Honda Research Institute EU
Honda Research Institute Europe GmbH
https://www.honda-ri.de/

A Maximum Likelihood Method for Driver-Specific Critical-Gap Estimation

Dennis Orth, Milton Orlando Sarria Paja, Kersten Schaller, Andreas Pech, Dorothea Kolossa, Martin Ernst Heckmann

2016

Preprint:

This is an accepted article published in 2017 IEEE Intelligent Vehicles Symposium (IV). The final authenticated version is available online at: https://doi.org/10.1109/IVS.2017.7995776

© 2017 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

DOI: https://doi.org/10.1109/IVS.2017.7995776

A Maximum Likelihood Method for Driver-Specific Critical-Gap Estimation

Dennis Orth[1,4], Dorothea Kolossa[1], Milton Sarria Paja[2,4],
Kersten Schaller[3,4], Andreas Pech[3], Martin Heckmann[4]

***Abstract*— In this work, we introduce a maximum likelihood (ML) method to estimate the smallest accepted gap of a specific driver, the so-called critical gap. Previous methods, like Troutbeck's or Raff's method, are well known and widely used but require a consistently behaving driver, which is usually not met. The methods will be investigated for the personalization of an intersection assistant, which we are currently developing. We start with the hypothesis that the size that constitutes an acceptable gap is driver-dependent. To test this hypothesis, we have performed a simulator study with 9 participants. The results reveal that there is a significant inter-individual difference in the critical gap between the drivers, as postulated. Next we investigate how well we can predict which gap the driver will take when we use the previously estimated personalized critical gap versus a driver independent one. Using the personalized gap compared to a driver independent one reduces the error from $11.8\,\%$ to $9.8\,\%$. These results are a clear indication that personalization can be utilized to increase the effectiveness and usability of an intersection assistant.**

I. INTRODUCTION

In the automotive context, personalization is a relatively recent topic [1]. Only a few years ago, the necessary sensors and computing devices to observe and store the user's behavior to subsequently learn a user model were only available in cars to a very limited extent. Early work by Daimler-Benz concentrated on personalized route recommendation systems [2]. Since then, car makers have largely extended these approaches to the entire area of in-vehicle infotainment systems and HVAC (Heating, Ventilation and Air Conditioning). At the CES 2014, Mercedes [3] presented a prototype called "Predictive User Experience", an interactive infotainment system, which learns user behaviors and predicts future choices. Jaguar-Landrover announced that they are working on a self-learning car [4], which not only learns the driver's preferences with respect to infotainment but also will adapt its Adaptive Cruise Control (ACC) to the individual driver. The expectation is that such a personalization will increase the effectiveness and usability of these systems and thereby reduce the distraction of the driver and lead to better acceptance of the systems [5]. In the realm of Advanced Driver Assistance Systems (ADAS) the personalization almost exclusively focuses on a personalized ACC. Methods to estimate the necessary driver models have e.g. been proposed in [6]. In [7], such a personalized ACC, which adapts in real-time to the driver, has been implemented in a car.

In this paper, we extend the idea of a personalized ADAS to an intersection assistant that we are currently developing. This intersection assistant will support the drivers when turning from a subordinate road onto a superordinate road by informing them of gaps of sufficient size in the crossing traffic to perform the turn [8]. Our assumption is that the appropriate gap size recommended by the system should depend on the individual driver. Therefore, we have performed a driving simulator study where we observed participants' gap choices when turning at an intersection. Due to simulator sickness and exhaustion the participants suffer from, one usually can not obtain an arbitrary amount of data in such experiments. In addition, it is necessary to be able to work with data of individual drivers in order to investigate the personalization aspect mentioned above.

Thus, the available amount of data is naturally quite low, motivating the use of Bayesian inference, which would allow for the incorporation of prior models in the estimation process. However, the standard approach to critical gap estimation by Troutbeck [9] is not per se a truly probabilistic model, as it does not work with a likelihood function that is well-defined under all circumstances. Therefore, we propose in the following a new likelihood function, which can be used to replace the state-of-the-art critical gap estimation method in the framework of Bayesian inference. After the definition of this new method, we analyze the results of our new and Troutbeck's method with respect to inter-individual differences in the gap choices. Finally, we will show that a prediction of the driver turn behavior will be more accurate if we use a personalized critical gap instead of a driver-independent critical gap.

