experiments: Experiment 1, standard multi-load case optimization runs (a), experiment 2, only top subdomain considers LC2 (b), experiment 3, equal consideration of both load cases in lower subdomain (c), and experiment 4, LC2 is prioritized in bottom subdomain (d).

Experiment 2: Figure 4(b) shows the compliances for experiment 2, for which LC2 is only introduced in the top design domain and only LC1 is considered in the bottom subdomain. Compared to a multi-load case optimization, the results provide a different design for a low preference on LC2. Starting from the single load case design, when we increase p_{22} to 0.1 a new load path that considers LC2 is formed. Compared to Figure 5(a), the new design considers LC2 without a direct north south connection, but instead with a sharp bend of the material in subdomain 2 such that subdomain 1 is less affected by LC2. Increasing up to $p_{22} = 1$, each load case ends up individually in its own subdomain. We obtain a design with slightly higher compliance than the multi-load case, which can be explained due to the constraint on the design freedom. However, we obtain a different structural concept compared to experiment 1, i.e. when comparing Fig. 5(a) where $p_{22} = 0.5$ to Fig 5(b) where $p_{22} = 1.0$. The new structure shows a north-south connection, smaller left-to-mid diagonals, emphasized east-west connections at top and bottom, and a clear triangle-like structure in the right part of the domain. Although compliance is higher, we have a different structural concept, more similar to two single load case designs in each of the domains connected in the center.

Experiment 3: This experiment revisits the multi-load case design for $p_{22} = 0.5$, but otherwise provides different trade-off structures between the load cases that result from the additional parameters and their variations. Interesting is the design for $p_{22} = 0.1$ in Fig. 5(c) in comparison to Fig. 5(b). This solution realizes a design criterion that puts priority on LC1 (the optimization considers this load in subdomain 1 with $p_{11} = 0.5$ and in subdomain 2 with $p_{12} = 0.9$), and in the same time take account for LC2 in subdomain 1, i.e. we transfer

LC2 to subdomain 1. Hence the optimization reinforces the left bottom to mid connection, to support LC2 in subdomain 1 and accumulates material in the center-right area. Additionally, letting p_{22} approach a value of 1, the optimization yields a new kind of (asymmetric) multi-load case design. This design is possible due to a shift towards higher preference for LC2 compared to the regular multi-load case optimization.

Experiment 4: This experiment considers the effect of reversed domains, i.e. LC2 is prioritized in design domain 1 on the opposite side. For low values of p_{22} a new concept for a structure is obtained, for which the influence of each load on the closer design domain is minimized and the load is preferred in the domain on the opposite side. There is an interesting effect from excluding LC1, when increasing p_{22} from 0.9 to 1.0. This step removes the north-south connection, i.e. it removes the LC1 effect from subdomain 2, and we obtain the mirrored variant of the design in experiment 2 i.e. Fig. 5(b) for $p_{22} = 0.1$.

The experiments demonstrate that we obtain new structural concepts with the proposed design domain dependent preferences. These new concepts are not obtained in the conventional multi-load case optimization. Also we can observe examples that indicate that the method enables to restrict the load paths to certain domains. Fundamentally, this demonstrates that the proposed method is able to at least some extend serve as a tool for load path preference realization. The new concepts represent different trade-offs between the load cases, but they have higher compliance values (if both load case compliance values are averaged and compared to the 50/50 multi-load case.) Although the new concepts might be suboptimal, the richer choice of could support a more creative concept finding process. More research and case studies are required as well as more practical applications. A first potential application show case is presented in the next section.

