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2016

Preprint:

This is an accepted article published in IEEE Congress on Evolutionary Computation. The final authenticated version is available online at: https://doi.org/[DOI not available]

Hybrid Evolutionary Approach for Level Set Topology Optimization

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***Abstract*—Although Topology Optimization is widely used in many industrial applications, it is still in the initial phase of development for highly nonlinear, multimodal and noisy problems, where the analytical sensitivity information is either not available or difficult to obtain. For these problems, including the highly relevant crashworthiness optimization, alternative approaches, relying not solely on the gradient, are necessary. One option are Evolutionary Algorithms, which are well-suited for this type of problems, but with the drawback of considerable computational costs. In this paper we propose a hybrid evolutionary optimization method using a geometric Level-Set Method for an implicit representation of mechanical structures. Hybrid optimization approach integrates gradient information in stochastic search to improve convergence behavior and global search properties. Gradient information can be obtained from structural state as well as approximated via equivalent state or any known heuristics. In order to evaluate the proposed methods, a minimum compliance problem for a standard cantilever beam benchmark case is considered. These results show that the hybridization is very beneficial in terms of convergence speed and performance of the optimized designs.**

I. Introduction

In the last few years, Computer-Aided Design (CAD) methods, together with numerical simulations with the Finite Element Method (FEM), became a standard for the design and analysis of mechanical structures. Particularly, this led to a rapid development of methods for structural Topology Optimization (TO) [5], which aim to develop optimal structural concepts within a defined design space and under specified boundary conditions and are applied in early phases of a product development. TO plays a vital role in many industrial branches. In particular, TO methods are widely used in lightweight design of structures in aerospace and automotive industry, they have started to gain acceptance in civil engineering, and are applied in materials science to develop smart materials, as well as in biomechanics to simulate bone remodeling when orthopaedic implants are inserted.

In most of the structural TO approaches, the optimization algorithms operate on a discrete mesh of cells, which occupies the entire design domain. Throughout the optimization process, the densities of the elements are changed to minimize a given objective function, subject to a constraint on the total mass. Typically, boundary conditions such as concentrated loads, body forces, pressures and supports (i.e. constraints on the nodal displacements of the FEM mesh) are imposed. Figure 1 illustrates an example of a typical structural TO problem, namely the compliance minimization (stiffness maximization) of the cantilever beam.

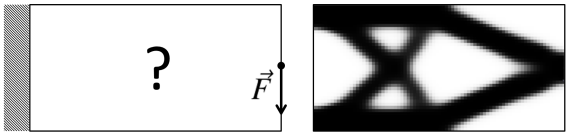

Fig. 1. Compliance minimization of the cantilever beam - typical TO problem. Design domain and boundary conditions (left) and the optimized design [14] (right).

An alternative class of TO approaches is constituted by so-called Level-Set Methods [8] that use an implicit parameterization of material boundaries. As a result, unlike in the other TO approaches (e.g. SIMP) [5], the material interface is clearly defined by the iso-contours of a level-set function, which is very important in the context of manufacturability of the optimized designs. For some problems, e.g. crashworthiness TO [5], this property is crucial because intermediate densities in the computational model would cause high deviations from realistic crash behavior, so in this case the standard density-based approaches should be avoided.

In most of the standard TO approaches, sensitivity analysis is used to obtain analytical gradients and efficient gradient-based methods are applied to optimize the material distribution. However, in case of some objective functions and physical phenomena, e.g. material nonlinearities, contact modeling or material failure, analytical sensitivities are not available and either stochastic or heuristic methods have to be used. However, the heavy losses of efficiency of the optimization process prohibited currently the application of these algorithms in industrial relevant optimization problems. In this work we target to overcome this problem by the integration of state information from intermediate solutions in the stochastic search process.

We focused mainly on Evolution Strategies that turned out to be very successful in many industrial applications, because they are gradient-free and work very well even for highly nonlinear, noisy or discontinuous problems. We follow the assumption that state information of the material can be integrated by approximated gradients, which can be

estimated by simplified models or by finite differencing and generic models that utilize information obtained from the physical structural state [2]. Taking advantage of the implicit representation of material boundaries with limited number of design parameters, we combine Level-Set Method with Hybrid Evolution Strategies, which allow for an efficient search of global optima.

In order to evaluate and validate proposed hybrid evolutionary optimization methods, they are applied to the cantilever beam benchmark case. The problem of compliance minimization with volume constraint is considered. Since the analytical gradients for this case are available, the hybrid methods can be directly compared with the gradient-based methods and no gradient approximation is necessary. By comparing different hybrid optimization methods for the cantilever beam benchmark case, the best hybridization concepts can be identified and used in the optimization of the problems where the analytical gradients cannot be obtained.

The paper is structured as follows. Section II describes the formulation of the optimization problem and the geometric Level-Set Method as well as the sensitivity analysis for the compliance minimization problem. In Section III the discretization of the continuous problem is described. Section IV presents the hybrid evolutionary optimization methods used in this research. Experimental setup and obtained results are described in Section V. The paper is concluded in Section VI.

