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Predicting Driver Left-Turn Behavior from Few Training Samples Using a Maximum A Posteriori Method

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***Abstract*—In this work, we introduce a novel maximum a posteriori (MAP) method, which can predict driver left-turn behavior from only a few training samples. For the prediction of the driver behavior in this scenario we utilize the so-called critical gap. It signifies how large a gap minimally has to be for the driver to accept it and take the turn. The latter is especially important for the personalization of an intersection assistant, which we are currently developing. In contrast to Troutbeck's critical gap estimation method, we define a likelihood over all observable accepted and rejected gaps, thus we provide a model that does not require the driver to behave consistently. Subsequently, we extend it to a MAP estimation by incorporating prior knowledge of the critical gap. Using this approach, we obtain a maximum prediction error of $13.8\,\%$ if only one training sample is used, which is a relative improvement of $35.2\,\%$ compared to Troutbeck's method.**

I. Introduction

In the year 2000, 25% of the police-reported light vehicle crashes in the United States occurred at intersections [1] and 26% in crossing path situations [2]. The authors of [2] argue that the reason for these crashes is the high cognitive complexity of the corresponding maneuvers, e. g. left turns. They showed that participants perceived a significantly higher cognitive workload when turning left in a complex urban scenario compared to turning left on rural roads. Motivated by insights like this, researchers have continued to investigate approaches that could potentially make the left-turn maneuver at intersections safer: In [3], they analyzed the gap selection of 29 drivers in a driving simulator, when the participants were supported by a dynamic signage at unsignalized T-intersections. The researchers of this work found that the sign providing the most detailed information leads to more conservative gap acceptance behavior at the more safety-critical shorter time gaps.

In the work of [4], augmented reality cues were used, which should assist especially middle-aged and older drivers with a range of UFOV (Useful Field of View) impairments in the left-turn scenario with oncoming traffic. They observed that the drivers' estimated Time-To-Collision (TTC) was substantially shorter in the cued case. However, the mean TTC never fell below the minimum time required to safely perform a left-turn.

The previously mentioned traffic situation is clearly not the only one that has great potential for accidents. Therefore car manufacturers have started to provide more advanced driver assistance systems such as collision avoidance systems, blind spot detection, adaptive cruise control, lane departure warning systems or driver drowsiness detection. In addition, there are more and more non-safety-related functions provided in the car via the infotainment system, besides navigation. But due to the multitude of audible and visual information delivered by these systems, which are often not coordinated and adapted to the drivers' needs, the risk of distraction and annoyance of the driver by these systems is increasing, too [5]. Unfortunately, this insufficiency also leads to a high risk that the user stops using them.

We have recently started to investigate the idea of a speech-based intersection assistant in the context of our *Assistance on Demand* (AOD) concept ([6], [7]). The central idea is that the drivers can activate the ADAS via speech commands when they desire support. As a consequence, the ADAS should be less annoying and distracting, thus the system should work with a higher efficiency and should be more widely accepted by the users.

The first scenario we have chosen to focus on in the context of our AOD concept, is the above mentioned left-turn maneuver especially at TWSC (Two-Way Stop-Controlled) intersections. Here, the system gives feedback on gaps in traffic that are suitable to allow the left turn. For doing so the system has to model the driver's left-turn behavior. In [8], we have already used the so-called critical gap to predict the driver's left-turn behavior in a driving simulator. For the critical gap estimation, we have introduced a new maximum likelihood (ML) method and compared the predictions using this method and the predictions of the well-known method by Troutbeck [9] for critical gap estimation. In addition, we have estimated a driver-independent and a driver-specific (personalized) critical gap. We were able to show that using a driver-specific critical gap leads to a more accurate prediction and that there is a significant difference between the individual critical gaps. In contrast to Troutbeck, we have also provided a probabilistic model for the critical gap, which does not assume a consistent

driver and is well suited for the incorporation of additional prior knowledge on the critical gap. The latter is required if the amount of training data is limited and we still want to predict the driver behavior with a high accuracy. We assume that there are several factors which influence the driver's behavior. In this work, we will extend the maximum likelihood critical gap estimation to a maximum a posteriori estimation and evaluate the latter by predicting the left-turn behavior of individual drivers using only a few training samples.

