Honda Research Institute EU
Honda Research Institute Europe GmbH
https://www.honda-ri.de/

Evolutionary Computation for Topology Optimization of Mechanical Structures: An Overview of Representations

Nikola Aulig, Markus Olhofer

2016

Preprint:

This is an accepted article published in Proceedings of: IEEE Congress on Evolutionary Computation. The final authenticated version is available online at: https://doi.org/[DOI not available]

Evolutionary Computation for Topology Optimization of Mechanical Structures: An Overview of Representations

Nikola Aulig
Honda Research Institute Europe GmbH
Carl-Legien-Straße 30
63073 Offenbach/Main, Germany
Email: nikola.aulig@honda-ri.de

Markus Olhofer
Honda Research Institute Europe GmbH
Carl-Legien-Straße 30
63073 Offenbach/Main, Germany
Email: markus.olhofer@honda-ri.de

***Abstract*—During the past decade, continuum topology optimization became an important industrial tool for the conceptual design of mechanical structures. The field of evolutionary computation provides suitable stochastic optimization algorithms for problems involving strong non-linearities or black-box simulations, for which existing gradient-based methods are not feasible. Due to the high design freedom of the phenotypic space, the encoding of the structural design is a critical aspect when applying evolutionary algorithms. Currently, the encoding approaches are scattered throughout different literature fields. This paper gathers them and provides a contemporary overview on the various structural representations used in conjunction with evolutionary computation for topology optimization. The important influence of the representation on the scalability of the approaches motivates the proposed categorization in three groups: Grid, Geometric and Indirect Representations. The existing representations are described and discussed on a conceptual level and chances and challenges are outlined.**

I. Introduction

About a decade ago, [1] provided an overview on Evolutionary Computation (EC) and structural design, including approaches for continuum topology optimization. Since then, a large variety of specialized methods has been developed in the field of EC, yet currently no contemporary survey exists.

Existing approaches can be found in the only loosely connected fields of evolutionary computation and structural design. Researchers in structural design are often not aware of the available EC methods as well as researchers from the EC community are often unfamiliar with the topology optimization problem. This paper contributes to both fields by proposing definitions for representations and by summarizing and categorizing existing EC approaches for topology optimization. We define three fundamental classes of representations with different advantages and drawbacks.

The next section introduces the topology optimization problem. Section III introduces the phenotype to be represented and different classes of representations. Then, the different representations are defined and approaches from literature are described in more detail, starting with the Grid Representation in Sec. IV, followed by the Geometric Representation in Sec. V and finally the Indirect Representation in Sec. VI. The paper is concluded in Sec. VII.

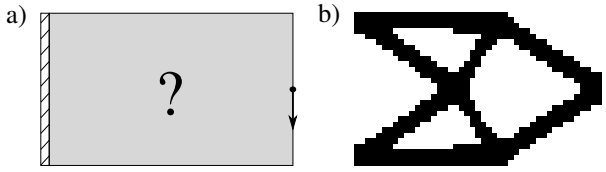

Fig. 1. Example for a) design space and boundary conditions of a cantilever topology optimization problem and b) optimized design [2].

II. Topology Optimization

The field of topology optimization addresses the task of finding a concept for a mechanical structure. Subject of the optimization is the basic layout and geometric connectivity of the design, thus its topology. These methods are applied by engineers and designers to generically obtain a concept early in a structural design process.

The focus of this paper is continuum topology optimization, in which the target is to find the optimum distribution of material and void regions within a predefined design space. This problem can typically be formulated as:

$$\begin{aligned} &\min_{\rho} \; f(\rho) \\ &\text{s.t.} : G_l(\rho) \leq 0, \; l = 1 \dots L, \\ &\qquad \rho(\mathbf{x}) \in \{0, 1\} \; \forall \mathbf{x} \in \Omega. \end{aligned} \tag{1}$$

The task is to find the distribution of material and void for all points $\mathbf{x}$ in the design space Ω such that the objective function $f(\rho)$ is minimized. At each point the density variable $\rho(\mathbf{x})$ can either be zero for representing void or one for representing material, respectively. Practically the problem can have additional constraints $G_l(\rho)$, e.g. commonly at least the total structural mass is restricted.

An example of a design space with loads and supports is shown in Fig. 1a). When considering minimizing compliance (i.e. maximum stiffness) as objective function, subject to a

fixed mass constraint the cantilever shown in Fig. 1b) can be obtained by state of the art topology optimization [2] and subsequent thresholding.

Linear or slightly non-linear problems that can be formulated analytically, can be addressed by gradient-based topology optimization methods [3], which are nowadays a standard tool in industry. Popular are density-based methods [4], which operate on a design space discretized into many small elements and relax the binary problem to enable the use of gradients. Binary solutions are obtained by penalizing interpolated material values. A distinct approach are level set methods [5], which parameterize the boundary between material and void regions by the contour of a level set function. This results in a smoothly shaped structure with a clear boundary. In the gradient-based methods optimization is performed by using heuristic update rules or mathematical programming methods, using parameter or shape sensitivities.

Although gradient based methods are very efficient with respect to function evaluations [6], the practical applicability is limited. The most obvious restriction is the necessity of a gradient, which cannot be obtained for every problem. In practical problems, analytic modeling can be difficult due to problem complexity or due to black box simulations. The determination of numerical gradients can be rendered infeasible by numerical noise or the high number of variables. Furthermore, practical problems often contain severe non-linearities resulting in multi-modal, rugged or discontinuous search spaces, such that a gradient-based search does not yield satisfying results. An example is the area of crashworthiness topology optimization, for which strong assumptions or model simplification have to be made, in order to perform a heuristic optimization, see for instance [7].