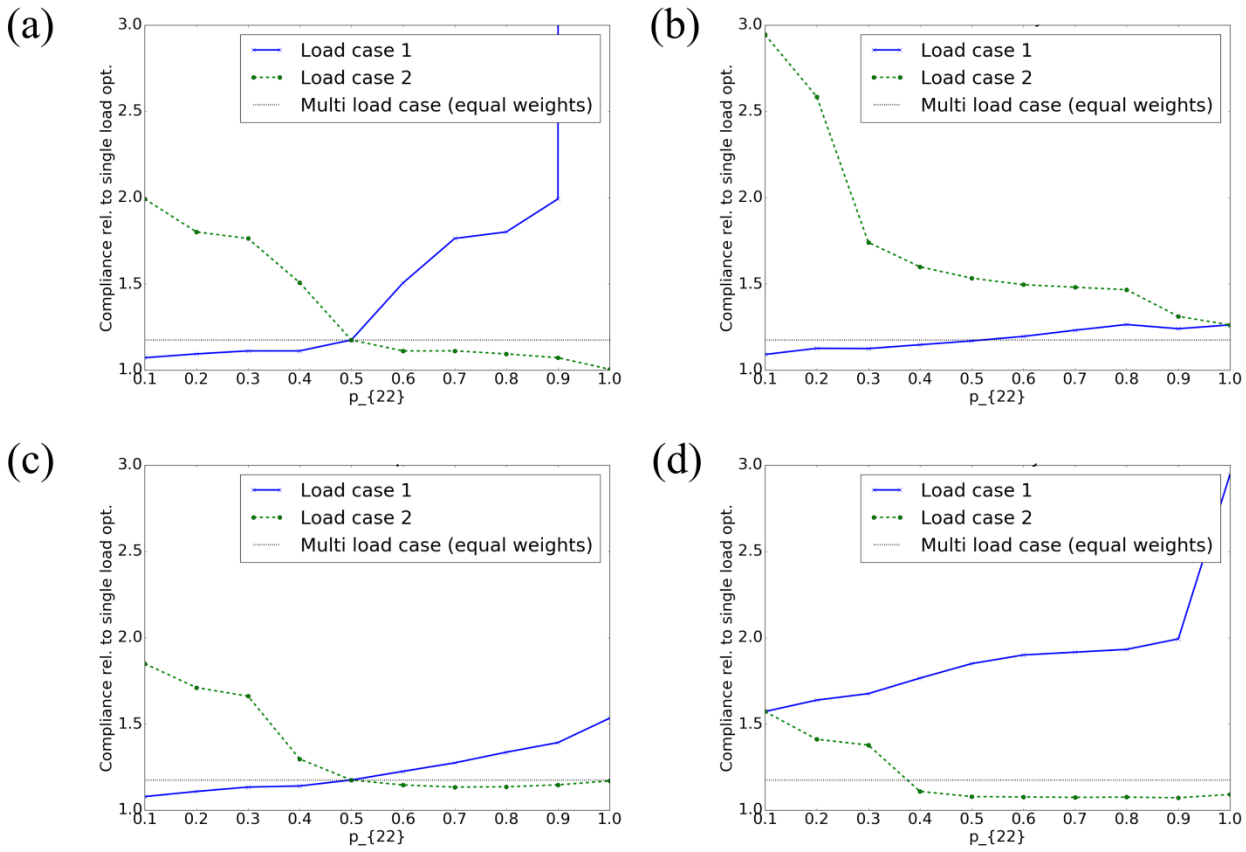

Figure 4: The compliances of the optimized designs for the different experiments: Experiment 1, standard multi-load case optimization runs (a), experiment 2, only top subdomain considers LC2 (b), experiment 3, equal consideration of both load cases in lower subdomain (c), and experiment 4, LC2 is prioritized in bottom subdomain (d).