In the next section, we introduce the notion of critical gaps and give an overview of the state-of-the-art methods for their estimation. After that, our novel maximum a posteriori approach is presented and compared to the very popular critical gap estimation method of Troutbeck. In Section III the driving simulator experiment is described which generated the gap acceptance data. The latter is used for the prediction of the drivers' turning behavior in Section IV. Here, the prediction accuracies of Troutbeck's method, our maximum likelihood method and maximum a posteriori method are compared. Finally, we give a conclusion and and outlook on future extensions and optimizations in Section V.

II. Critical Gap Estimation

The so-called *critical gap* is a parameter, which is often used to calculate the capacity and delay of a minor road, especially for two-way stop-controlled intersections. It signifies how large a gap minimally has to be for the driver to accept it and take the turn, thus vacating the minor road. Previously, this value was primarily used to measure the capacity of a specific intersection and hence was calculated from a large pool of drivers all passing this intersection [10]. In our case, however, we use it as a criterion to quantify the behavior of one individual driver. To facilitate this, for the moment, we assume that the layout of the intersections and the traffic density at the intersections are all identical. In the following, we give a short outline of the work on critical gap estimation, before we present Troutbeck's and our novel method in more detail.

A. Methods for critical gap estimation

The estimation of critical gaps goes back to the year 1950, when Raff [11] proposed his method. In principle, the critical gap is the intersection defined by:

$$1 - \mathsf{F}_{\mathsf{rej}}(t) = \mathsf{F}_{\mathsf{acc}}(t)\ , \tag{1}$$

where $\mathsf{F}_{\mathsf{rej}}(t)$ is a cumulative distribution function (CDF), giving the probability of rejecting a gap of t seconds. Similarly, $\mathsf{F}_{\mathsf{acc}}(t)$ is the CDF of the accepted gaps. It does not provide a probabilistic model since only a single value is given which denotes the critical gap. Because of its simplicity it has frequently been used. Troutbeck recently evaluated a revised method [12], which uses the largest rejected gap of a driver instead of all rejected gaps. He summarizes that in this way critical gaps with a much smaller bias than the original method are obtained. Thus, the revised Raff method leads to similar results compared to Troutbeck's method but the latter should still be preferred. However, because of the missing probabilistic model of Raff's method, it is not suited for our maximum a posteriori approach.

TABLE I
Notation for the maximum likelihood equations.

Symbol	*Description*
a_i	logarithm of gap accepted by the driver at intersection i
r_i	logarithm of the largest gap rejected by the driver at intersection i
μ_c	mean of the distribution of the logarithms of the individual driver's critical gaps
σ_c^2	variance of the distribution of the logarithm of the individual driver's critical gaps
$\mathsf{f}_c()$	probability density function for the normal distribution of the critical gap
$\mathsf{F}_c()$	cumulative distribution function for the normal distribution of the critical gap

In the year 1999, Brilon et. al. [10] gave a comprehensive overview of critical gap estimation methods published until this point and concluded that it is not a trivial task to compare all methods. Some of them are very different in their theoretical concept and lead to different results. They expected that it is very likely that more methods will be proposed. At the time of their work they already counted 20-30 published methods. However, they reported that the so-called *maximum likelihood method* from Troutbeck and the method proposed by Hewitt achieved the best results. In 2014, [13] gave a more recent overview of novel methods and specifically Troutbeck's method, also concluding that Troutbeck's method leads to the best results but that the Mode Central Gap method performs similarly well. They recommended that the latter should be subject to further studies in order to clarify the question whether this method should be preferred in the future. Due to its frequent use and its success in the mentioned comparisons, we decided to benchmark our novel approach against Troutbeck's method. We will first detail Troutbeck's method in the next section and then introduce our novel MAP approach.

