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Predictive Risk Maps

Florian Damerow[1] and Julian Eggert[2]

***Abstract*— This paper addresses the problem of risk assessment in dynamic traffic environments for future behavior evaluation and planning. Risk assessment has to be driven from behavioral needs of the acting scene entity to evaluate the best possible future behavior. We present a general approach for a temporally continuous future risk estimation. Using this risk estimation, we introduce predictive risk maps based on the variation of the acting entities' possible dynamics. The predictive risk maps indicate how critical a certain behavior will be in the future and we use them for future behavior evaluation and planning. Explicitly introducing risk indicators for the collision case of moving entities we show the generality of our approach by applying it to several different traffic scene types.**

I. Introduction

Future Advanced Driver Assistance Systems (ADAS) aim to relief the strain on the driver by providing him with elaborate warning systems or by overtaking more and more responsibility up to a level of partially automated driving. We strive for a system that is able to securely support driving tasks, especially in inner-city traffic scenarios where traffic scene analysis is inherently difficult if more than a few scene entities are involved.

For complex scenarios, traffic scene analysis has to be guided from behavioral needs of the acting entity in order to determine what to do next. This implies an evaluation of possible dynamics in the scene and its consequences in terms of behavioral risk. For this purpose, we introduce an estimation of behavioral risk by evaluating the relation between the own and other scene entities' predicted dynamics. As a result, this allows to use the estimation of behavioral risk in order to improve the own behavior.

In previous related work, risk is modeled in terms of collision probability in order to use it for trajectory evaluation, selection or planning. In [1] the Time-To-Collision (TTC) is used to determine the collision probability as a risk measure. In [2] a stochastic reachable set of states is calculated for each traffic participant. The collision probability is then derived using those stochastic reachable sets of states. [3] follows a similar approach, where inevitable collision states (ICS) are used as a risk measure. A robot movement is determined in order to avoid those ICS. For a large number of scene entities the set of ICS becomes unacceptably large and no safe movement can be determined for the robot anymore. Thus they extend the idea of ICS to probabilistic collision states (PCS) allowing a certain estimation probability of collision for the robot movement. In [4] a risk measure is introduced, that is also based on reachable space but takes the expected collision damage in terms of an impact factor into account. In [5] risk is used in order to improve the motion prediction of other traffic scene entities. The risk measure is in terms of collision probability and allows the generation of risk functions in the action space. These functions can then be used to determine the probability that a traffic scene entity will move according to a certain trajectory, assuming that traffic participants preferably choose low risk trajectories. The time is considered in the trajectory prediction, however, the risk functions do not have an explicit temporal component, so that a temporal behavior planning is not directly possible.

The estimation of collision probability alone is not always sufficient to generate the best possible behavior. As an example consider [6], where in the case of unavoidable car-to-pedestrian collision a risk function in terms of pedestrian injury is used in pedestrian protection scenarios. The injury risk function is based on the collision velocity. A similar approach is taken in the presented paper: A risk estimation always involves the calculation of an event probability along with the estimation of the expected damage in case of a critical event.

For behavior estimation, many approaches use risk evaluation to directly select a trajectory out of a set of given alternatives. On the contrary, spatial risk maps in form of occupancy grids are often used for full path planning (see e.g. in [7], which uses an A* planning algorithm). In these approaches, motion planning for highly dynamic situations is not straightforward, since occupancy grids have only limited capabilities to include temporal content, e.g. by temporal smearing of the occupancy probabilities using the predicted object motion.

To address behavior evaluation in dynamic traffic environments, in this paper we propose (i) an approach towards temporally continuous future risk estimation with several moving scene entities, based on their predicted states, (ii) "predictive risk maps" as means to represent and superpose future dynamic risks of various types from different sources, (iii) a behavior evaluation, selection and planning scheme based on the predictive risk maps, which minimizes risk and maximizes efficiency.

The paper is organized as follows: In section II we introduce a general method for future risk estimation in dynamic scenes. Followed by section III where we apply the risk estimation to the collision case of moving entities. In section IV we build a risk map by varying the ego-

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car trajectory defined by a variation parameter. Finally in section V we use risk maps to evaluate and select the ego-car behavior in different traffic situations. Simulations results are presented and discussed.