In addition to the gradient-based approaches, non-gradient stochastic search approaches based on EC can be applied. Evolutionary algorithms have various advantages like being relatively robust and general, having the potential to identify global optima for multi-modal objective functions, and operating without gradient information. EC can also address problems with strong non-linearities or black box simulations, while having the disadvantage of considerably higher computational demands. Therefore, we consider non-gradient approaches not in direct competition with gradient-based approaches but rather as complementary approaches for problems, in which gradient based methods do not provide satisfactory results.

III. Representing the Structure

Especially with respect to the high computational demands of EC methods, an important aspect is the parameterization of the problem, i.e. the relation between the geno- and the phenotype. Following the biological inspiration, the phenotype is the solution in the context of the original problem that is subject to evaluation. In the evolutionary optimization however, solutions are encoded in the genotype to which variational operators are applied. In [8], representation is defined as "*mapping from the phenotypes onto a set of genotypes*".

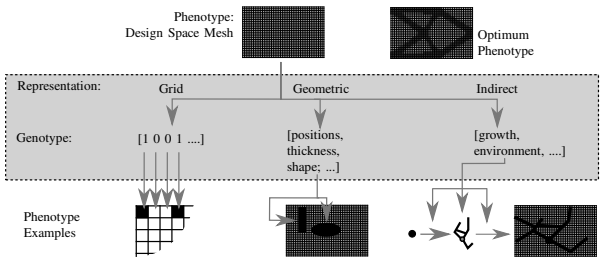

Fig. 2. Proposed categories of representations for topology optimization.

For an evolutionary algorithm, the choice of representation is essential. It not only determines the dimensionality of the search space but also its structure, thus the representation directly determines the probability for the identification of improved solutions and consequently the progress rate of the optimization. It also determines how close the optimization can get to the optimum phenotype. It is therefore essential to not only differentiate approaches based on the optimization algorithm, but also based on the applied representation.

In topology optimization the physics of the problem is described by a governing equation. Practically, this results in an evaluation of the structure using a finite element analysis solver. For this purpose the structure is discretized in a fine grid (with possibly up to millions) of finite elements, where the fineness is practically limited by the computational capabilities. Following this argumentation, the phenotype in structural optimization is formed by a finite element mesh, that is subject to evaluation. This paper proposes categories for the existing representations of this phenotype. Concretely, the representations that are used in the field of EC are categorized in three classes: Grid, Geometric and Indirect Representation. An overview of the proposed concepts is shown in Fig. 2.

IV. Grid Representation

We propose to classify a representation as "Grid Representation", if the genotype encodes properties of fixed locations in a grid, that discretizes the design space. In a typical case, a maximum of design flexibility is reached, when a variable is assigned to each cell of a finite element mesh discretization of the design space.

A. Bit-Array Encoding

The straightforward case is a bit-array encoding of the grid, i.e. the genotype encodes a binary variable, representing either void or material, for each grid cell. This is illustrated in Fig. 3. In the most common case, each variable of the grid cell corresponds to material in one finite element of the analysis model.

An early work on optimizing a bit-array representation using a genetic algorithm can be found in [9], minimizing the weight of a cantilever plate, subject to displacement and stress constraints. A connectivity analysis is required: material elements that do not share an edge with at least one neighboring material element or that are not directly or indirectly connected to the boundary conditions are switched to void. In [10] the approach

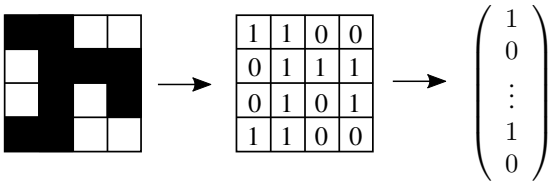

Fig. 3. Example for a Grid Representation.

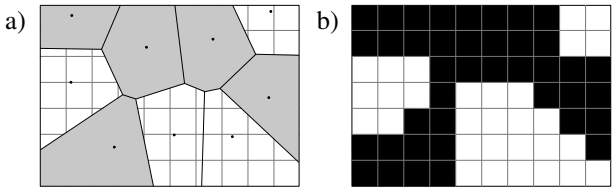

Fig. 4. Example for (a) Voronoi representation of structure and (b) corresponding structural design, as used in [31].

has been extended for slightly more finely discretized solutions by introducing a hierarchical design domain subdivision: After optimizing based on a coarse finite element representation, the design space is subdivided to refine the optimized solution and the optimization is continued with several populations.

Plenty of different domain-specific optimization approaches have been proposed since then, based on genetic algorithms [11]–[13], artificial immune system algorithms [14], particle swarm optimization [15], [16] or modified binary differential evolution [17]. Also multi-objective genetic algorithms combined with local search operators have been applied [18]–[20].

To the most part, the methods in literature have been applied to minimum compliance problems, i.e. problems for which the stiffness of a structure is maximized subject to a static load [10]–[17] or compliant mechanism design, i.e. maximizing the output displacement for an input load [18], [20].

Some approaches combine EC with state respectively gradient information. In [21] simulated annealing is combined with information on the elemental stress. In [22] recombination of solutions is done based on the stress contours of the design in order to guide the optimization. Also an ant-colony optimization algorithm is modified to include gradient information by biasing the pheromone concentration based on elemental sensitivity information, and is applied e.g. for the compliance topology optimization problem [23] and compliant mechanism design [24]. Using the results of gradient-based methods for population initialization, a multi-objective topology optimization method is proposed in [25], [26].