B. Troutbeck's method

Troutbeck assumes that a driver's critical gap lies somewhere between the largest rejected gap and the gap accepted by the driver [9],[14]. To model the behavior of the drivers, [14] uses a log-normal distribution for the critical gaps. The transformation of the observed gaps in the logarithm domain allows to use the normal distribution for the core computations. The basic notations for the method used in the following are given in Table I. In [15], Brilon recently motivated the approach of Troutbeck by analogies to a classical problem in survival theory. More concretely, he argues that it corresponds to the failure time estimation of interval censored data [15], [16]. The critical gap, i. e. the failure event, can not be observed directly. It can only be observed that it lies in the semi-closed interval $(r_i, a_i]$ when r_i indicates the logarithm of the largest gap rejected by the driver at intersection i and a_i the logarithm of the gap accepted by the driver at intersection

i. The hidden parameters μ and σ of the distribution of μ_c can then be found by maximizing the log-likelihood:

$$\mathcal{L} = \sum_{i=1}^{N} \ln \mathsf{p}(r_i < \mu \leq a_i | \mu, \sigma) \ , \tag{2}$$

for all observations N. Eq. 2 can then be written as [15]:

$$\mathcal{L} = \sum_{i=1}^{N} \ln[\mathsf{F}_c(a_i) - \mathsf{F}_c(r_i)] \ , \tag{3}$$

where $\mathsf{F}_c()$ denotes the cumulative distribution function of the normal distribution with parameters μ and σ. Hence the parameters can be determined via:

$$(\mu_c, \sigma_c) = \arg\max_{\mu,\sigma} \sum_{i=1}^{N} \ln[\mathsf{F}_c(a_i) - \mathsf{F}_c(r_i)] \tag{4}$$

As a final step, the critical gap t_c and variance s^2 in linear scale are computed according to

$$t_c = \mathrm{e}^{\mu_c + 0.5\sigma_c^2}, \qquad s^2 = t_c^2(\mathrm{e}^{\sigma_c^2} - 1). \tag{5}$$

Note that the index i in the tuple (r_i, a_i) indicates the i-th intersection of the current driver. This is in contrast to the original formulation in [14], where the index i is used to denote the i-th driver at the current intersection.

The failure time estimation of interval censored data requires that $r_i < \mu_c \leq a_i$. Consequently, the method of Troutbeck assumes a consistent and homogeneous driver. This means that the largest rejected gap r_i recorded at intersection pass i must always be smaller than the corresponding accepted gap a_i. This is however often not fulfilled. In cases where the assumption is not met, the original method in [14] diminished the value of the accepted gap by a small number and set this as the value for the largest rejected gap. We obtained better results by using the next smaller rejected gap from intersection pass i instead. If the next smaller gap is still larger than the accepted gap, we repeat this step. If we do not find a rejected gap smaller than the accepted gap, we discard the measurement for intersection i. We assume that in this way we obtain more meaningful observations from the recorded intersections. Yet both heuristics to conciliate the observations with the underlying probabilistic model require a modification of the observations. The consequences of this on the validity of the derived probabilistic inferences are difficult to judge.

C. Novel maximum a posteriori method

When we need to model an individual driver, we frequently only have very limited data available per driver. Hence methods, which yield good results with few observations, are highly desirable. One such approach is Bayesian inference, as it allows the incorporation of prior knowledge, based on other drivers, to model the current driver. The validity of the inferences made with this approach strongly depend on the validity of the underlying probability distributions. The method developed by Troutbeck is not well suited for this for two reasons. First, only the largest rejected gaps and the corresponding accepted gaps are taken into account and in addition, the largest rejected gap must be smaller than the accepted gap of a specific intersection. Thus, there are rejected gaps and accepted gaps, which potentially could serve as training data but can not used. Second, the previously mentioned heuristics to conciliate the model and the observations can potentially strongly impair the validity of the required probability distributions. Therefore, we have developed a new maximum likelihood method to estimate the critical gap, which is able to take all observations into account and does not require the driver to behave fully consistently [8]. Applying Bayesian methods, we extend this approach to a maximum a posteriori approach. In Bayesian inference, it is no longer the likelihood $p(\mathbf{Y}|\theta)$, which is maximized for the unknown parameter θ and known training set $\mathbf{Y}$. Instead, the posterior distribution density is maximized, which can be expressed with the help of Bayes' theorem as follows:

$$\mathsf{p}(\theta|\mathbf{Y}) = \frac{\mathsf{p}(\mathbf{Y}|\theta)\mathsf{p}(\theta)}{\mathsf{p}(\mathbf{Y})} \ . \tag{6}$$

Here, $\mathsf{p}(\theta)$ is the prior, which lets us incorporate the prior knowledge on the critical gaps. Note that the parameter θ can be a vector, which will be the case for our method later.