II. Risk Estimation for Dynamic Scenes

Risk is the expectation value of the cost or benefit related to critical future events (see [8], [9] for a definition). In traffic scenarios, this implies estimating the probability of the future events from some starting point in time, as well as the damage that the critical event implies if it occurs. We then have two factors contributing to the risk estimation [1]:

$$P(\text{damage}|\text{states}) := \\ P(\text{damage}|\text{states}, \text{event} = \text{true})\, P(\text{event} = \text{true}|\text{states}). \quad (1)$$

With "states", we usually mean the current states of the involved scene elements and their future extrapolation using some chosen prediction model. E.g. in the case of cars, we would choose a kinematic model with longitudinal and lateral components. If an event has kicked in, the damage is usually a function of the states at the moment of the event.

The damage probability $P(\text{damage}|\text{states}, \text{event} = \text{true})$ can be described at a desired degree of accuracy based on physical and empirical models. For example, for the car-to-car collision event, we can approximate the damage calculation based on a partially elastic collision model, with the resulting damage being a function of the involved kinetic energy and the force evoked by the momentum change at collision [2].

More important for planning in dynamic environments is the second term $P(\text{event} = \text{true}|\text{states})$, which describes the probability that a future critical event happens, starting from the current states. For its implementation we assume that we can calculate the risk for different prediction times from the time-course of the states of the involved entities.

As an example, let us consider the car-to-car collision risk of two cars. Starting at $t = 0$, we look at the generalized "trajectories" (i.e., the sequence of states over time)

$$\text{tr} = \{\text{state(t)} \,|\, t = 0, ..., T\} \ , \quad (2)$$

which are gained starting from state(0) (the current state) using an appropriate prediction model. For the two cars, the states are mainly composed of their kinematic components, so that we use **e**go car and **o**ther car trajectories [3]

$$\begin{aligned} \text{tr}_e &= \{x_e(t), v_e(t), a_e(t), t \,|\, t = 0, ..., T\} \ , \\ \text{tr}_o &= \{x_o(t), v_o(t), a_o(t), t \,|\, t = 0, ..., T\} \ . \end{aligned} \quad (3)$$

We apply a continuous risk estimation on these trajectories by comparing the states pairwise at the same moments of predicted time t, under the assumption that a critical event will happen at that moment. For the collision case, the risk will e.g. depend on the distance, the relative orientation at impact, and the involved velocities of the two cars. We then get

$$\text{risk}(t) = F[x_e(t) - x_o(t), v_e(t), v_o(t), t] \text{ for } t = 0, ..., T \ . \quad (4)$$

For the behavior planning as introduced in the following sections, a continuous spatial risk function is required. Thus we convert this time-dependent risk function into a risk function over traveled ego-car distance, $R(x_e) := \text{risk}[t(x_e)]$, which can be done as long as $x_e(t)$ is invertible, meaning that we transform between $t(x_e)$ and $x_e(t)$.

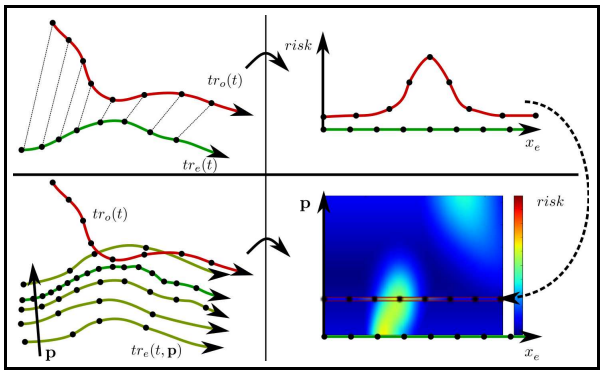

Fig. 1. Upper left: Two predicted spatio temporal trajectories (e.g. of **e**go and **o**ther car tr_e and tr_o. Upper right: By comparison at same time points (upper left, connecting lines between trajectories), we calculate the risk function $R(x_e)$ over ego-car trajectory distance. Lower left: Risk function calculated for different ego-car trajectories parametrized by $\mathbf{p}$. Lower right: Composition into one *predictive risk map* $R(x_e, \mathbf{p})$. Spots in the map indicate areas of future high risk that should be avoided.