II. Problem Formulation and Sensitivity Analysis

Let us define a global level-set function having the following property:

$$\begin{cases} \Phi(\mathbf{x}) > 0, & \mathbf{x} \in \Omega, \\ \Phi(\mathbf{x}) = 0, & \mathbf{x} \in \partial\Omega, \\ \Phi(\mathbf{x}) < 0, & \mathbf{x} \in D \backslash \Omega. \end{cases} \tag{1}$$

Similarly, let us define a local level-set function of the ith structural component:

$$\begin{cases} \phi_i(\mathbf{x}) > 0, & \mathbf{x} \in \Omega_i, \\ \phi_i(\mathbf{x}) = 0, & \mathbf{x} \in \partial\Omega_i, \\ \phi_i(\mathbf{x}) < 0, & \mathbf{x} \in D \backslash \Omega_i, \end{cases} \tag{2}$$

where Ω_i is the region occupied by an elementary structural component. In particular, it holds:

$$\Omega = \bigcup_{i=1}^{m} \Omega_i, \tag{3}$$

where m is the number of structural components. Following the idea presented by Guo et al. [9], for $D = \mathbb{R}^2$, $\mathbf{x} = (x, y)^T$, we introduce a level-set function (describing an elementary beam) of the following form:

$$\begin{aligned} \phi_i(\mathbf{x}) = -\Bigg(\left(\frac{\cos\theta_i \left(x - x_{0i}\right) + \sin\theta_i \left(y - y_{0i}\right)}{l_i/2} \right)^r \\ + \left(\frac{-\sin\theta_i \left(x - x_{0i}\right) + \cos\theta_i \left(y - y_{0i}\right)}{t_i/2} \right)^r - 1 \Bigg), \end{aligned} \tag{4}$$

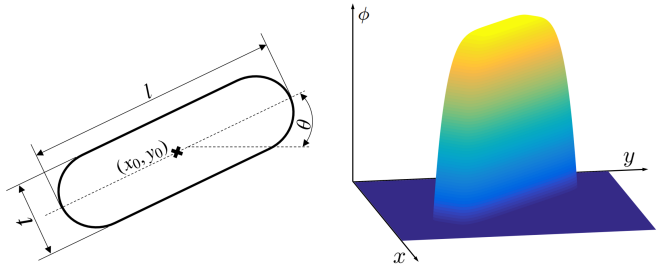

Fig. 2. Parametrization and a plot of the level-set function given by (4). For readability, values $\phi < 0$ were set to 0.

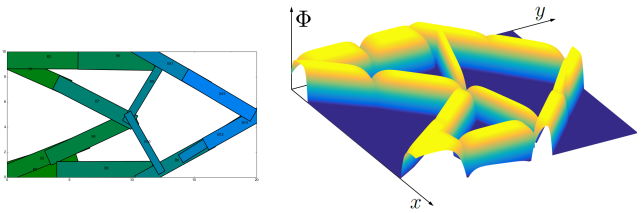

Fig. 3. Possible layout of the elementary beam components for the cantilever beam problem (left) and the corresponding level-set field (right).

where (x_0, y_0) denotes the position of the center of a beam with length l and thickness t. θ is the rotation angle measured counterclockwisely from the horizontal axis. Like in the original approach [9], we take $r = 6$. The parametrization and a plot of the level-set function described by (4) is shown in the Figure 2.

In order to obtain a structure corresponding to the implicit level-set representation, we use a density-based geometry mapping:

$$\begin{aligned} \mathbf{E}(\mathbf{x}) &= \rho(\mathbf{x})\, \mathbf{E}^0 \\ 0 \leq \rho(\mathbf{x}) &\leq 1, \end{aligned} \tag{5}$$

where $\mathbf{E}^0$ is the reference stiffness tensor and $\rho(\mathbf{x})$ is the density at the point $\mathbf{x} \in D$. The density depends in turn on the value of the level-set function:

$$\rho(\mathbf{x}) = H\left(\Phi(\mathbf{x})\right), \tag{6}$$

where:

$$\Phi(\mathbf{x}) = \max\left(\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), ..., \phi_m(\mathbf{x})\right), \tag{7}$$

and $H(x)$ is the Heaviside function:

$$H(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0. \end{cases} \tag{8}$$

The compliance minimization (stiffness maximization) problem is considered and neglecting body forces, it is formulated as follows:

$$\min_{\mathbf{u} \in \mathbf{U},\, \mathbf{E}} C(\mathbf{u}) = \min_{\mathbf{u} \in \mathbf{U},\, \mathbf{E}} \int_{\Gamma_t} \mathbf{t}\mathbf{u}\, d\Gamma, \tag{9}$$

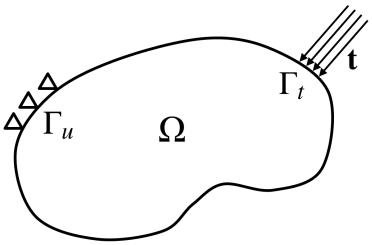

Fig. 4. Mechanical problem formulation.

where $\mathbf{t}$ are the boundary tractions acting on the surface Γ_t (see Figure 4), $\mathbf{U}$ denotes the space of kinematically admissible displacements and $\mathbf{E}$ is the stiffness tensor.