The so-called Evolutionary Structural Optimization (ESO) [27], [28] approaches have to be mentioned in this overview for clarification. ESO starts from a completely filled design space and gradually removes material from elements of the grid. Although the term "Evolutionary" is included in the name, it relies on gradient information and it does not implement any concept of population, random variation or selection and thus does not classify as evolutionary algorithm.

B. Real-Valued Array Encoding

When operating directly on the mesh, gradient-based algorithms as in [4] apply a continuous variable to each mesh element, due to the necessity of gradual changes. This is rarely done in the existing EC approaches, most likely since genetic algorithms are able to directly address the discrete problem.

An exception is [29], in which each element of the mesh is assigned a real-valued cost variable. Each element is defined as node in a graph, connected by edges between neighboring elements. Still, the representation involves one variable per element in the mesh, hence it classifies as a Grid Representation. The problem of finding the optimum structure for a given volume constraint is transformed to the task of finding the optimum subtree, including the correct number of vertices with the lowest cost. An advantage of the method is that the subtree finding process only generates connected structures and the application of crossover and mutation to the cost variables with a genetic algorithm is straightforward. The representation was applied to compliance minimization and eigenvalue maximization.

V. Geometric Representation

We propose to classify a representation as "Geometric Representation", if the genotype encodes properties of a set of movable shape primitives, that define the geometry of the structure. Properties of the shape primitives are for instance position, shape and thickness. When decoding, a geometry mapping is applied to obtain the phenotypic finite element mesh. The search space dimensionality, as well as the potential structural complexity, depends on the number of primitives, but both are independent of the number of finite elements used in the structural analysis.

A. Voronoi-cells

In [30], a Voronoi cell representation for topology optimization is proposed, motivated by the idea of overcoming the direct coupling of the finite element mesh and the number of variables by a geometry described by Voronoi cells. In the genotype a set of coordinates for the Voronoi sites is encoded. Voronoi sites divide the design space into polyhedral-shaped subsets, termed Voronoi cells, in which each point of the subset is assigned to the nearest Voronoi site. Every Voronoi site is assigned a binary variable, i.e. material or void. In this way the geometry of any structure formed from polyhedral subsets can be represented, and mapped to the underlying structural model. The Voronoi representation and a structural mapping are illustrated in Fig. 4. In [30] the Voronoi representations are evaluated on the minimum compliance problem using genetic algorithms and evolutionary strategies. The idea was picked up by several researchers. It was applied to weight minimization with maximum displacement constraints [31] and also applied to the design of electromechanical devices [32].

B. Material-Mask Overlay

A representation, based on optimizing the position and shape of rectangular holes in the design space that is otherwise

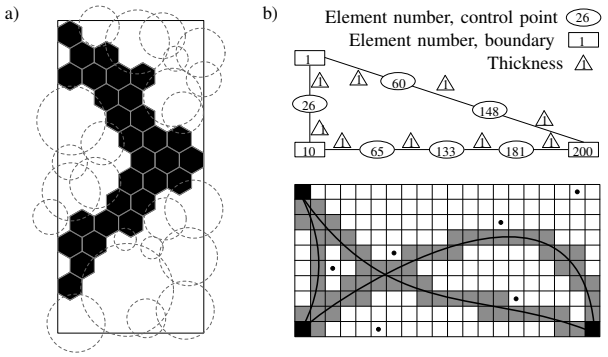

Fig. 5. Representation with (a) Material mask overlay [33], (b) graph corresponding to Bézier curves [38].

filled with material is proposed in [30]. The approach can be considered as early version of the material mask overlay strategy proposed in [33], [34]. In this case, the geometric form described by the material-masks are circles. For each mask, the Cartesian coordinates of the circle center, its radius and its material state are encoded as variables. A user defined number of masks is mapped to the structure by assigning material or void to the elements of a hexagonal element mesh based on the mask material state. The material state is a binary variable, such that elements whose center lies within the radius of a mask, are turned into material or void elements respectively. If masks overlap, the topmost mask decides whether an element in the intersection is filled with material or void. The material-mask overlay strategy is applied to the design of a compliant mechanism. An important parameter is the number of masks that are applied, which decides on the search dimensionality and implicitly on the structural complexity that can be represented. Optimization is performed with a multi-objective particle swarm optimization. In [35] an adaptive variant is proposed, in which the number of masks is adapted during the optimization, by removing redundant masks. Furthermore only negative i.e. void masks are applied, such that material is by default put in any regions of the design space without masks. An illustration of the concept can be seen in Fig. 5a). It is applied to the problem of designing compliant mechanisms with stress constraints using a parallel mutation-only search. Also other geometric masks may be used such as elliptic or rectangular ones, although circular masks yield solutions more efficiently [36]. The efficiency of the method can be increased when using gradient information, which has been demonstrated on the minimum compliance problem [37].

C. Graph Representation

In contrast to having independent components, several researches have explored the idea of representing the phenotype as a graph. In these cases the genotype encodes positions of nodes and characteristics of the edges such as form or thickness. Examples are graphs in which edges represent beam elements or spline curves that are connected in the nodes.

1) Bézier Curves: In [39] locations at which the structure interacts with boundary conditions such as loads and supports are connected by Bézier curves. Each element of the mesh, through which the curve passes through, is assigned material and forms the skeleton of the structure. The complexity of the represented structure can be controlled by the number of control points. An additional thickness parameter defines the thickness of the connection, by adding layers of elements around the skeleton. Thus the structure is defined by the location of the control points and the thickness values along the curves. A genetic algorithm is applied for optimization and applicability for compliant mechanisms and compliance cantilever is demonstrated.