Fig. 1 upper left shows the state predictions of two traffic participants, with the trajectories being used in a pairwise and point wise comparison to calculate the risk function. In Fig. 1 upper right, the risk function $R(x_e)$ is shown, which exhibits a pronounced maximum at the point where the trajectories come closest to each other, and with a width that depends on how far the predictions reach into the future.

There are two issues to consider here. (i) We assume that the risk function is continuous, even if e.g. the distances are such that they exceed the cars' dimensions and a spatial overlap (required for collision) is not given. This takes into account that state estimations may be uncertain, either due to prediction variability or to measurement and sensor effects, so that a collision may even be possible (although with a very small probability) for larger distances. (ii) We include an explicit time-dependence that accounts for the effect that states which are further away in the future are more uncertain than states with a close prediction horizon (with the consequence that the risk will also be more delocalized in space and time).

III. Risk Indicators for the Collision Case

For the collision case, the probability of a collision event increases with small distances and is maximal when the cars get closest. We therefore introduce the closest encounter

[1] Here we consider only one event type, however critical events can be of several different types, with the most common ones (for traffic domains) being the collision between two traffic participants or between a traffic participant and the environment.

[2] In this work we are only interested in a qualitative model of the damage term, so that we will not go into further detail here.

[3] If the other traffic participant is stationary, then we take e.g. only its position and its spatial extension for calculating if the ego-car gets close to it.

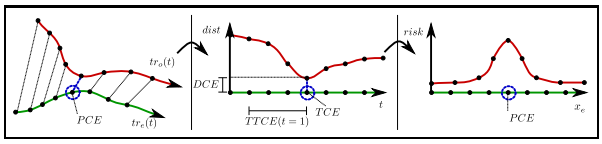

Fig. 2. Left: Two spatio temporal trajectories. Middle: calculation of the predicted spatial distance over time and extraction of characteristic risk parameters (PCE, TCE, DCE and TTCE). Right: Resulting continuous risk function evaluated by those parameters. Collision risk is maximal where the distance DCE between both cars is minimal (at $x_e = \text{PCE}, t = \text{TCE}$).

between a pair of trajectories as a criticality indicator for collision risk. We use the following parameters:

- DCE: Distance of Closest Encounter

$$\text{DCE} = \min_t ||x_e(t) - x_o(t)|| \tag{5}$$

- TCE: Time of Closest Encounter

$$\text{TCE} = \text{argmin}_t ||x_e(t) - x_o(t)|| \tag{6}$$

- TTCE: Time To Closest Encounter

$$\text{TTCE}(t) = TCE - t \tag{7}$$

- PCE: Point of Closest Encounter

$$\text{PCE} = x_e(\text{TCE}) \tag{8}$$

Starting from (1), we now model the probability of a future collision event and the damage probability at the collision as functions of DCE and TCE. For the risk as a function of prediction time, we assume the collision event probability to be a Gaussian with a maximum around PCE, with an amplitude that depends on DCE and TCE, in a way that closer DCEs lead to a higher peaks (higher maximal collision probability at PCE) and longer TCEs lead to a broader width. According to (1) we set

$$\begin{aligned} &\text{risk}(t) = \\ &P(\text{event} = \text{true}, \text{t}_{event} = t|\text{states})\, P(\text{damage}|\text{event}_t, \text{states}), \end{aligned} \tag{9}$$

where

$$\begin{aligned} &P(\text{event} = \text{true}, \text{t}_{event} = t|\text{states}) = \\ &P(\text{event} = \text{true}|\text{states}) P(\text{t}_{event} = t|\text{states}) \ , \end{aligned} \tag{10}$$

with the event and damage probabilities

$$P(\text{event} = \text{true}|\text{states}) = e^{-\frac{1}{2\sigma_1^2 \text{TCE}^2}\text{DCE}^2}, \tag{11}$$