In the next section we will first introduce the intersection assistant in more detail. Next we will outline the two methods that have been used to estimate the individual drivers' gap choices and a set of experiments for their validation. In the last part of the paper, we will present the experimental results and discuss them, drawing conclusions for future work on the personalization of intersection assistance systems.

A. An on demand intersection assistant

Despite the general usefulness of ADAS, there are frequent situations where these systems produce false alarms. Such

[1]Institute of Communication Acoustics, Ruhr-Universität Bochum, Bochum, Germany {dennis.orth,dorothea.kolossa}@rub.de
[2]INRS-EMT, Canada and Santiago de Cali University, Colombia sarria@emt.inrs.ca
[3]Frankfurt University of Applied Sciences, Frankfurt, Germany kschalle@stud.fh-frankfurt.de, pech@fb2.fra-uas.de
[4]Honda Research Institute Europe GmbH, Offenbach am Main, Germany martin.heckmann@honda-ri.de

false alarms can distract and annoy the driver. This can ultimately lead to a deactivation of the system by the driver and hence eliminate the possible safety benefits of the system. As a countermeasure, different authors have proposed systems which monitor the driver's behavior and only present relevant warnings [10]. However, the estimation of the driver's awareness is notoriously difficult. As an alternative approach, we have recently proposed an *Assistance on Demand* (AoD) concept [11], [8]. Following the on-demand idea, the system is not always active, but is only activated by the drivers when they desire support in the current situation. System and driver interact with each other via speech. Hence the system acts like a co-pilot, with which the driver can communicate.

We currently apply the AoD concept to a speech-based personalized intersection assistant which supports the driver in an inner-city scenario. The scenario we have chosen is turning left from a subordinate road onto a superordinate road at an unsignalized intersection. This has been reported to be one of the most challenging tasks for drivers [12], [13]. Upon arriving at the intersection, the driver can delegate the task of monitoring the right side traffic to the system via a speech command, e.g. "please watch right." The system will then give feedback to the driver upon suitable gaps in the traffic, e.g. via the speech output "gap after next car."

In [14], a very similar approach was applied for the use case "turning left at a rural intersection with oncoming traffic". In contrast to [14], our AoD concept puts more emphasis on the collaborative sharing of tasks between the driver and the system.

In a driving simulator study we have previously performed, the AoD concept was clearly preferred to manual driving, reduced the workload and facilitated the monitoring of the traffic [11]. For this study, we have suggested gaps to the drivers based on values reported in [15]. These values were specific for the type of intersection and averaged over a large set of drivers. Despite the high acceptance of the AoD system by the participants of our previous study, many mentioned that the gaps suggested by the system did not fit to their own driving behavior. From this, we conclude that an adaptation of the gaps suggested by the system to the individual driver should lead to an increase of system acceptance.

II. CRITICAL GAP ESTIMATION

The so-called *critical gap* is a parameter which is often used to calculate the capacity and delay of a minor road, especially for unsignalized T-intersections. It signifies how large a gap minimally has to be for the driver to accept it and take the turn, thus vacating the minor road. Previously, this value was primarily used to measure the capacity of a specific intersection and hence was calculated from a large pool of drivers all passing this intersection [16]. In our case, however, we use it as a criterion to quantify the behavior of one individual driver. To facilitate this, for the moment, we assume that the layout of the intersections and the traffic density at the intersections are all identical.

In the next section at first we describe a critical gap estimation method which is well known in the field of intersection theory and widely used [16]. We will show that this method is not well suited for naturalistic driving data, since it makes wrong assumptions on the individual driver behavior. After that analysis, we propose our maximum likelihood method for estimating critical gaps and go into the differences of both methods.

TABLE I
NOTATION FOR THE MAXIMUM LIKELIHOOD EQUATIONS.