Analytical (or approximate) sensitivities are necessary for hybridization of the evolutionary algorithms. By applying the adjoint method [5] and the chain rule to the problem above, we obtain the partial derivative of compliance with respect to the level-set function:

$$\frac{\partial C}{\partial \Phi} = -\iint_D \delta\left(\Phi\left(\mathbf{x}\right)\right) \boldsymbol{\varepsilon}^T\left(\mathbf{x}\right) \mathbf{E}^0 \boldsymbol{\varepsilon}\left(\mathbf{x}\right)\, d\Omega, \tag{10}$$

where $\boldsymbol{\varepsilon}$ is the elastic strain and $\delta(x)$ is the Dirac delta function:

$$\delta\left(x\right) = \begin{cases} +\infty, & \text{if } x = 0 \\ 0, & \text{if } x \neq 0. \end{cases} \tag{11}$$

The general form of a derivative of the level-set function with respect to a design variable p_i (e.g. x_{0_i}, θ_i, t_i) is given by the following formula:

$$\frac{\partial \Phi}{\partial p_i}\left(\mathbf{x}\right) = \begin{cases} \frac{\partial \phi_i}{\partial p_i}\left(\mathbf{x}\right), & \text{if } \Phi\left(\mathbf{x}\right) = \phi_i\left(\mathbf{x}\right) \\ 0, & \text{otherwise,} \end{cases} \tag{12}$$

where the partial derivative $\frac{\partial \phi_i}{\partial p_i}$ can be easily derived analytically based on (4). Plots of these derivatives (normalized with the norm of the gradient of the level-set function) are shown in Figure 5[1].

Therefore, using the Lagrangian function of the form,

$$L = C\left(\mathbf{u}\right) + \lambda V, \tag{13}$$

where the volume (area in case of $D = \mathbb{R}^2$) occupied by the material is

$$V = \iint_D H\left(\Phi\left(\mathbf{x}\right)\right)\, d\Omega. \tag{14}$$

The analytical sensitivities are given by

$$\begin{aligned} \frac{\partial L}{\partial p_i} = &-\iint_D \delta\left(\Phi\left(\mathbf{x}\right)\right) \frac{\partial \Phi}{\partial p_i}\left(\mathbf{x}\right) \boldsymbol{\varepsilon}^T\left(\mathbf{x}\right) \mathbf{E}^0 \boldsymbol{\varepsilon}\left(\mathbf{x}\right)\, d\Omega \\ &+ \lambda \iint_D \delta\left(\Phi\left(\mathbf{x}\right)\right) \frac{\partial \Phi}{\partial p_i}\left(\mathbf{x}\right)\, d\Omega. \end{aligned} \tag{15}$$

[1]The idea of normalization with the norm of the level-set function gradient and the intuitive interpretation of the normalized derivatives is explained in Section III.

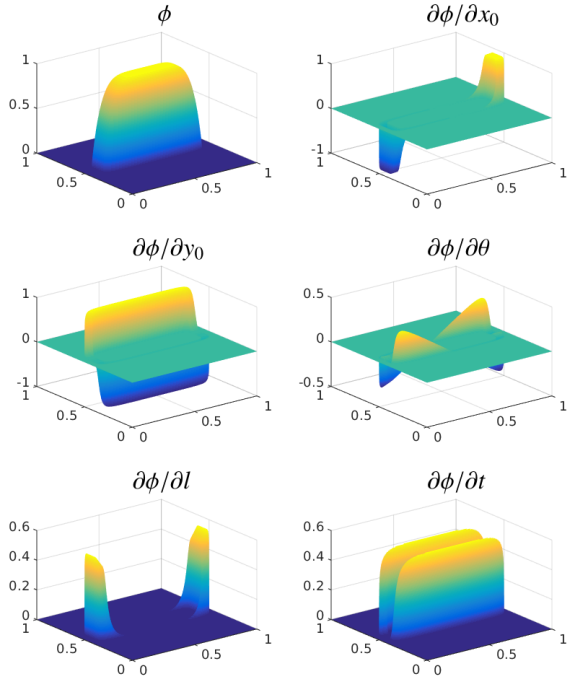

Fig. 5. Level-set function and its derivatives with respect to the design variables. Just the values in the neighborhood of the interface are shown (the rest has been clamped to zero). The derivatives are normalized with the norm of level-set function gradient $\left(\nabla\phi = \left[\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right]\right)$.

III. Problem Discretization

After discretization, the compliance minimization problem satisfying the equilibrium conditions can be formulated as follows:

$$\begin{aligned} &\min_{\mathbf{u},\rho_e} \mathbf{f}^T\mathbf{u} \\ &\text{s.t.}: \left(\sum_{e=1}^{N} \rho_e \mathbf{K}_e\right)\mathbf{u} = \mathbf{f} \\ &\qquad 0 < \rho_{min} \leq \rho_e \leq 1, \;\; e = 1, ..., N, \end{aligned} \tag{16}$$

where $\mathbf{f}$ and $\mathbf{u}$ are the load and displacement vectors, respectively, $\mathbf{K}_e$ is the element's stiffness matrix and ρ_e is the element's density.