The first step is to properly define a likelihood in the sense of Bayesian statistics. A proper likelihood needs to be a joint density function of all observations, so

$$\mathsf{p}(\mathbf{Y}|\theta) = \mathsf{p}(\mathbf{y}_1, ..., \mathbf{y}_M|\theta) \ , \tag{7}$$

where $\mathbf{Y}$ are the observations composed of all $\mathbf{y}_i$, the feature vectors with dimension D_o. θ is the parameter vector with dimension D_θ, we are attempting to find.

In the following a new likelihood is described, which fulfills our previously defined targets and does not require that the accepted gap has to be higher than the largest rejected gap. Suppose that one has a data vector $\mathbf{y}$, which consists of all accepted gaps $a_1, ..., a_K$ and all rejected gaps $r_1, ..., r_L$ extracted from the recorded intersections. For this data vector, we can define the likelihood $\mathsf{p}(\mathbf{y}|\mu, \sigma)$ that is the probability of seeing the recorded data given a critical gap μ and its standard deviation σ:

$$\mathsf{p}(\mathbf{y}|\mu, \sigma) = \mathsf{p}(a_1, ..., a_K, r_1, ...r_L|\mu, \sigma) \ . \tag{8}$$

We assume that the accepted gaps a_i and rejected gaps r_j are conditionally independent given the critical gap μ and its variance σ. This means that one can factorize the joint distribution from Eq. 8 to yield a product of two joint distributions:

$$\mathsf{p}(\mathbf{y}|\mu, \sigma) = \mathsf{p}(a_1, ..., a_K|\mu, \sigma)\mathsf{p}(r_1, ...r_L|\mu, \sigma) \ . \tag{9}$$

Furthermore, we assume conditional independence of the accepted gaps and of the rejected gaps among each other. So, finally we can write the likelihood as:

$$\mathsf{p}(\mathbf{y}|\mu, \sigma) = \prod_{k=1}^{K} \mathsf{p}(a_k|\mu, \sigma) \prod_{l=1}^{L} \mathsf{p}(r_l|\mu, \sigma) \ . \tag{10}$$

Hence, in Eq. 10 we have two distributions we want to determine: $\mathsf{p}(a|\mu,\sigma)$, which describes the probability of seeing an accepted gap a given a specific critical gap μ with standard deviation σ and $\mathsf{p}(r|\mu,\sigma)$, which describes the probability of seeing a rejected gap r given the same parameters. For this purpose, we follow Troutbeck and assume a log-normal distribution for the accepted and rejected gaps. Hence a_k and r_l describe the logarithm of the accepted and rejected gaps. Consequently we also use the cumulative distribution function (CDF) $\mathsf{F}(x)$ with the parameters μ, σ to model the accepted and rejected gaps. This implies that $\mathsf{F}(x)$ serves as the probability density function (PDF) describing the likelihood $\mathsf{p}(a|\mu,\sigma)$. Based on equivalent reasoning, $1-\mathsf{F}(x)$ describe the probability density function for $\mathsf{p}(r|\mu,\sigma)$.

To arrive at valid probability density functions, $\mathsf{p}(r|\mu,\sigma)$ and $\mathsf{p}(a|\mu,\sigma)$ must be normalized. The integrals of $\mathsf{F}(x)$ and of $1-\mathsf{F}(x)$ over x diverge if we choose an unbounded integral. To circumvent this, we assume that the gaps we observe fall into the range $[e^{-\tau}, \mathrm{e}^{\tau}]$, which corresponds to $[-\tau,\tau]$ in the logarithmic domain. The probability density functions for the accepted gaps and the rejected gaps hence take the following form:

$$\mathsf{p}(a|\mu,\sigma) = \frac{\mathsf{F}(a|\mu,\sigma)}{\int_{-\tau}^{\tau} \mathsf{F}(x|\mu,\sigma)\mathsf{d}x} \,, \tag{11}$$

$$\mathsf{p}(r|\mu,\sigma) = \frac{1-\mathsf{F}(r|\mu,\sigma)}{\int_{-\tau}^{\tau} 1-\mathsf{F}(x|\mu,\sigma)\mathsf{d}x} \,. \tag{12}$$