Symbol	*Description*
a_i	logarithm of gap accepted by the driver at intersection i
r_i	logarithm of the largest gap rejected by the driver at intersection i
μ_c	mean of the distribution of the logarithms of the individual driver's critical gaps
σ_c^2	variance of the distribution of the logarithm of the individual driver's critical gaps
$\mathsf{f}_c()$	probability density function for the normal distribution of the critical gap
$\mathsf{F}_c()$	cumulative distribution function for the normal distribution of the critical gap

A. Troutbeck's estimation method

Troutbeck assumes that a driver's critical gap lies somewhere between the largest rejected gap and the gap accepted by the driver [9],[17]. To model the behavior of the drivers, [17] uses a log-normal distribution for the critical gaps. The transformation of the observed gaps in the logarithmic domain allows using the normal distribution for the core computations. The basic notations for the method used in the following are given in Table I. In [18], Brilon recently motivated the approach of Troutbeck by analogies to a classical problem in survival theory. More concretely, he argues that it corresponds to the failure time estimation of interval censored data [18], [19]. The critical gap, i. e. the failure event, can not be observed directly. It can only be observed that it lies in the semi-closed interval $(r_i, a_i]$ when r_i indicates the logarithm of the largest gap rejected by the driver at intersection i and a_i the logarithm of the gap accepted by the driver at intersection i. The hidden parameters μ and σ of the distribution of μ_c can then be found by maximizing the log-likelihood:

$$\sum_{i=1}^{N} \ln \mathsf{p}(r_i < \mu_c \leq a_i | \mu, \sigma) \;, \tag{1}$$

for all observations N. Eq. 1 can then be written as [18]:

$$\sum_{i=1}^{N} \ln[\mathsf{F}_c(a_i) - \mathsf{F}_c(r_i)] \;, \tag{2}$$

where $\mathsf{F}_c()$ denotes the cumulative distribution function of the normal distribution with parameters μ and σ. Hence the parameters can be determined via:

$$(\mu_c, \sigma_c) = \arg\max_{\mu,\sigma} \sum_{i=1}^{N} \ln[\mathsf{F}_c(a_i) - \mathsf{F}_c(r_i)] \tag{3}$$

As a final step, the critical gap t_c and variance s^2 in linear scale are computed according to

$$t_c = \mathrm{e}^{\mu_c + 0.5\sigma_c^2}, \qquad s^2 = t_c^2(\mathrm{e}^{\sigma_c^2} - 1). \tag{4}$$

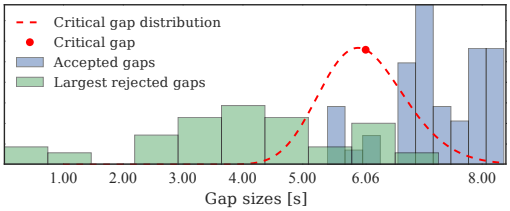

Fig. 1. Histogram of largest rejected gaps, accepted gaps and the estimated critical gap.

In contrast to [17], in our case, the index i in the equations above is not used to indicate a tuple (r_i, a_i) of the i-th driver but of the i-th intersection.

Figure 1 shows an example histogram of both largest rejected gaps and accepted gaps. The red dotted line indicates the log-normal distribution, which is derived from this observations via the previously described algorithm. The corresponding t_c is marked with a thick red point on the curve.

The failure time estimation of interval censored data requires that $r_i < \mu_c \leq a_i$. Consequently, the method of Troutbeck assumes a consistent and homogeneous driver. This means that the largest rejected gap r_i recorded at intersection pass i must always be smaller than the corresponding accepted gap a_i. This is however often not fulfilled. In cases where the assumption is not met, the original method in [17] diminished the value of the accepted gap by a small number and set this as the value for the largest rejected gap. We obtained better results by using the next smaller rejected gap from intersection pass i instead. If the next smaller gap is still larger than the accepted gap, we repeat this step. If we do not find a rejected gap smaller than the accepted gap, we discard the measurement for intersection i. We assume that in this way we obtain more meaningful observations from the recorded intersections. Yet both heuristics to conciliate the observations with the underlying probabilistic model require a modification of the observations. The consequences of this on the validity of the derived probabilistic inferences are difficult to judge.