In order to estimate the analytical sensitivities of the Lagrangian function (15), the discrete form of the gradient has to be found, as well. Due to the Dirac delta term the numerical approximation of the integrals is difficult and several approaches to solve this problem were proposed. The most straightforward approach is to introduce an approximate Heaviside function, whose derivative is continuous and is treated as the approximate Dirac delta function [4]. This approach, however, is usually applied to signed distance level-set functions[2] and did not work correctly for the parametrization proposed in this work. This is also the case of other approaches

[2]A signed distance function ϕ is an implicit function having the following property: $|\phi\left(\vec{x}\right)| = d\left(\vec{x}\right)$, where d is a distance function defined as $d\left(\vec{x}\right) = \min\left(|\vec{x} - \vec{x}_I|\right) \;\; \forall \vec{x}_I \in \partial\Omega$ [12]. In particular, for signed distance functions, it holds: $|\nabla\phi| = 1$.

that either could not be applied to a non-signed distance level-set functions [15] or were giving inaccurate results. As a result, an alternative, simple and accurate method was proposed and is described below.

Analytical sensitivities are given by integrals (15) of the following form:

$$I = \iint_{\mathbb{R}^2} \delta\left(\Phi\left(x,y\right)\right) f\left(x,y\right) d\Omega \tag{17}$$

The integral I, after the change of variables, is rewritten as [11]:

$$I = \int_0^{\infty} \int_{K:\Phi=t} \frac{\delta\left(t\right)}{\left|\nabla\Phi\left(x(l,t),y(l,t)\right)\right|} f\left(x(l,t),y(l,t)\right)\, dl\, dt \tag{18}$$

The Dirac delta term is nonzero just for one value of t, i.e. $\Phi = t = 0$. Therefore, it can be written:

$$I = \int_{K:\Phi=0} \frac{f\left(x(l),y(l)\right)}{\left|\nabla\Phi\left(x(l),y(l)\right)\right|} dl \tag{19}$$

One possible way of estimating the integral above would be to find all the finite elements that the 0th iso-line of the level-set function passes through. Once those elements are found, the value of the fraction $f(x,y)/\left|\nabla\Phi\left(x,y\right)\right|$ can be evaluated in the center of each element and multiplied by the length of the interface inside the finite element. By summing up all those values, the integral (17) can be efficiently approximated as:

$$I \approx \sum_{e\in G} \frac{f\left(x_{C_e},y_{C_e}\right)}{\left|\nabla\Phi\left(x_{C_e},y_{C_e}\right)\right|} l_e, \tag{20}$$

where G is the set of indices of the elements that the 0th level-set is passing through, (x_{C_e}, y_{C_e}) are the coordinates of the center of the element and l_e is the length of the interface inside the element. The formula can be easily applied to calculate the analytical sensitivities (15)[3].

Due to the above mentioned simplifications in the numerical estimation of the integral (18), the accuracy of the gradient approximation might be lowered and would decrease with coarsening of the FEM mesh. Also in context of the increased nonlinearity of the Lagrangian function introduced by the used parametrization (e.g. corresponding to connecting (disconnecting) of the elementary beam components), the local validity of the gradient information might pose additional difficulties for the optimization algorithm. Nevertheless, since in this paper we focus mainly on the hybrid algorithms, which in general case should work properly even for very rough gradient estimates (e.g. crashworthiness), the accuracy of the gradient approximation is not of principal interest and simplicity of the proposed approach is favored.

[3]It should be noted that the term $\frac{\partial\Phi}{\partial p}(x,y)/\left|\nabla\Phi\left(x,y\right)\right|$, where p is a parameter of a beam (e.g. x_0, θ), has a very easy, intuitive interpretation. As shown in Figure 5, it acts as a weighting factor that takes into account just those parts of the beam that contribute to a derivative with respect to the given parameter. For instance, in case of the compliance derivative with respect to the x_0 parameter, this would involve taking strain energies ($\frac{1}{2}\boldsymbol{\varepsilon}^T\mathbf{E}^0\boldsymbol{\varepsilon}$) at one end of a beam with weight -1 and at the opposite end with weight 1. As a result, depending on which side of the beam the strain energies are bigger, the derivative is either positive or negative and the optimization algorithm moves the beam in the direction of higher energies.

IV. Hybrid Evolutionary Methods

In this section the hybrid algorithms used in this research are described. Two first hybrid methods are based on the standard Evolution Strategy [3], chosen for simplicity, and having the following core structure:

$t := 0$;
initialize $P(0) := \{\vec{a}_1(0), ..., \vec{a}_\mu(0)\} \in I^\mu$;
evaluate $P(0) : \{\Phi\left(\vec{a}_1(0)\right), ..., \Phi\left(\vec{a}_\mu(0)\right)\}$;
while $(\iota\left(P(t)\right) \neq true)$ **do**
  recombine: $P'(t) := r_{\Theta_r}\left(P(t)\right)$;
  mutate: $P''(t) := m_{\Theta_m}\left(P'(t)\right)$;
  evaluate $P''(t) : \{\Phi\left(\vec{a}''_1(t)\right), ..., \Phi\left(\vec{a}''_\lambda(t)\right)\}$;
  select: $P(t+1) := s_{\Theta_s}\left(P''(t)\right)$;
  $t := t + 1$;
end while