The method is applied to path generating compliant mechanisms, with a constraint handling evolutionary algorithm in [40]. In [38] and following publications the Bézier curves are encoded in a graph in which the vertices are the end- and control points and a graph specific crossover is applied. Fig. 5b) shows an example for the Bézier curves representing a structure and the according graph. A more recent version [41] distinguishes between active and inactive curves, that can change their state during the optimization. Only active curves contribute to the structure, such that the amount of variables that contribute to the design is adaptive during the optimization.

2) Bit-Array and Skeleton: In [42] a Geometric Representation is combined with a bit-array representation in a two phase approach. In the first phase a bit-array encoding is applied and optimized by a two-stage adaptive genetic algorithm. A criterion for skeleton convergence is applied as stopping criterion for the first phase. Originating from image processing "skeletonization" refers to the extraction of a thin skeleton structure from the bit-array solution, such that connectivity and extent of the structure are preserved. In the second phase the Geometric Representation is constructed from the optimized skeleton. Horizontally connected elements are aggregated to a rectangle with height of one element. The geometric variables are the left and right boundary elements of the rectangles. Rectangles are nodes in a graph and edges are defined from the connections of the converged skeleton such that graph specific cross-overs can be applied. A genetic algorithm is used for optimization of minimum compliance cantilevers.

3) Complex Shaped Beam Elements: Complex shaped beam elements [43] can be applied as basic geometric unit in a topology optimization. Start and end nodes of the beams are the nodes of the graph and parameters corresponding to the edges define shape and thickness. Straight, variable-thickness, and curved variable-thickness beams are applied. Variable thickness and curves are realized by a cubic thickness distribution and center line, respectively. Specialized operators are proposed such as splitting and merging of beams. Due to the capability of changing the displacement path via shape changes along the center line, the representation is especially suited for the design of compliant mechanism and is applied to the design of an adaptive car seat concept prototype.

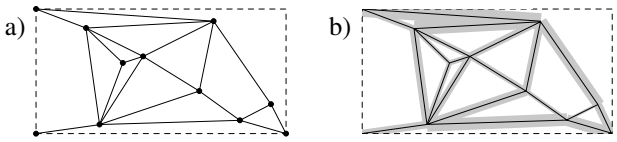

Fig. 6. Representation by constructive solid geometry as in [44]: (a) nodes and triangulation, (b) beam shapes along edges.

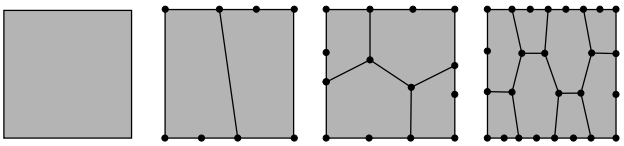

Fig. 7. Starting point and the first three developmental steps of the cellular division process, shown at equilibrium [49].

4) Constructive Solid Geometry: A constructive solid geometry is proposed in [44]. Primitives are combined to obtain more complex geometries by using solid modeling operators such as the union operator. The authors use rectangular beam primitives, defined by two nodes and a thickness parameter. A skeleton topology, i.e. the edges of the graph are obtained by Delaunay triangulation. The coordinates of the nodes, besides fixed support and load nodes, are optimized. The triangulated mesh edges are decoded into a structure with rectangular beams with the corresponding thickness. For overlapping bars the constructive solid modeling union operator is applied such that the complete area covered by the bars forms the structure. Fig. 6 illustrates the representation. A volume constraint can be met by scaling the beam thickness values. A genetic algorithm is applied to the optimization of minimum compliance structures.

D. Level Set Methods

In classical level set methods [5], the shape of the boundary between void and material is defined by the contours of a level set function. Due to the large number of basis functions composing the level set, usually gradient-based optimization is applied, but also genetic algorithms have been used [45]. Although the level set approach is not explicitly formulated, most Geometric Representations presented in the preceding subsections work on a similar principle. There also exist level set approaches that are explicitly based on a smaller number of geometric basis functions. Hybrid evolutionary strategies have been proposed to optimize a level set function based on beam-shaped geometric basis functions [46].

VI. Indirect Representation

In Grid and Geometric Representations a fixed descriptive model is used to describe the structure directly. In contrast to these cases, we propose to classify a representation as an "Indirect Representation", if the genotype encodes variable properties of a generative model, which implicitly defines material locations or a geometry. The rules or development processes, that create the structure, are often inspired by onto- or morphogenesis in nature and mimic various aspects at different levels of abstraction.

A. Lindenmayer System

The grammar based Lindenmayer system (L systems) is a model for embryonic cellular division of organisms, originally developed for modeling branched topology in plants. A planar graph is termed map L system and defined as a finite set of regions, separated by edges that are connected at vertices. Biologically the edges represent the cell boundaries. In [47] it is used to evolve a family of three-dimensional table-like structures. Although no explicit reference is made to topology optimization and the structure is evaluated without a finite element analysis, effectively this is an early example for addressing a topology optimization problem with an Indirect Representation.

A more recent approach [48], [49] explicitly applies a map L system to topology optimization. The model features cell division in two cells without vanishing of cells. A first phase, applies markers to all edges. In a second phase matching markers are connected, realizing the cell divisions. The two phases are repeated several times. Each division stage is followed by a dynamic stage until a force equilibrium is reached. Only the equilibrium result is used for the structural optimization. The development of a topology in a cellular division process is illustrated in Fig. 7. Additionally, a property that controls the thickness of the edges is introduced. The rules of the cellular division encoded in the genotype are optimized by a genetic algorithm.