B. Towards a critical gap likelihood

In our experiments we found that Troutbeck's assumption of a consistently behaving driver does not hold in the inner-city left turn scenario. Thus, the largest rejected gap recorded for a driver at an intersection, was often larger than the corresponding accepted gap. As described before, in these cases the recorded data had to be modified or actually to be discarded, to fit Troutbeck's assumptions. Thus, there are rejected gaps and accepted gaps, which potentially could serve as training data but are not used. So, there would be the wish to

1) be able to use all recorded data for training and
2) to have a mathematically properly defined likelihood for all observable accepted and rejected gaps in the sense of Bayesian statistics.

Especially the second point will be important if we want to use the likelihood in the context of Bayesian inference [20]. This would enable us to incorporate prior knowledge about the density of critical gaps in a structured way. A proper likelihood needs to be a joint density function of all observations, so

$$\mathsf{p}(\mathbf{Y}|\theta) = \mathsf{p}(\mathbf{y}_1, ..., \mathbf{y}_M|\theta), \tag{5}$$

where $\mathbf{Y}$ are the observations composed of all $\mathbf{y}_i$, the feature vectors with dimension D_o. Here, θ is the parameter vector with dimension D_θ, we are attempting to find.

In the following a new likelihood is described, which fulfills our previously defined targets and does not require that the accepted gap has to be higher than the largest rejected gap. Suppose that one has a data vector $\mathbf{y}$, which consists of all accepted gaps $a_1, ..., a_K$ and all rejected gaps $r_1, ..., r_L$ extracted from the recorded intersections. For this data vector we can define the likelihood $\mathsf{p}(\mathbf{y}|\mu, \sigma)$, that is the probability of seeing the recorded data given a critical gap μ and its standard deviation σ:

$$\mathsf{p}(\mathbf{y}|\mu, \sigma) = \mathsf{p}(a_1, ..., a_K, r_1, ...r_L|\mu, \sigma)\ . \tag{6}$$

We assume that the accepted gaps a_i and rejected gaps r_j are conditionally independent given the critical gap μ and its variance σ. This means that one can factorize the joint distribution from Eq. 6 to yield a product of two joint distributions:

$$\mathsf{p}(\mathbf{y}|\mu, \sigma) = \mathsf{p}(a_1, ..., a_K|\mu, \sigma)\mathsf{p}(r_1, ...r_L|\mu, \sigma)\ . \tag{7}$$

Furthermore, we assume conditional independence of the accepted gaps and of the rejected gaps among each other. So, finally we can write the likelihood as:

$$\mathsf{p}(\mathbf{y}|\mu, \sigma) = \prod_{k=1}^{K} \mathsf{p}(a_k|\mu, \sigma) \prod_{l=1}^{L} \mathsf{p}(r_l|\mu, \sigma)\ . \tag{8}$$

Hence, in Eq. 8 we have two distributions we want to determine: $\mathsf{p}(a|\mu, \sigma)$, which describes the probability of seeing an accepted gap a given a specific critical gap μ with standard deviation σ and $\mathsf{p}(r|\mu, \sigma)$, which describes the probability of seeing a rejected gap r given the same parameters. For this purpose, we follow Troutbeck and assume a log-normal distribution for the accepted and rejected gaps. Hence a_k and r_l describe the logarithm of the accepted and rejected gaps. Consequently we also use the cumulative distribution function (CDF) $\mathsf{F}(x)$ with the parameters μ, σ to model the accepted and rejected gaps. This implies that $\mathsf{F}(x)$ serves as the probability density function (PDF) describing the likelihood $\mathsf{p}(a|\mu, \sigma)$. Based on equivalent reasoning, $1-\mathsf{F}(x)$ describe the probability density function for $\mathsf{p}(r|\mu, \sigma)$.