where $\Phi : I \to \mathbb{R}$ is the fitness function (I denotes the genotype space) and $\vec{a} \in I = \mathbb{R}^n$ is an individual. The size of the parent population is denoted by $\mu \geq 1$, whereas $\lambda \geq 1$ stands for the offspring population size. A population at generation t consists of all the parent individuals $\vec{a}_i(t) \in I$ in the current generation, i.e. $P(t) := \{\vec{a}_1(t), ..., \vec{a}_\mu(t)\}$. The recombination operator is defined as a mapping $r_{\Theta_r} : I^\mu \to I^\lambda$, whereas mutation operator takes the form: $m_{\Theta_m} : I^\lambda \to I^\lambda$. Both Θ_r and Θ_m are sets of operator parameters controlling recombination and mutation, respectively. In order to choose the next generation of the parent population, the selection operator $s_{\Theta_s} : I^\lambda \to I^\mu$ is used. The termination condition is denoted by $\iota : I^\mu \to \{true, false\}$.

All of the strategy parameters are mutated according to the Schwefel's self-adaptation idea [13]:

$$\begin{aligned} \sigma' &= \sigma \cdot \exp\left(\tau' \cdot N(0,1)\right) \\ \vec{a}' &= \vec{a} + \vec{N}\left(\vec{0}, \sigma'\right) \\ \tau' &= \frac{1}{\sqrt{2n}} \end{aligned} \tag{21}$$

A. Global Hybrid Evolution Strategy with Gradient-based Local Search (GHES)

In the GHES method, a local gradient-based improvement step was introduced into a standard Evolution Strategy. The improvement step performs a local search acting on the solutions produced in the mutation and recombination step. In order to enable the optimization to converge to a minimum even if the accuracy of the gradient approximation is low, the improvement acts only on a part of the offspring population. As a result, just the individuals with the best fitness after a joint recombination-mutation-improvement step survive. A simplified algorithmic description is presented below:

$t := 0$;
initialize $P(0) := \{\vec{a}_1(0), ..., \vec{a}_\mu(0)\} \in I^\mu$;
evaluate $P(0) : \{\Phi(\vec{a}_1(0)), ..., \Phi(\vec{a}_\mu(0))\}$;
while $(\iota(P(t)) \neq true)$ **do**
  recombine: $P'(t) := r_{\Theta_r}(P(t))$;
  mutate: $P''(t) := m_{\Theta_m}(P'(t))$;
  improve: $P'''(t) := i_{\Theta_i}(P''(t))$;
  evaluate $P'''(t) : \{\Phi(\vec{a}'''_1(t)), ..., \Phi(\vec{a}'''_\lambda(t))\}$;
  select: $P(t+1) := s_{\Theta_s}(P'''(t))$;
  $t := t + 1$;
end while

Where i_{Θ_i} is the improvement operator with parameters Θ_i, carrying out a gradient step on a subset of individuals (including necessary evaluations for performing the line search [1]).

In this method, we use an Evolution Strategy as a global optimization method and the Steepest Descent method as a local search algorithm. An optimal step length is found by a quadratic interpolation method [1]. Although the computational cost of a single iteration rises considerably, as can be seen in the Figure 8, the hybrid method can exhibit much better performance than each method separately, which justifies the use of such an approach.

B. Concurrent Hybrid Evolution Strategy with Gradient Individual (CHES)

In the CHES method, a concept of a gradient individual was utilized. A similar approach was proposed by Woo et al. [16]. An outline of the algorithm is presented below:

$t := 0$;
initialize $P(0) := \{\vec{a}_1(0), ..., \vec{a}_{\mu-1}(0), \vec{a}_g(0)\} \in I^\mu$;
evaluate $P(0) : \{\Phi(\vec{a}_1(0)), ..., \Phi(\vec{a}_{\mu-1}(0)), \Phi(\vec{a}_g(0))\}$;
while $(\iota(P(t)) \neq true)$ **do**
  improve gradient individual: $\vec{a}'_g(t) := i_{\Theta_i}(\vec{a}_g(t))$;
  recombine: $P'(t) = \{\vec{a}'_1(t), ..., \vec{a}'_{\lambda-1}(t)\} :=$
  $r_{\Theta_r}(\{\vec{a}_1(t), ..., \vec{a}_{\mu-1}(t), \vec{a}'_g(t)\})$;
  mutate: $P''(t) = \{\vec{a}''_1(t), ..., \vec{a}''_{\lambda-1}(t)\} :=$
  $m_{\Theta_m}(\{\vec{a}'_1(t), ..., \vec{a}'_{\lambda-1}(t)\})$;
  evaluate $P''(t) \cup \vec{a}'_g(t) :$
  $\{\Phi(\vec{a}''_1(t)), ..., \Phi(\vec{a}''_{\lambda-1}(t)), \Phi(\vec{a}'_g(t))\}$;
  select: $P(t+1) := s_{\Theta_s}(P''(t) \cup \vec{a}'_g(t))$;
  select gradient individual:
  $\vec{a}_g(t+1) := argmin_{\vec{a} \in P(t+1)} \Phi(\vec{a})$;
  $t := t + 1$;
end while

The optimization method works as follows: $(\mu - 1)$ individuals follow a standard optimization procedure with the Evolution Strategy. Additionally, a gradient individual ($\vec{a}_g$), undergoing a single Steepest Descent step (as in Section IV-A) in each optimization iteration, is introduced. As the individual to improve, always the best offspring from the whole population is chosen. However, in this approach, neither mutation nor recombination is applied to the improved individual, which follows a pure gradient-based optimization path. In this way a strategy that, in each iteration, chooses between evolutionary and gradient-based optimization method is obtained. This seems to be reasonable since the performance of both methods at different optimization stages might be different. It is commonly believed that the Evolutionary Algorithms are better for identifying good areas of the search space, while they are worse at the end of optimization, in the vicinity of an optimum, due to the stochastic character of the optimization. In this approach merits of both methods can be combined.