$$P(\text{t}_{event} = t|\text{states}) = \frac{1}{2\pi\sigma_2^2 \text{TCE}^2} e^{-\frac{1}{2\sigma_2^2 \text{TCE}^2}\text{TTCE}^2(\text{t})}, \tag{12}$$

$$P(\text{damage}|\text{event}_t, \text{states}) = e^{-\frac{1}{2\sigma_3^2 \text{TCE}^2}[v_e'(t) - v_e(t)]^{-2}}, \tag{13}$$

where $\sigma_1, \sigma_2, \sigma_3$ are model parameters. In the event probability terms (11) and (12), TTCE is the time left for the system to react before point of closest encounter (and therefore, maximal risk of collision) is reached. In combination with the DCE-term, this is used as a general, continuous indicator of collision risk similar to TTC. The DCE-term describes how close two entities will get at the PCE, with a large DCE resulting in a low maximal probability of collision and therefore a low maximal risk value.

In the damage term (13) we used the velocities of the ego-car after and before the collision $v_e'(t)$ and $v_e(t)$ to model the threat for a human driver proportional to the acceleration that he/she encounters caused by the impact (approximated by the velocity change).

The TCE terms in the denominators model the risk dependency over time, under the assumption that risky events which are still further away in the future have a lower probability of occurrence, because for larger TCEs there is a larger uncertainty in the possible future behaviors, that would lead to an avoidance of the predicted risk. E.g., if the ego-car or the other car break or accelerate differently than according to their predicted trajectory, a collision may be avoided.

Fig. 2 middle shows the calculation of the indicators DCE and TCE from the comparison of the positions at predicted trajectories (left), as well as the resulting risk function $R(x_e)$ based on (9). If there is more than one minimum of the distance (which in standard longitudinal road traffic is rarely the case), we additively superpose several factors corresponding to the respective DCEs and TCEs.

IV. Predictive Risk Maps

A major target of the estimation of future risk in terms of the own and other scene entities' predicted behavior is the evaluation, selection and planning of the ego-cars' future behavior.

To analyze different behavior alternatives of the ego-car, we vary the ego-car trajectory systematically using its prediction parameters $\mathbf{p}$. This could be e.g. the ego-car velocity $\mathbf{p} = v_e$ to evaluate different velocity alternatives. For each selection of $\mathbf{p}$, we estimate the predicted trajectory and calculate the future risk function $R(x_e, \mathbf{p})$ according to section II. Each of the gained risk functions can now be evaluated according to overall risk and efficiency considerations. In Fig. 1 lower left, we show a set of possible trajectories of the other traffic participant. From each of them, in interaction with the ego-car trajectory, we gain a risk function, and the set of risk functions is then composed into a *predictive risk map* that allows to localize the future risks for different ego-car behavior options (Fig. 1 lower right).

The risk maps can then be used for behavior planning, by searching for the most favorable path across the map from the current state to a desired target zone, considering risk and efficiency minimization constraints along the path, as illustrated in Fig. 3. This can be achieved by using standard planning algorithms or other minimization techniques. In section V we explain a gradient based evaluation of predictive risk map based behavior for different typical traffic situations.

In the risk maps, each traffic scene entity results in a risk spot or risk area. Due to the TCE-influence, risk spots further away in time are lower and broader and over time, as they come closer, they get higher and sharper, i.e., more localized. This has the desirable effect that it allows to plan more

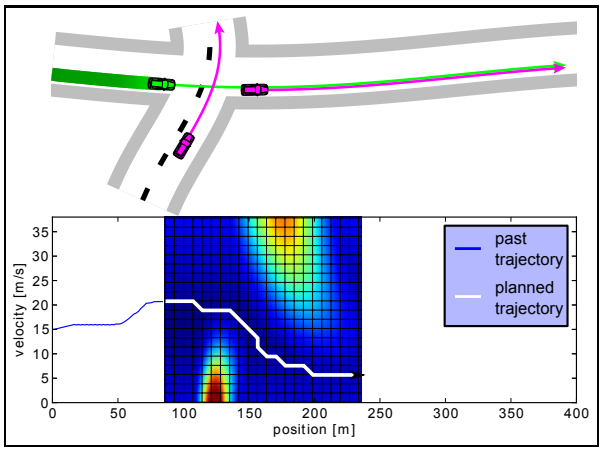

Fig. 3. Behavior Planning using Predictive Risk Maps. Blue curve: The past driven trajectory. White curve: The future cost-optimized trajectory resulting from the selection or planning using the predictive risk map which allows the evaluation of future risk for different behavior alternatives.

coarsely for the distant time horizon and readjust planning when the risks become more sharply localized.