To arrive at valid probability density functions, $\mathsf{p}(r|\mu, \sigma)$ and $\mathsf{p}(a|\mu, \sigma)$ must be normalized. The integrals of $\mathsf{F}(x)$ and of $1 - \mathsf{F}(x)$ over x diverge if we choose an unbounded integral. To circumvent this, we assume that the gaps we observe fall into the range $[\exp(-\tau), \exp(\tau)]$, which corresponds to $[-\tau, \tau]$ in the logarithmic domain. The probability density functions for the accepted gaps and the rejected gaps hence

take the following form:

$$\mathsf{p}(a|\mu,\sigma) = \frac{\mathsf{F}(a|\mu,\sigma)}{\int_{-\tau}^{\tau} \mathsf{F}(x|\mu,\sigma)\mathrm{d}x} \,, \tag{9}$$

$$\mathsf{p}(r|\mu,\sigma) = \frac{1-\mathsf{F}(r|\mu,\sigma)}{\int_{-\tau}^{\tau} 1-\mathsf{F}(x|\mu,\sigma)\mathrm{d}x} \,. \tag{10}$$

To solve the integral over $\mathsf{F}(x|\mu,\sigma)$ and $1-\mathsf{F}(x|\mu,\sigma)$, we use the antiderivative of the standard normal CDF $\Phi(x)$ with $\mu = 0, \sigma = 1$, which is given by:

$$\int \Phi(x)\mathrm{d}x = x\Phi(x) + \phi(x) \,, \tag{11}$$

where $\phi(x)$ is the corresponding standard normal PDF. The generic forms of the normal PDF and CDF are given by:

$$\mathsf{f}(x|\mu,\sigma) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right) \,, \tag{12}$$

$$\mathsf{F}(x|\mu,\sigma) = \Phi\left(\frac{x-\mu}{\sigma}\right) \,. \tag{13}$$

Utilizing Eq. 11 together with Eq. 12 and Eq. 13, we can integrate by substitution to obtain the following probability densities:

$$\mathsf{p}(a|\mu,\sigma) = \frac{\mathsf{F}(a|\mu,\sigma)}{\left((x-\mu)\mathsf{F}(x|\mu,\sigma) + \sigma^2\mathsf{f}(x|\mu,\sigma)\right)\Big|_{-\tau}^{\tau}} \tag{14}$$

$$\mathsf{p}(r|\mu,\sigma) = \frac{1-\mathsf{F}(r|\mu,\sigma)}{\left((x-\mu)[1-\mathsf{F}(x|\mu,\sigma)] - \sigma^2\mathsf{f}(x|\mu,\sigma)\right)\Big|_{-\tau}^{\tau}} \tag{15}$$

At this point, we are able to compute the likelihood in Eq. 8 for a given training data set. Thus, we obatin the critical gap by maximizing this likelihood as follows:

$$(\mu_c, \sigma_c) = \arg\max_{\mu,\sigma} \mathsf{p}(\mathbf{y}|\mu,\sigma) \,. \tag{16}$$

III. Experimental Setup

To test the hypothesis that there are differences in the critical gaps different drivers accept, and to compare both critical gap estimation methods, we have built an inner-city intersection scenario in a driving simulator. We have set up a road layout with four consecutive T-intersections, which allows the participants to drive through an infinite number of successive intersections. The buildings have been placed in such a way that the participants are forced to stop at the intersection in order to see if there are cars approaching from the left or right. The layout of all intersections is identical, as the goal was to observe the behavior of an individual driver in identical situations. The only difference between the intersections is the arrangement of the buildings. Nevertheless, the visibility of the driver onto the crossing traffic is identical for the different intersections.

A. Apparatus

As simulation software, we have used IPG CarMaker 4.5, which is a simulation environment mainly intended for the physical simulation of cars. The software runs on a computer with an NVIDIA Geforce GTX 980 graphics card, an Intel(R) Core(TM) i5-6600 CPU @ 3.30 GHz and 16 GiB RAM. As demonstrated in Figure 2 there are three displays used for the simulation view: Two 24 inch displays (1920×1200 pixels) and one 46 inch display (1360×768 pixels), where the 3D view of the middle screen is scaled down to fit in size to the other displays. A Logitech G27 steering wheel and corresponding pedals were used. But there was no force feedback support for this software in conjunction with the Linux operating system. The participant's line of sight has to be orthogonal to every simulator screen because the CarMaker 4.5 software has no functionality to adapt the camera view to a different line of sight of the viewing participant. Nevertheless, the positions of the screens remained the same for all participants. Figure 2 gives an overview of this setup and also shows the lighting conditions under which the entire test procedure was conducted.