C. Hybrid CMA-ES Method (HCMA-ES)

The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [10] turned out to be very successful in many practical applications. An interesting idea for hybridization of the CMA-ES was proposed by Chen et al. [7].

In this method, the gradient information is incorporated by adjusting the weighted mean $\langle \vec{x} \rangle_w^{(g)}$ along the gradient direction, i.e.:

$$\langle \vec{q} \rangle_w^{(g)} = \langle \vec{x} \rangle_w^{(g)} - \alpha \nabla f\left(\langle \vec{x} \rangle_w^{(g)}\right) \tag{22}$$

In this work, a dynamic adaptation of the step length α according to the line search algorithm as in the previous sections was additionally introduced.

Besides replacing $\langle \vec{x} \rangle_w^{(g)}$ with $\langle \vec{q} \rangle_w^{(g)}$, the method remains unchanged with respect to the standard CMA-ES.

V. Experiments

A. Setup

In this paper the cantilever beam benchmark test was considered. The beam is fixed at one end and a static, point, unit load is applied in the middle of the opposite end. The objective function minimized in this case is the compliance of the structure. The design domain and boundary conditions are shown in the Figure 6. A simplified representation of the initial design and the corresponding mapping to the FEM mesh are shown in the Figure 7.

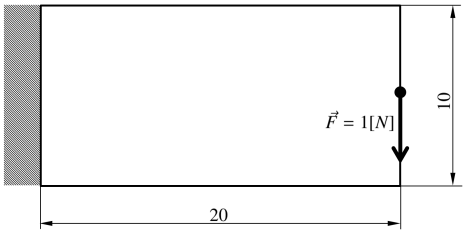

Fig. 6. Design space and loads of the cantilever beam test case.

Exact configuration of the case is shown in Table I.

Property	Symbol	Value	Unit
Young's modulus	E	$2.1 \cdot 10^5$	MPa
Poisson's ratio	ν	0.3	-
Load	F	1	N
Mesh resolution	-	100 x 50	-

TABLE I
CONFIGURATION OF THE CANTILEVER BEAM TEST CASE.

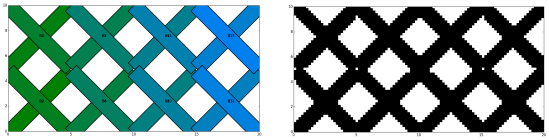

Fig. 7. Initial layout of the beams and the corresponding mapping to the FEM mesh.

Method	Description
SD	Steepest Descent optimization method (as a reference).
ES	Standard (20,100) Evolution Strategy with a single standard deviation.
CMA-ES	Covariance Matrix Adaptation (8,17) Evolution Strategy.
GHES	Global Hybrid (20,100) Evolution Strategy with a single standard deviation and 10% of improved individuals.
CHES	Concurrent Hybrid (20,100) Evolution Strategy with a single standard deviation.
HCMA-ES	Hybrid CMA-ES(8,17) Method.

TABLE II
TESTED OPTIMIZATION ALGORITHMS.

Parameter	Symbol	Value
Lagrange multiplier for volume	λ	0.02
Reference step length in the line search	α	0.2
Initial standard deviation in the mutation step	σ_{init}	0.1

TABLE III
PARAMETRIZATION OF THE OPTIMIZATION ALGORITHMS.

The FEM mesh consists of 5000 four-node shell finite elements that, depending on the value of the level-set function, are assigned Young's modulus either equal to the value specified in Table I (material) or equal to 1% of this value (non-material). Material properties as for steel are introduced.

The optimization framework was implemented in Python. A python implementation of the CMA-ES algorithm [10] was used and modified for implementing the HCMA-ES, as well. The free, open-source CalculiX solver was used to carry out the finite element analyses.

In order to compare different optimization methods, an extensive statistical testing was carried out. Comparison of the means was based on the two-sample t-test for normally distributed random variables with unequal variances. Equality of variances was checked with use of the two-sample F-test. Normality of the compared random variables was checked with the Jarque-Bera test. For all tests, a 5% significance level was assumed.

B. Results

In this section a comparison of different optimization algorithms for the geometric Level-Set Method is presented. The evaluation of the methods is based on the results of 30 optimization runs for each of the algorithms presented in Table II.

In Table III the parametrization of the optimization algorithms is presented.

Convergence of the averaged Lagrangian function for different optimization methods is shown in the Figure 8. Corresponding statistical evaluation of the methods is presented in the Figure 9.