To solve the integral over $\mathsf{F}(x|\mu,\sigma)$ and $1-\mathsf{F}(x|\mu,\sigma)$, we use the antiderivative of the standard normal CDF $\Phi(x)$ with $\mu=0, \sigma=1$, which is given by:

$$\int \Phi(x)\mathsf{d}x = x\Phi(x) + \phi(x) \,, \tag{13}$$

where $\phi(x)$ is the corresponding standard normal PDF. The generic forms of the normal PDF and CDF are given by:

$$\mathsf{f}(x|\mu,\sigma) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right) \,, \tag{14}$$

$$\mathsf{F}(x|\mu,\sigma) = \Phi\left(\frac{x-\mu}{\sigma}\right) \,. \tag{15}$$

Utilizing Eq. 13 together with Eq. 14 and Eq. 15, we can integrate by substitution to obtain the following probability densities:

$$\mathsf{p}(a|\mu,\sigma) = \frac{\mathsf{F}(a|\mu,\sigma)}{\left((x-\mu)\mathsf{F}(x|\mu,\sigma) + \sigma^2\mathsf{f}(x|\mu,\sigma)\right)\Big|_{-\tau}^{\tau}} \tag{16}$$

$$\mathsf{p}(r|\mu,\sigma) = \frac{1-\mathsf{F}(r|\mu,\sigma)}{\left((x-\mu)[1-\mathsf{F}(x|\mu,\sigma)] - \sigma^2\mathsf{f}(x|\mu,\sigma)\right)\Big|_{-\tau}^{\tau}} \tag{17}$$

At this point, we are able to compute the likelihood in Eq. 10 for a given training data set.

Now, an appropriate prior density $\mathsf{p}(\mu,\sigma)$ has to be chosen and its parameters, the *hyperparameters*, have to be set to parametrize the prior distribution. This is the central step, which allows us to incorporate our prior knowledge about the critical gaps. In this work, we choose a 2-dimensional Gaussian distribution as prior:

$$\mathsf{p}(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^2 \det \mathbf{S}}} \cdot e^{-\frac{1}{2}(\mathbf{x}-\mathbf{m})^{\mathsf{T}}\mathbf{S}^{-1}(\mathbf{x}-\mathbf{m})} \,, \tag{18}$$

with $\mathbf{x} = (\mu,\sigma)^{\mathsf{T}}$. The mean of the prior distribution and hence for μ and σ is $\mathbf{m} = (m_\mu, m_\sigma)^{\mathsf{T}}$. The covariance matrix $\mathbf{S}$ contains the variances of μ and σ. Here, we assume that the cross-terms are zero; thus the covariance matrix can be defined as:

$$\mathbf{S} = \begin{pmatrix} \mathrm{s}_\mu^2 & 0 \\ 0 & \mathrm{s}_\sigma^2 \end{pmatrix} \,. \tag{19}$$

Given the set $\mathcal{J}$, containing the critical gaps of J drivers, we calculate the prior distribution for the estimation of the current driver's critical gap $g \in \mathcal{J}$, based on the critical gaps of all other drivers in the set. Thus, at first, we calculate the $J-1$ critical gaps μ_i and corresponding standard deviations σ_i for each of those drivers with the maximum likelihood method from Eq. 10 and compute the means via

$$m_\mu = \frac{1}{J-1}\sum_{i\in\mathcal{J}\setminus\{g\}} \mu_i \,, \quad m_\sigma = \frac{1}{J-1}\sum_{i\in\mathcal{J}\setminus\{g\}} \sigma_i \,. \tag{20}$$

With the calculated means of μ and σ we can calculate the variances using

$$\mathrm{s}_\mu^2 = \frac{1}{J-1}\sum_{i\in\mathcal{J}\setminus\{g\}} (\mu_i - m_\mu)^2 \,, \tag{21}$$

$$\mathrm{s}_\sigma^2 = \frac{1}{J-1}\sum_{i\in\mathcal{J}\setminus\{g\}} (\sigma_i - m_\sigma)^2 \,. \tag{22}$$