Fig. 2. Low-fidelity driving simulator setup.

B. Participants

There were 9 participants (two female) with a mean age of 32 years and a standard deviation of 4 years. Their driving experience ranged from 3 to 18 years, while their traveled distance per year lay between $7.000\,\mathrm{km}$ and $40.000\,\mathrm{km}$.

C. Procedure

Before the core part of the experiment, each participant had a training session in order to obtain simulator driving practice and to become familiar with the environment. The road layout contained some initial curves followed by the intersections of the actual test course. This session took approximately 10 minutes.

In the core experiment, each participant had to drive 3 scenarios with 16 intersections each. The distribution of the presented gaps was different for each intersection and simulated medium to high traffic density from left and right. The gap sizes presented to the participants were between two and eight seconds. All traffic cars were driving at $50\,\mathrm{km\,h^{-1}}$. Hence the scenario simulated a typical urban traffic. Each participant drove the same scenario, i. e. the traffic was controlled in a fully deterministic way. Variations occurred as participants approached the intersections at different speeds, stopped at different points in front of the intersections and decided to take different gaps. After each of the 3 scenarios a short break took place. With this setting we could obtain a

TABLE II
ANALYSIS OF VARIANCE

Method	ANOVA
Troutbeck	$F(8,385) = 25.988$, $p < 0.001$
New method	$F(8,385) = 19.381$, $p < 0.001$

maximum number of 48 (largest rejected gap, accepted gap)-tuples per participant from the recordings. To ensure that no habituation effects to the observed gap sizes could occur, different gap sizes were used in the three scenarios.

After the experiment, the critical gap and the corresponding variance (Eq. 4) were calculated. For a better comparison of both critical gap estimation methods, we only used the largest rejected gap and accepted gap of every intersection, even if our method is suitable for incorporating all rejected gaps from an intersection. Typically, as described in Sec. II, it was not possible to use the gap recordings from all 48 intersections of each participant. The average number of used intersections for a participant was 44.9 with a standard deviation of 2.9.

IV. RESULTS

In this section, we analyze the inter-individual differences of the estimated critical gaps and we determine the estimation accuracy obtained by Troutbeck's method and by our suggested approach. The accuracy is measured by predicting whether a driver will take a gap and by comparing the prediction to the actual behavior.

A. Analysis of estimated critical gaps

Figure 3 shows the estimated critical gap t_c and the corresponding standard deviation s for every participant and both methods. The red and yellow line show the estimated critical gap when jointly using the recordings of all participants. One can see that the results of both methods are very similar. The standard deviations of our method are always slightly larger than the corresponding standard deviations of Troutbeck's method.

Looking at the individual critical gaps, one can see that, for both methods, there are clear differences between the critical gaps of the nine participants. A one-way ANOVA (Table II) of the results of both methods has also confirmed that there is a significant inter-individual difference in the critical gaps of the different participants. Note that the explanatory power of the ANOVA result is reduced due to the fact that we are forced to use the estimated means and variances of the critical gaps, since we don't possess any observations of the latter. More conclusive evidence is obtained from the prediction process in the next section.

B. Estimation accuracy

One way of evaluating the accuracy of an estimated critical gap is by using it to predict the behavior of a specific driver. Consistent drivers will reject every gap smaller than their critical gap and accept all the others. In order to investigate whether a personalized critical gap leads to a better prediction accuracy than a driver-independent critical gap, we have computed the following two types of cross-validations for both methods:

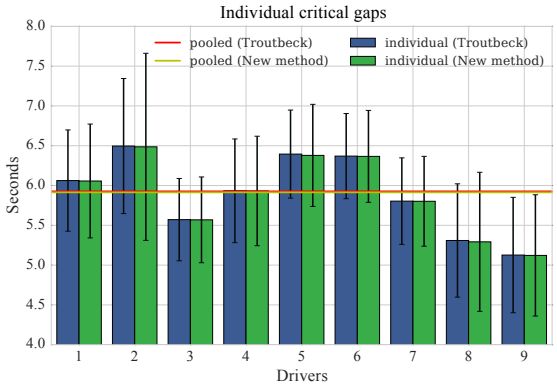

Fig. 3. Critical gap for each participant (blue and green bars), standard deviation of critical gap (black lines) and critical gap using all participants for estimation (red and yellow line).