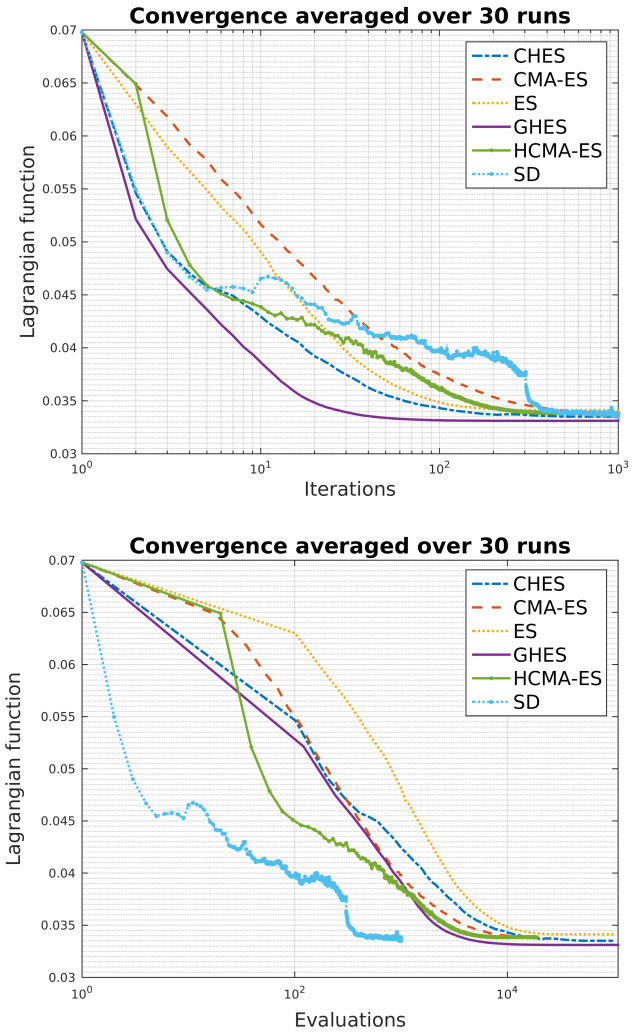

Fig. 8. Convergence of the Lagrangian function.

The results show that the GHES method has superior convergence properties compared to both methods composing it, i.e. ES and SD. This is true both in terms of optimization iterations and number of evaluations of the objective function (each involving one static FEM analysis). It should be noted that the increase of numerical costs associated with the hybridization is relatively low and requires just 20% more evaluations than in case of the ES. The GHES much more frequently leads to the best topologies (Table IV) and involves much lower variance of the Lagrangian function (see Figure 9). The method converges fast at the beginning, which is a crucial property in practical applications, in which the number of available evaluations is often heavily limited, and much more often tends to the global optimum. The main advantage of the GHES method is that it does not only switch between different methods composing it, but it can indeed benefit from a joint mutation-improvement phase by finding better starting

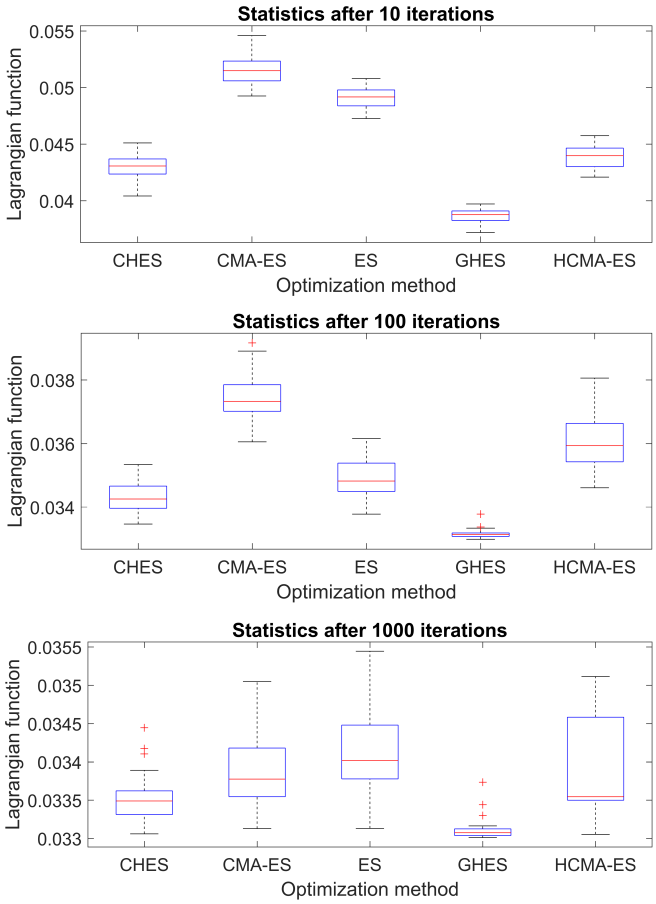

Fig. 9. Statistical evaluation of the evolutionary optimization methods.

Topology type	ES	CMA-ES	GHES	CHES	HCMA-ES
	7%	0%	80%	37%	23%
	27%	63%	0%	43%	50%
	10%	10%	17%	7%	0%
	3%	0%	3%	3%	3%

TABLE IV
FOUR MAIN TOPOLOGY TYPES AND THE FREQUENCY WITH WHICH THEY WERE DEVELOPED. TOPOLOGY TYPES ORDERED ACCORDING TO THE INCREASING VALUE OF THE MINIMIZED LAGRANGIAN FUNCTION.

points for the Steepest Descent improvements. In this way, additional speedup is achieved.