The final step to obtain a maximum a posteriori estimate is to incorporate the prior distribution of our parameters $\theta = (\mu,\sigma)$ as shown in Eq. 6 but neglecting the marginal distribution $p(\mathbf{y})$ in the denominator, since it does not depend on μ or σ:

$$(\mu_c, \sigma_c) = \arg\max_{\mu,\sigma} \mathsf{p}(\mathbf{y}|\mu,\sigma)\mathsf{p}(\mu,\sigma) \,. \tag{23}$$

Finally, the critical gap μ_c is converted back into linear scale as in Eq. 5

III. Experimental Setup

To collect training data and evaluate our method, we have built an inner-city intersection scenario in a driving simulator, where each participant in the experiment drove several rounds. We have set up a road layout with four consecutive T-intersections, which allows the participants to drive through an infinite number of successive intersections. The buildings have been placed in such a way that the participants are forced to stop at the intersection in order to see if there are cars approaching from the left or right. The layout of all intersections is identical, as the goal was to observe the behavior of an individual driver in identical situations. The only difference between the intersections is the arrangement of the buildings. Nevertheless, the visibility of the driver onto the crossing traffic is identical for the different intersections.



Fig. 1. Low-fidelity driving simulator setup.

A. Apparatus

As simulation software, we have used IPG CarMaker 4.5, which is a simulation environment mainly intended for the physical simulation of cars. As one can see in Figure 1 there are three displays used for the simulation view. A Logitech G27 steering wheel and corresponding pedals were used for control. The participant's line of sight has to be orthogonal to every simulator screen because the CarMaker 4.5 software has no functionality to adapt the camera view to a different line of sight of the viewing participant. Nevertheless, the positions of the screens remained the same for all participants.

B. Participants

There were 9 participants (two female) with a mean age of 32 years and a standard deviation of 4 years. Their driving experience ranged from 3 to 18 years, while their traveled distance per year lay between $7.000\,\mathrm{km}$ and $40.000\,\mathrm{km}$.

C. Procedure

Before the core part of the experiment, each participant had a training session in order to obtain simulator driving practice and to become familiar with the environment. The road layout contained some initial curves followed by the intersections of the actual test course. This session took approximately 10 minutes.

In the core experiment, each participant had to drive 3 scenarios with 16 intersections each. The distribution of the presented gaps was different for each intersection and simulated medium to high traffic density from left and right. The gap sizes presented to the participants were between two and eight seconds. All traffic cars were driving at $50\,\mathrm{km\,h^{-1}}$. Hence the scenario simulated a typical urban traffic. Each participant drove the same scenario, i. e. the traffic was controlled in a fully deterministic way. Variations occurred as participants approached the intersections at different speeds, stopped at different points in front of the intersections and decided to take different gaps. After each of the 3 scenarios a short break took place. With this setting we could obtain a maximum number of 48 (largest rejected gap, accepted gap)-tuples per participant from the recordings. To ensure that no habituation effects to the observed gap sizes could occur, different gap sizes were used in the three scenarios.

After the experiment, the critical gap and the corresponding variance (Eq. 5) were calculated.

IV. Analysis

The results of our previous study [8] have shown that there are clear differences in the gap sizes different drivers accept when repeatedly driving identical intersections. In addition, we have shown that the use of driver-dependent (personalized) critical gaps for the prediction of a driver's turning behavior, can reduce the error rate by $13\,\%$[1] relative from $14.3\,\%$ to $12.4\,\%$ compared to the use of driver-independent critical gaps if our ML method is used.

In this paper we evaluate our novel MAP method for driver-specific critical gap estimation, in particular when only few training samples are available. To do so, we have implemented a subset cross validation. That means, we chose ten subset sizes (1, 2, 3, 4, 5, 10, 15, 20, 30) and drew a random subset of training samples for every subset category from the driver's data set and used the rest of this data set of the driver for prediction. For a better comparison, only the largest rejected gaps and corresponding accepted gaps were used for training and prediction for all methods, even though our method could use all rejected gaps. This procedure was repeated 30 times for every subset size and every driver. Finally, the mean error rate was calculated for every subset size. As depicted in Fig. 2, all three methods achieve a minimum error rate of around $12\,\%$ with 30 training samples.

The mean error rate of Troutbeck's method and the ML method increases as soon as subset sizes with fewer than 15 samples are used. Troutbeck's method achieves lower mean error rates than the ML method if subset sizes smaller than 15 samples are used. It achieves a maximum mean error rate

[1]The error rates are slightly different compared to [8] because we have updated our evaluation process.