The method has first been applied to aircraft structural design [48] followed by validation on a compliance cantilever [49] showing that the approach provides close to optimal solutions and that the number of evaluations is significantly lower than in the approaches from [12], [13], [31]. Further applications, for instance to aeroelastic topology optimization of flapping wing venation, followed [50].

B. Gene Regulatory Network

In [51], a relatively detailed biologically inspired cellular model for the evolutionary development of structures is applied. It features cell division and physical interaction of the cells, such as adhesion and repellence. The genotype encodes a simple form of a Gene Regulatory Network (GRN) consisting of a gene with a regulatory and a structural unit. The state of the regulatory unit is based on the cell type and the environment. It influences the structural unit, which determines the type of the cell and eventually controls a cell division. In [52] a more complex cell model is implemented based on the same GRN structure, but with a higher number of units and including additional physical forces for adherence and repulsion and chemical gradients used for chemotaxis. The chemical gradient causes an additional force that is acting on the cells, and can also be read by the regulatory units for gene activation.

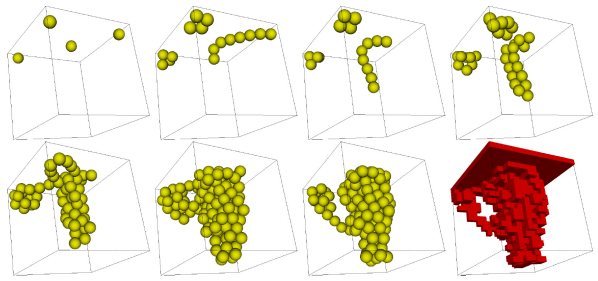

Fig. 8. Cellular Growth model based on motile polarized cells and voxelization (from Steiner et.al. [52]).

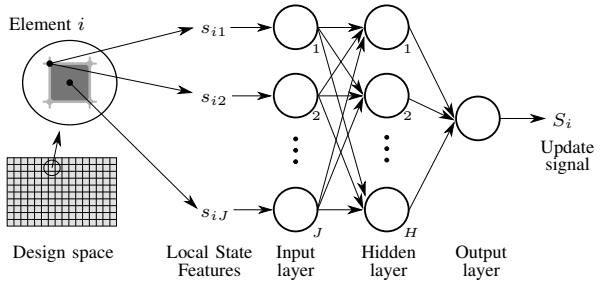

Fig. 9. Neural network update signal representation, by using Local State Features [53].

The growing process and mapping to a three-dimensional structure is illustrated Fig. 8. The development process is started from a single or few cells. Based on the GRN encoded rules, the cells repeatedly divide and specialize in material and void cells and at the end of the development process, when the calculation area is filled, void cells are removed and material cells are mapped to the structure. An evolutionary multi-objective algorithm is applied in order to minimize weight and inner stress [51] or bi-linear energy [52] within the elements of a three dimensional structure. The results show that the model is able to devise complex lightweight structures, also in regions of the Pareto front of very low weight, in which classical gradient based algorithms failed to find solutions.

C. Neuro-evolution

1) Neuro-evolutionary Topology Optimization: In [53] a hybrid combination of a neuro-evolutionary and a classical density-based optimization method is proposed. It uses physical information, termed Local State Features, obtained from the structural analysis to generically substitute gradient information. The mapping of the Local State Features by an artificial neural network model to an update signal is illustrated in Fig. 9. The update signal is optimized by an evolution strategy and used to modify the structure, which itself is represented by one continuous density variable per element of the discretized design space. However, instead of the structure, the parameters of the model are encoded in the genotype, thus encoding an estimated search direction. In this way, mesh resolution and search space of the evolutionary optimizer are decoupled. The approach has been tested on the minimum compliance problem, showing that the efficiency depends strongly on the type of Local State Features used.

Recently the idea is developed towards a state-based representation [54]. The smooth update signal function is replaced by a piecewise constant model, assuming that elements with similar state features also have a similar influence on the objective function and therefore can be modified together. It is applied to intrusion minimization in a crashworthiness problem.

2) Compositional Pattern Producing Network: A state of the art method in the field of generative encoding are Compositional Pattern Producing Network (CPPN) [55] as model for a developmental process, often optimized by Neuro Evolution

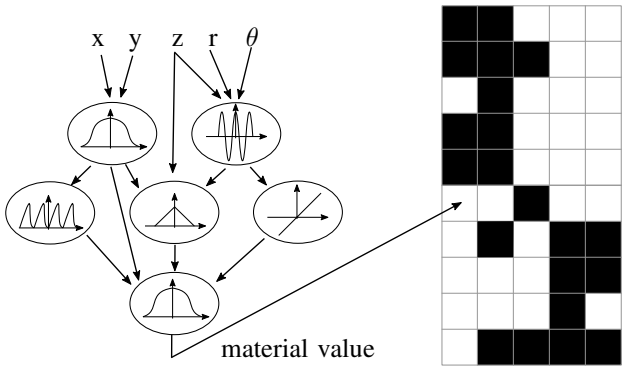

Fig. 10. CPPN representation for structure, adapted from [56].

for Augmenting Topologies (NEAT), hence CPPN-NEAT. It is important that topology in this context refers to the topology of a neural network model. A CPPN is a model for embryonic development, more precisely it implements the specialization of a cell based on its local position. The nodes encode various mathematical activation functions, that process the input coordinates of the cell. By e.g. Gaussian or periodic functions, desirable features such as symmetry or repetition can be realized. NEAT evolves a neural network model by starting from a simple network and increasing its complexity.