In case of the CHES method, the hybridization also leads



Fig. 10. Topologies optimized with use of the SIMP approach [14] (left) and a standard Level-Set Method [6] (right).

to significant improvements with respect to ES and SD. This time, however, the algorithm just chooses between ES and SD, what no longer leads to the additional increase of convergence speed associated with hybridization. As a result, the algorithm is as good as the best of the methods composing it. However, this is still a desirable behavior since a method performing better in a given search space region can be chosen. Moreover, the CHES method needs just one additional evaluation of the objective function per iteration than the ES. Therefore, the computational cost associated with the hybridization is negligible compared to the speedup that is obtained. As in the GHES method, CHES much more frequently leads to the best topologies.

The last hybrid approach, HCMA-ES, at least in the initial phase of the optimization process, exhibits the best performance among all of the hybrid approaches as long as the number of FEM evaluations is considered[4]. Nevertheless, the method exhibits much higher variance of the Lagrangian function throughout the whole optimization process and is considerably more noisy than any other method considered in this section. The method seems to be very sensitive to any inaccuracies of the gradient approximation, which might be a severe drawback when applying it to the problems where the analytical sensitivities are not available.

The topologies optimized with the hybrid methods are consistent with the designs obtained with use of the state-of-the-art methods for structural TO (see Figure 10). However, those approaches cannot be used to optimize arbitrary objective functions, which was the main motivation for using evolutionary optimization methods in this work.

C. Discussion

The results show that with hybridization significant improvements of convergence properties and global search capabilities can be achieved. The GHES has proven that the hybridization can lead to methods not only as good as their components, but also having superior convergence speed with respect to every method composing them. This was the main difference between GHES and CHES approach, where the algorithm can just switch between the methods and is as good as its best component, but involves considerably lower computational costs. Finally, the HCMA-ES was the most promising

[4]Ideally, when comparing different optimization methods, just the algorithms in their optimal configurations should be considered. This can result in completely different population sizes for the ES and the CMA-ES. In this paper, however, the main focus is to compare the standard optimization algorithms with their hybrid versions, justifying the choice of different population sizes for the ES and the CMA-ES.

method in terms of the number of the FEM evaluations, but involved an increased noisiness of the Lagrangian function during the optimization.

It seems that the GHES represents the best hybridization concept also for the optimization problems where the sensitivities cannot be derived analytically. This was demonstrated also in the considered test case, where due to the inaccuracy of the gradient estimation caused either by an insufficient mesh resolution or by an imprecise minimum estimation in the line search, both SD and HCMA-ES were strongly disturbed, while GHES remained unaffected. In GHES, the evolutionary process can decide when to use the external knowledge sources and therefore, more local improvement techniques can be incorporated without having negative influence on the optimization process. Finally, similar hybridization concept could be used together with the CMA-ES, which turned out to be very successful in solving the optimization problem considered in this paper.

VI. Conclusion

The main focus of this paper was a development of efficient, hybrid, level-set-based evolutionary methods for structural TO. For evaluation, a standard cantilever beam benchmark case was used. However, the principal goal was to find a hybridization concept that could be further used for optimization of problems, where analytical sensitivities are not available.

In particular, the application of the proposed methods to crashworthiness TO seems to be very interesting and is currently under investigation. The crashworthiness problems are characterized by very strong nonlinearities, discontinuities and noise, arising from modeling of highly complex physical phenomena ranging from contact to material failure. As a result, in general case, analytical sensitivities for crash problems cannot be easily found. However, simplified models or generic models utilizing local state features [2] can be used to obtain approximations of the gradients. The hybrid evolutionary algorithms proposed in this paper offer ways to use these rough approximations and enhance the performance of the evolutionary search.

Within the research, performance of the following hybrid optimization algorithms was evaluated:

- Global Hybrid Evolution Strategy with Gradient-based Local Search (GHES), where after the mutation step, a part of the offspring population is improved by a single step in the negative gradient direction.
- Concurrent Hybrid Evolution Strategy with Gradient Individual (CHES), where after each selection step, the best individual becomes the gradient individual, optimized with the Steepest Descent method.
- Hybrid CMA-ES Method (HCMA-ES), where the mean of the search distribution in the CMA-ES is moved along the direction of the negative gradient.

The hybrid methods were compared with both standard Evolution Strategy and the Covariance Matrix Adaptation Evolution Strategy.

The hybrid approaches presented in this paper showed that the evolutionary methods can benefit from incorporation of the external knowledge sources. Due to the increased capability of finding global optima, they might be considered as an attractive alternative even for the gradient-based methods in case if the analytical sensitivities are available. Nevertheless, even if the information that should support the convergence process (e.g. gradient estimate) is incorrect, evolution can use the selection mechanism to eliminate the negative effects (e.g. GHES, CHES). Those properties broaden significantly the scope of possible extensions and make the further research very promising.

Finally, the hybridization concept used in the GHES method seems to be the most promising in terms of convergence speed, ability to find global optima and applicability to the problems, where only a rough approximation of the gradient information can be obtained.

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