For the Leave-One-Intersection-Out (LOIO) cross validation, one intersection is left out from the data set of driver i, and the rest of the intersections of data set i are used for the estimation of the individual critical gap. Then, a prediction is done for the held-out intersection. This procedure is performed for every intersection in the data set of driver i. At the end, the true positives, true negatives, false positives and false negatives are extracted to compute the overall accuracy for driver i. We repeat this evaluation for every driver to obtain nine different accuracies.

For the Leave-One-Driver-Out (LODO) cross validation, all intersections of all drivers except for driver i are used to estimate a critical gap. Afterwards, all intersections of driver i are used for prediction and an overall accuracy for this driver is computed. This cross validation can be computed for every driver, as well, leading to nine different accuracies.

The results are summarized in Table III. Evidently, in the case of Troutbeck's method, a $20.1\,\%$ lower error rate was achieved with the personalized critical gap (LOIO) compared to the driver-independent critical gap. The error rates of both critical gap estimation methods are very similar, but our suggested approach achieves a $4.1\,\%$ lower error rate for the driver-independent critical gap. Since the personalized critical gaps of both methods are almost the same (Fig. 3), there is no difference in the error rates for the LOIO cases. Both methods achieve an error rate of $9.8\,\%$, which is sufficient for the application in the suggested intersection assistant. However, we expect an extension of the proposed method to a full maximum-a-posteriori estimation, incorporating a prior as mentioned in Sec II-B, to lead to better accuracies than Troutbeck's method, especially if critical gaps need to be estimated from a small initial amount of data.

V. CONCLUSIONS

The results of the current study show that there are clear differences in the gap sizes different drivers accept when repeatedly driving identical intersections. They varied between $5.1\,\mathrm{s}$ and $6.5\,\mathrm{s}$. In our scenario, where the traffic cars

TABLE III

ERROR RATES OF BOTH CRITICAL GAP ESTIMATION METHODS

Method	Error rates		Rel. Improvement
	driver-independent	driver-dependent	
Troutbeck	12.3 %	9.8 %	20.1 %
New method	11.8 %	9.8 %	16.7 %
Rel. Improvement	4.1 %	0 %	

were driving at $50\,\mathrm{km\,h^{-1}}$, this corresponds to a difference of $19\,\mathrm{m}$ between the driver with the smallest critical gap to the one with the largest critical gap. In addition, we could show that a subsequent use of the personalized critical gaps for the prediction of the driver turn behavior could reduce the error rate by $16.7\,\%$ relative from $11.8\,\%$ to $9.8\,\%$ compared to the use of driver-independent critical gaps. From this we conclude that it can be expected that a personalization of the gap suggestions to the individual driver is beneficial for our targeted on demand intersection assistant.

Furthermore, we could define a new critical gap estimation method, which uses a proper likelihood function in the sense of Bayesian statistics and does not require a consistently behaving driver. We are able to extend the method to a maximum a posteriori estimation, which allows one to incorporate prior knowledge into the process. The latter will enable one to estimate valid and sensible critical gaps with a low amount of data, which will be needed as soon as we want to use personalized critical gaps already when we have only observed few driving maneuvers of a specific driver [21].

We are currently continuing the analysis of the data with the goal of differentiating the gaps accepted from the left and right. So far, we have not investigated how different traffic layouts and visibility influence the gap acceptance. We expect that the values of the accepted gaps will change depending on the situation but that the relative differences, we observed between the participants, will not change. The next step will then be to investigate how the personalization of the suggested gaps influences the acceptance of our Assistance on Demand concept. This will be a significant step towards personalized ADAS. Our ultimate goal is then to build an online demonstrator and to evaluate our system in real traffic.

ACKNOWLEDGMENT

We would like to thank Heiko Wersing, Martina Hasenjäger and Tobias Rodemann for fruitful discussions. Many thanks go to our participants for their patience and effort.

REFERENCES

[1] M. Hasenjäger, "Personalization in the automotive context," Honda Research Institute Europe GmbH, Tech. Rep., 2016.