Recently, CPPN-NEAT has been applied to topology optimization. In [56] the Cartesian and polar coordinates of the design space mesh are used as inputs to a CPPN. The output of the network determines the materials type, and a multi-material structure is obtained. The objective function is defined as the ability of the structure to match a frequency profile. The representation is depicted in Fig. 10.

In [57] two CPPN nets encode on one hand presence and absence of material and on the other hand type of the material, respectively. Although the problem is not explicitly considered as topology optimization, it considers the distribution of the materials in a 3-dimensional voxel space domain. Again coordinates are used as inputs. The method is applied to optimize the ratio of generated energy to total cost of a photovoltaic collector.

VII. Discussion

The preceding sections present an overview and categorization of the existing representations that can be used for the

problem of continuum topology optimization of mechanical structures with EC. Three categories of representations are identified: Grid, Geometric and Indirect Representations. In this section, the representations are discussed and chances and challenges are outlined.

Grid Representation: A Grid Representation is relatively simple to realize, since it does not require implementation of geometric boundaries. The most common and simple mapping utilizes the computational grid of a finite element analysis model. This simplicity results in a reduced scalability, since the resolution of the discretization directly influences the number of variables and therefore Grid Representations become prohibitive expensive for fine grids. The search space dimensionality is higher and a lot of solutions can be generated which are not relevant. Hence, care has to be take when defining variational operators in order to reduce the amount of infeasible candidates, for instance solutions that show numerical instabilities or a lack of connectedness. Yet, within the practical limit on the number of elements, the optimum phenotype can always be represented in genotype-space, since full design freedom is maintained and no assumptions on the solutions are implied. Thus Grid Representations might have the highest potential for small-sized problems, for which little or no previous knowledge is available.

Geometric Representation: A Geometric Representation provides a direct control over the potential complexity of the structural design, by choosing the number of shape primitives. It decouples the mesh resolution from the dimensionality of the genotype by limiting the number of components within the structure. This indicates that prior knowledge about properties of good solutions is required, thus each Geometric Representation is most likely suited for a limited set of objective functions. In order to fully utilize the potential of this method, suitable shape primitives have to be defined, that are a good trade-off between design flexibility and search space dimensionality. Also, Geometric Representations may be chosen in order to bias the solution such that optimized structures can be easily fabricated from components. However, Geometric Representations tentatively involve a higher implementation effort, since a geometry mapping from the genotype to the finite element mesh is required. Geometric Representations might be most feasible for evolving structures with limited number of features, when valid assumptions on choosing shape primitives are available.

Indirect Representation: Optimizing a generative model instead of the structure might be able to exploit the potential of evolutionary computation, by creation and re-usage of problem-specific design patterns for evolving creative solutions. Tentatively, many structural details can be encoded with few optimization variables, when compared to Grid or Geometric Representations. However, finding and implementing a suitable Indirect Representation can be a challenging task and the simulation can be computationally expensive and it can be unclear which phenotypes are in fact represented. Currently, the literature field on Indirect Representations is the smallest of the three categories. More research is required to propose and evaluate new approaches.

Although the majority of approaches in literature applies Grid Representations, many interesting alternatives exist. Generally, considering reference and application, might be an academically and practically rewarding direction, as indicated by the applications of the cellar division representation in Sec. VI-A. The minimum compliance problem can serve as a first standard reference test case (see Example in Fig. 1).

The authors hope that by the review of literature and by outlining strengths and weaknesses of the approaches, interesting directions for research will be picked up by the EC community, and potentially lead to new approaches for the topology optimization problem.

REFERENCES

[1] R. Kicinger, T. Arciszewski, and K. De Jong, "Evolutionary computation and structural design: A survey of the state-of-the-art," *Computers & Structures*, vol. 83, no. 23, pp. 1943–1978, 2005.

[2] E. Andreassen, A. Clausen, M. Schevenels, B. Lazarov, and O. Sigmund, "Efficient topology optimization in matlab using 88 lines of code," *Structural and Multidisciplinary Optimization*, vol. 43, no. 1, pp. 1–16, 2011.

[3] O. Sigmund and K. Maute, "Topology optimization approaches," *Structural and Multidisciplinary Optimization*, vol. 48, no. 6, pp. 1031–1055, 2013.

[4] M. Bendsøe and O. Sigmund, *Topology Optimization Theory, Methods and Applications*, 2nd ed. Springer Verlag Berlin, 2004.

[5] N. van Dijk, K. Maute, M. Langelaar, and F. van Keulen, "Level-set methods for structural topology optimization: a review," *Structural and Multidisciplinary Optimization*, vol. 48, no. 3, pp. 437–472, 2013.

[6] O. Sigmund, "On the usefulness of non-gradient approaches in topology optimization," *Structural and Multidisciplinary Optimization*, vol. 43, pp. 589–596, 2011.

[7] C. Ortmann and A. Schumacher, "Graph and heuristic based topology optimization of crash loaded structures," *Structural and Multidisciplinary Optimization*, vol. 47, no. 6, pp. 839–854, 2013.

[8] A. Eiben and J. Smith, *Introduction to Evolutionary Computation*, 2nd ed., ser. Natural Computing. Springer Berlin Heidelberg, 2007.

[9] E. Sandgren, E. Jensen, and J. Welton, "Topological design of structural components using genetic optimization methods," in *Winter Annual Meeting of the American Society of Mechanical Engineers*, ser. Sensitivy Analysis and Optimization with Numerical Methods, vol. AMD-115, 1990, pp. 31–43.