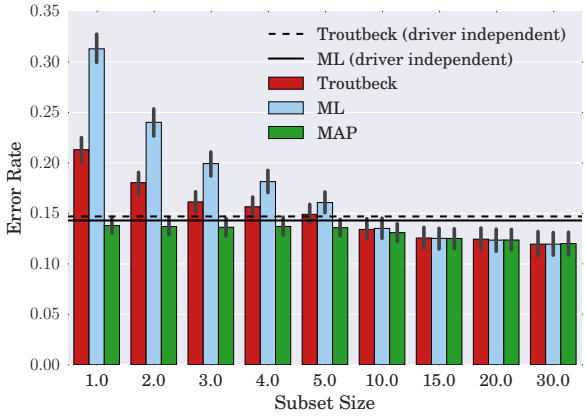

Fig. 2. The mean error rate for every cross-validation subset size using Troutbeck's method, our ML and MAP method are depicted by the colored bars. The vertical black lines on every bar show the variances. The mean error rate obtained by using driver-independent critical gaps is depicted by the horizontal lines.

of $21.3\,\%$ for a training subset with 1 sample. In contrast, we see that the mean error rate of the MAP method does not rise above $13.8\,\%$ in any case. The MAP approach thus achieves a $35.2\,\%$ lower relative error rate compared to Troutbeck's method in the case of 1 training sample. Note, that the MAP approach not only yields lower mean error rates but also lower variances of error rates with small subset sizes compared to the other approaches. The variances are indicated by the vertical black lines on top of the colored bars. Furthermore, we show the mean error rates obtained by using the driver-independent critical gaps of Troutbeck and our ML approach with two horizontal black lines. We can see that the MAP approach already leads to a lower error rate than the driver-independent approaches when a subset size of 1 sample is used. In this case, the difference between these error rates is small. However, a further advantage of the MAP approach is that we are not confronted with the decision at which training data set size we should switch from the driver-independent to the driver dependent critical gap. As we can see, for a subset size of 5 samples the other methods still lead to a higher error rate compared to the use of the driver-independent critical gap. In this case, the MAP methods leads to an error rate of $13.7\,\%$, which is still a relative improvement of $4.2\,\%$ compared to the error rate of the driver-independent model.

V. Conclusion

After defining a new maximum likelihood method in our last work, we have now successfully extended it to a maximum a posteriori method, which is capable of estimating driver-specific critical gaps, even if only a few training observations are used. Subsequently, it was used to predict the left-turn behavior of a single driver with a maximum prediction error of $13.8\,\%$ if only one training sample is used. This is a relative improvement of $35.2\,\%$ compared to Troutbeck's method in this case. In addition, using the MAP approach, the prediction error is always lower than the error rate if a driver-independent critical gap is used for prediction. The latter makes it unnecessary to use a combination of a driver-independent and driver-specific critical gap estimation. A combination is not trivial, since it would require using an additional classifier, which is capable of making a perfect decision between those two critical gaps.

In contrast to Troutbeck's method, the proposed MAP model uses a properly defined likelihood of the observed accepted and rejected gaps, which does not assume homogenous drivers and thus is a more realistic model. In addition, the MAP approach is well suited for the incorporation of further prior information about factors influencing the driver's behavior.

The currently chosen prior follows a 2-dimensional normal distribution, thus the Gaussian prior allows σ to take negative values. This should not be possible, since σ denotes the standard deviation, which can not take negative values. But as the results show, using the Gaussian prior for the MAP estimation already yields a good prediction performance. However, in order to improve the performance further, in the future we will investigate other suitable priors, which only allow non-negative values for sigma.

As a next step, we will investigate which additional prior information should be incorporated, to estimate the individual driver's behavior even better. For example, according to the method of [17], we plan to include information about the waiting time of the driver. Thus, the critical gap estimate will not remain fixed over time at each intersection, but it will rather incorporate the number of rejected gaps and their respective sizes in a probabilistic way. In future work, it will also be interesting to model the driving behavior for other intersection layouts, e.g. multi-lane intersections, other light conditions and other weather conditions. This would be also incorporated as prior information into the estimation.

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