[2] S. Rogers and P. Langley, "Personalized driving route recommendations," in *Proc. of the AAAI Workshop on Recommender Systems*, 1998, pp. 96–100.

[3] Mercedes-Benz, "Interactive prototype: Predictive user experience," Presentation at CES, 2014. [Online]. Available: http://ces.mercedes-pressevents.com/content_predictive.php

[4] Jaguar-Landrover, "Jaguar land rover develops the self-learning intelligent car of the future," July 2014. [Online]. Available: http://newsroom.jaguarlandrover.com/en-in/jlr-corp/news/2014/07/jlr_self_learning_car_090714/

[5] N. Walter, T. Altmüller, and K. Bengler, "Concept of a reference architecture for an extendable in-vehicle adaptive recommendation service," ser. AutomotiveUI '15. ACM, 2015, pp. 88–93.

[6] A. Rosenfeld, Z. Bareket, C. V. Goldman, D. J. LeBlanc, and O. Tsimhoni, "Learning drivers' behavior to improve adaptive cruise control," *Journal of Intelligent Transportation Systems*, vol. 19, no. 1, pp. 18–31, 2015.

[7] G. N. Bifulco, L. Pariota, F. Simonelli, and R. Di Pace, "Development and testing of a fully adaptive cruise control system," *Transp. Research Part C: Emerging Technologies*, vol. 29, pp. 156–170, 2013.

[8] M. Heckmann and H. Wersing, "Method for controlling a driver assistance system," Patent EP 2 949 536 A1, 2015.

[9] Z. Tian, M. Vandehey, B. W. Robinson, W. Kittelson, M. Kyte, R. J. Troutbeck, W. Brilon, and N. Wu, "Implementing the maximum likelihood methodology to measure a driver's critical gap," *Transportation Research Part A: Policy and Practice*, vol. 33, no. 3-4, pp. 187–197, 1999.

[10] J. Coughlin, B. Reimer, and B. Mehler, "Driver wellness, safety & the development of an awarecar," MIT AgeLab, Tech. Rep., 2009.

[11] N. Schömig, M. Heckmann, H. Wersing, C. Maag, and A. Neukum, "Assistance-on-demand: A speech-based assistance system for urban intersections," ser. AutomotiveUI '16 Adjunct. ACM, 2016, pp. 51–56.

[12] P. A. Hancock, G. Wulf, D. Thom, and P. Fassnacht, "Driver workload during differing driving maneuvers," *Accident Analysis & Prevention*, vol. 22, no. 3, pp. 281–290, 1990.

[13] A. Stinchcombe and S. Gagnon, "Estimating workload demands of turning left at intersections of varying complexity," ser. Driving Assessment 2009, vol. 5. University of Iowa Public Policy Center, 2009, pp. 440–446.

[14] L. Biester, "Cooperative automation in automobiles," Ph.D. dissertation, Humboldt-Universität zu Berlin, Mathematisch-Naturwissenschaftliche Fakultät II, 2009.

[15] FGSV, *Handbuch für die Bemessung von Strassenverkehrsanlagen*. Köln, Germany: Forschungsgesellschaft für Strassen und Verkehrswesen, 2005.

[16] W. Brilon, R. Koenig, and R. J. Troutbeck, "Useful estimation procedures for critical gaps," *Transp. Research Part A*, vol. 33, no. 3-4, pp. 161–186, 1999.

[17] R. J. Troutbeck, *Estimating the Critical Acceptance Gap from Traffic Movements*, ser. Physical Infrastructure Centre research report. Physical Infrastructure Centre, Queensland University of Technology, 1992.

[18] W. Brilon, "Some Remarks Regarding the Estimation of Critical Gaps," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2553, pp. 10–19, 2016.

[19] J. Sun, *The statistical analysis of interval-censored failure time data*. Springer Science & Business Media, 2007.

[20] A. Gelman, *Bayesian Data Analysis*. Chapman and Hall/CRC; 3 edition, 2013, ch. 1.

[21] D. Orth, D. Kolossa, and M. Heckmann, "Predicting driver left-turn behavior from few training samples using a novel maximum-a-posteriori method," in *submitted to: 2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC)*.