[10] C. D. Chapman, K. Saitou, and M. J. Jakiela, "Genetic algorithms as an approach to configuration and topology design," *Journal of Mechanical Design*, vol. 116, no. 4, pp. 1005–1012, 1994.

[11] Y. Kim and O. De Weck, "Progressive structural topology optimization by variable chromosome length genetic algorithm," in *China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems, M3*, 2004.

[12] S. Wang and K. Tai, "Structural topology design optimization using genetic algorithms with a bit-array representation," *Computer Methods in Applied Mechanics and Engineering*, vol. 194, no. 36-38, pp. 3749 – 3770, 2005.

[13] R. Balamurugan, C. Ramakrishnan, and N. Singh, "Performance evaluation of a two stage adaptive genetic algorithm (tsaga) in structural topology optimization," *Applied Soft Computing*, vol. 8, no. 4, pp. 1607 – 1624, 2008, soft Computing for Dynamic Data Mining.

[14] G.-C. Luh and C.-H. Chueh, "Multi-modal topological optimization of structure using immune algorithm," *Computer Methods in Applied Mechanics and Engineering*, vol. 193, no. 36, pp. 4035–4055, 2004.

[15] G.-C. Luh, C.-Y. Lin, and Y.-S. Lin, "A binary particle swarm optimization for continuum structural topology optimization," *Applied Soft Computing*, vol. 11, no. 2, pp. 2833 – 2844, 2011, the Impact of Soft Computing for the Progress of Artificial Intelligence.

[16] J. Lu and Y. Chen, "Performance evaluation of particle swarm optimization and solid isotropic material with penalization in topology optimization," in *Computational Intelligence for Measurement Systems and Applications (CIMSA), 2012 IEEE International Conference on*. IEEE, 2012, pp. 97–101.

[17] C.-Y. Wu and K.-Y. Tseng, "Topology optimization of structures using modified binary differential evolution," *Structural and Multidisciplinary Optimization*, vol. 42, no. 6, pp. 939–953, 2010.

[18] D. Sharma, K. Deb, and N. Kishore, "Towards generating diverse topologies of path tracing compliant mechanisms using a local search based multi-objective genetic algorithm procedure," in *Evolutionary Computation, 2008. CEC 2008.(IEEE World Congress on Computational Intelligence). IEEE Congress on*. IEEE, 2008, pp. 2004–2011.

[19] D. Sharma, K. Deb, and N. Kishore, "Customized evolutionary optimization procedure for generating minimum weight compliant mechanisms," *Engineering Optimization*, vol. 46, no. 1, pp. 39–60, 2014.

[20] D. Sharma and K. Deb, "Generation of compliant mechanisms using hybrid genetic algorithm," *Journal of The Institution of Engineers (India): Series C*, vol. 95, no. 4, pp. 295–307, 2014.

[21] P. Y. Shim and S. Manoochehri, "Generating optimal configurations in structural design using simulated annealing," *International journal for numerical methods in engineering*, vol. 40, no. 6, pp. 1053–1069, 1997.

[22] C. Li, T. Hiroyasu, and M. Miki, "Stress-based crossover operator for structural topology optimization," *Journal of Computational Science and Technology*, vol. 2, no. 1, pp. 46–55, 2008.

[23] A. Kaveh, B. Hassani, S. Shojaee, and S. Tavakkoli, "Structural topology optimization using ant colony methodology," *Engineering Structures*, vol. 30, no. 9, pp. 2559 – 2565, 2008.

[24] S.-C. Kim, D.-H. Chang, K.-S. Yoo, and S.-Y. Han, "Development of modified ant colony optimization algorithm for compliant mechanisms," in *SIMUL 2012, The Fourth International Conference on Advances in System Simulation*, 2012, pp. 152–157.

[25] J. A. Madeira, H. Rodrigues, and H. Pina, "Multi-objective optimization of structures topology by genetic algorithms," *Advances in Engineering Software*, vol. 36, no. 1, pp. 21–28, 2005.

[26] J. Madeira, H. Rodrigues, and H. Pina, "Multiobjective topology optimization of structures using genetic algorithms with chromosome repairing," *Structural and Multidisciplinary Optimization*, vol. 32, no. 1, pp. 31–39, 2006.

[27] Y. Xie and G. P. Steven, "A simple evolutionary procedure for structural optimization," *Computers & structures*, vol. 49, no. 5, pp. 885–896, 1993.

[28] X. Huang and Y. Xie, *Evolutionary Topology Optimization of continuum structures*. John Wiley & Sons, 2012.

[29] J. Madeira, H. Pina, and H. Rodrigues, "Ga topology optimization using random keys for tree encoding of structures," *Structural and Multidisciplinary Optimization*, vol. 40, no. 1-6, pp. 227–240, 2010.

[30] M. Schoenauer, "Shape representations and evolution schemes." *Evolutionary Programming*, vol. 5, 1996.

[31] H. Hamda and M. Schoenauer, "Topological optimum design with evolutionary algorithms," *Journal of Convex Analysis*, vol. 9, no. 2, pp. 503–518, 2002.

[32] J. Denies, "Métaheuristiques pour l'optimisation topologique: application à la conception de dispositifs électromagnétiques," Ph.D. dissertation, École normale supérieure de Cachan-ENS Cachan, 2013.

[33] A. Saxena, "On optimization of 2d compliant mechanisms using honeycomb discretization with material-mask overlay strategy," in *ASME 2007 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 2007, pp. 1331–1341.