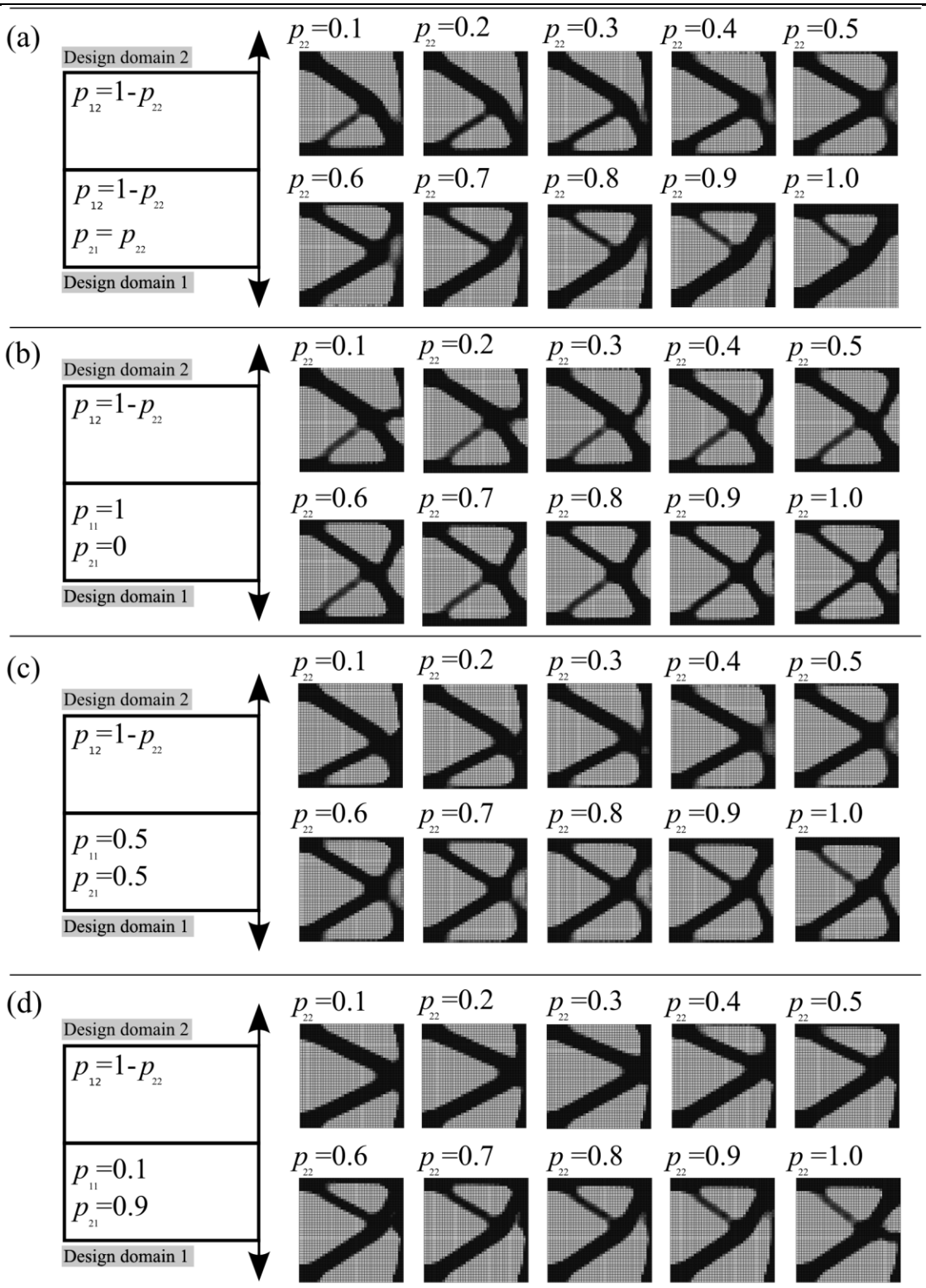

Figure 5: Overview on optimized designs obtained with the four different experiments: Experiment 1, standard multi-load case optimization runs (a), experiment 2, only top subdomain considers LC2 (b), experiment 3, equal consideration of both load cases in lower subdomain (c), and experiment 4, LC2 is prioritized in bottom subdomain (d).

4. Application to Vehicle Body Concept Design

As stated earlier, an important application for the method proposed in this paper is the optimization of vehicle body-in-white concepts. Previous work applied the SEW-HCA method for this type of multi-disciplinary problems where crash and stiffness loadings were considered concurrently in one optimization over the complete design domain [8]. The extension of the method as explored in Sec. 2 and 3 could be used to express the preference on where the energy of a crash should be absorbed. Usually the designer intends to restrict the front crash energy to components in the vehicle front with as little effect as possible on the remaining structure, especially the cabin. The proposed new method could contribute to restrict the front crash load paths to the front of the vehicle only.
An example of such a vehicle body-in-white design domain is shown in Fig. 6. The vehicle design domain is split into a front and a remaining subdomain. Typical stiffness loads will be optimized throughout the entire structure including both design domains. The front crash will only be considered in the front subdomain. Technically, for usage with SEW-HCA, additional loads may be necessary to support the crash loading along the interface of the subdomains. The experiment of this optimization is subject to future research.

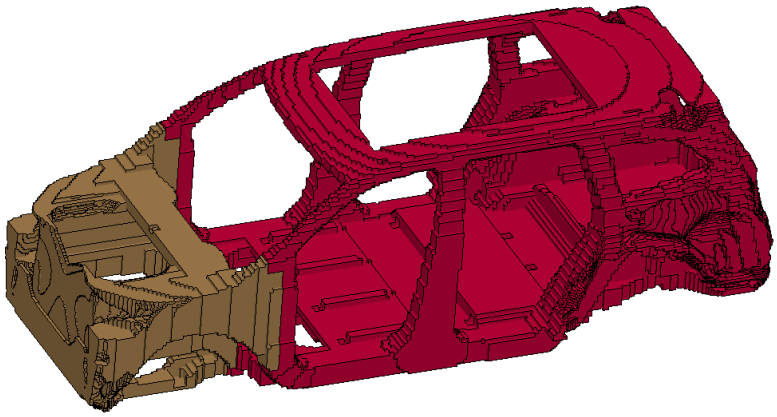

Figure 6: An example for a vehicle body in white design domain that is split into two subdomains (best viewed in color). The front design domain can be optimized to identify load paths for absorption of energy in a front crash, concurrently with other global load cases, while the remaining design domain does not consider the front crash during the optimization.

5. Conclusions

This article introduces a novel method for defining load path preferences into a topology optimization with a Scaled Energy Weighting Hybrid Cellular Automata (SEW-HCA) approach. In order to control the load paths throughout the structure, the global design domain is split into subdomains according to the a-priori preferences. The load path preferences are formulated by means of additional preference parameters in the load case weighting step, using design domain dependent preferences. Results on a minimum compliance cantilever problem show that the new method is able to control load paths and create new concepts that are not obtained by conventional multi-load case optimization. Future work is needed to perform more comprehensive experiments and to evaluate the method on practical use cases such as a proposed vehicle body design problem.

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