[34] N. Padhye, "Topology optimization of compliant mechanism using multi-objective particle swarm optimization," in *Proceedings of the 2008 GECCO conference companion on Genetic and evolutionary computation*, ser. GECCO '08. New York, NY, USA: ACM, 2008, pp. 1831–1834.

[35] A. Saxena, "An adaptive material mask overlay method: Modifications and investigations on binary, well connected robust compliant continua," *J. Mech. Des*, vol. 133, no. 4, p. 041004, 2011.

[36] A. Saxena, "Are circular shaped masks adequate in adaptive mask overlay topology synthesis method?" *Journal of Mechanical Design*, vol. 133, no. 1, p. 011001, 2011.

[37] A. Saxena, "Topology design with negative masks using gradient search," *Structural and Multidisciplinary Optimization*, vol. 44, no. 5, pp. 629–649, 2011.

[38] S. Wang and K. Tai, "Graph representation for structural topology optimization using genetic algorithms," *Computers & Structures*, vol. 82, no. 20-21, pp. 1609 – 1622, 2004.

[39] K. Tai and T. Chee, "Design of structures and compliant mechanisms by evolutionary optimization of morphological representations of topology," *Journal of Mechanical Design*, vol. 122, no. 4, pp. 560–566, 2000.

[40] K. Tai, G. Y. Cui, and T. Ray, "Design synthesis of path generating compliant mechanisms by evolutionary optimization of topology and shape," *Journal of Mechanical Design*, vol. 124, no. 3, pp. 492–500, 2002.

[41] N. Wang and K. Tai, "Target matching problems and an adaptive constraint strategy for multiobjective design optimization using genetic algorithms," *Computers & Structures*, vol. 88, no. 19-20, pp. 1064 – 1076, 2010.

[42] R. Balamurugan, C. Ramakrishnan, and N. Swaminathan, "A two phase approach based on skeleton convergence and geometric variables for topology optimization using genetic algorithm," *Structural and Multidisciplinary Optimization*, vol. 43, no. 3, pp. 381–404, 2011.

[43] M. Sauter, "A graph-based optimization method for the design of compliant mechanisms and structures," Ph.D. dissertation, Swiss Federal Institute of Technology Zurich, 2008.

[44] F. Ahmed, B. Bhattacharya, and K. Deb, "Constructive solid geometry based topology optimization using evolutionary algorithm," in *Proceedings of Seventh International Conference on Bio-Inspired Computing: Theories and Applications (BIC-TA 2012)*, ser. Advances in Intelligent Systems and Computing, J. C. Bansal, P. K. Singh, K. Deep, M. Pant, and A. K. Nagar, Eds. Springer India, 2013, vol. 201, pp. 227–238.

[45] M. De Ruiter and F. Van Keulen, "Topology optimization using a topology description function," *Structural and Multidisciplinary Optimization*, vol. 26, no. 6, pp. 406–416, 2004.

[46] M. Bujny, N. Aulig, M. Olhofer, and F. Duddeck, "Hybrid evolutionary approach for level set topology optimization," in *Evolutionary Computation (CEC), 2016 IEEE Congress on*, to be published (accepted), July 2016.

[47] G. S. Hornby, "Generative representations for evolving families of designs," in *Genetic and Evolutionary Computation - GECCO 2003*. Springer, 2003, pp. 1678–1689.

[48] M. H. Kobayashi, H.-T. C. Pedro, R. M. Kolonay, and G. W. Reich, "On a cellular division method for aircraft structural design," *The Aeronautical Journal*, vol. 113, pp. 821–831, 2009.

[49] H.-T. C. Pedro and M. H. Kobayashi, "On a cellular division method for topology optimization," *International Journal for Numerical Methods in Engineering*, vol. 88, no. 11, pp. 1175–1197, 2011.

[50] B. Stanford, P. Beran, and M. Kobayashi, "Aeroelastic optimization of flapping wing venation: A cellular division approach," *AIAA Journal*, vol. 50, no. 4, pp. 938–951, 2012.

[51] T. Steiner, Y. Jin, and B. Sendhoff, "A cellular model for the evolutionary development of lightweight material with an inner structure," in *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2008)*. New York, NY, USA: ACM, 2008.

[52] T. Steiner, J. Trommler, M. Brenn, Y. Jin, and B. Sendhoff, "Global shape with morphogen gradients and motile polarized cells," in *Proceedings of the 2009 IEEE Congress on Evolutionary Computation, Trondheim*. IEEE, 2009.

[53] N. Aulig and M. Olhofer, "Neuro-evolutionary topology optimization of structures by utilizing local state features," in *Proceedings of the 2014 Conference on Genetic and Evolutionary Computation*, ser. GECCO '14. New York, NY, USA: ACM, 2014, pp. 967–974.

[54] N. Aulig and M. Olhofer, "State-based representation for structural topology optimization and application to crashworthiness," in *Evolutionary Computation (CEC), 2016 IEEE Congress on*, to be published (accepted), July 2016.

[55] K. O. Stanley, "Compositional pattern producing networks: A novel abstraction of development," *Genetic programming and evolvable machines*, vol. 8, no. 2, pp. 131–162, 2007.

[56] N. Cheney, E. Ritz, and H. Lipson, "Automated vibrational design and natural frequency tuning of multi-material structures," in *Proceedings of the 2014 Conference on Genetic and Evolutionary Computation*, ser. GECCO '14. New York, NY, USA: ACM, 2014, pp. 1079–1086.

[57] R. V. R. Evins and S. Burges, "Multi-material compositional pattern-producing networks for form optimisation." in *European Conference on the Applications of Evolutionary Computation, EvoApplications*, 2014.