Generally, several risk functions that correspond to a variation of the other cars hypothetical behavior via its predicted trajectory can be superposed, leading to a larger spread of the peaks in the risk function. However, the risk functions are targeted for a prototypical situation, i.e., if the other traffic participants exhibit a completely different behavior (e.g. continuing straight at an intersection instead of turning) this should be considered as a separate case.

Nevertheless, different risk sources can be combined in one risk function. This applies to risks of a different type, like e.g. the risk of crashing without external interaction when driving too fast around a curve, or to collisions with additional traffic participants, like other cars, pedestrians or road and infrastructure elements. In a more rigorous treatment it can be shown that the critical event rates from different risks can be added, since at the event of the accident usually only a single risk kicks in.

V. Behavior Evaluation for Different Apllication Scenarios

A. Behavior Selection

Our system acts in dynamic and changing environments. Predictive risk maps rely on the prediction of scene entity trajectories. As the prediction becomes more uncertain for moments further away in time we need to re-evaluate the selected behavior from time to time. This includes the re-calculation of predictive risk and re-selecting or re-planning of further trajectories. As we assume a continuously changing environment the risk maps will change continuously as well. Thus the selected behavior will not differ too much if the time Δt between reevaluations is sufficiently small.

Given a predictive risk map as described above the target is to select the future behavior in a way that minimizes risk and maximizes efficiency at the same time.

As described in section IV we build a predictive risk map for the collision case by creating a variation of the ego-car trajectory using the ego car velocity as a variation parameter $\mathbf{p} = v_e$. The predictive risk map $R(x_e, v_e)$ indicates how risky the choice of a certain velocity v_e along the longitudinal position on the ego-car trajectory x_e will be.

Cost Function: For selecting the future behavior we combine the risk map with an efficiency measure. Here we use the deviation from a desired travel velocity as an efficiency measure. The cost function $\text{cost}(x_e, v_e)$ is thus a combination of risk R and travel cost tc. We use the max operator for combining those terms,

$$\text{cost}(x_e, v_e) = \max(R(x_e, v_e), tc(v_e)) \ , \tag{14}$$

with

$$\text{tc}(v_e) = tc_0 + m \cdot |v_{e,des} - v_e| \ , \tag{15}$$

where $v_{e,des}$ is the desired velocity, m the slew rate and tc_0 the minimal travel cost at the desired velocity. We choose tc_0 and m so that $\max(\text{tc}) \leq \max(R)$. The travel cost form a cost valley with minimal cost at $v_{e,des}$, increasing linearly with the deviation of the given velocity from the desired velocity.

Gradient-Descent-Like Behavior Selection: Using the cost function we can select the behavior at every time step according to a gradient-descent-like approach. First we determine the selected cost, the cost of a selected trajectory segment, as the maximum risk of each trajectory variation by

$$\text{cost}_{selected}(v_e) = \max_{x_e}(\text{cost}(x_e, v_e)) : x_e \in [x_{e,cur}, x_{e,cur} + \Delta x], \tag{16}$$

where $x_{e,cur}$ is the current longitudinal spatial position and Δx defines the length of the prediction horizon in terms of spatial position. The acceleration control input for the ego-car u can be determined as the negative selected cost gradient at the current velocity $v_{e,cur}$

$$u = -b \cdot \frac{\partial}{\partial v_e} \text{cost}_{selected}(v_{e,cur}) \ . \tag{17}$$

B. Simulation Results

In the following we apply the introduced predictive risk maps combined with the gradient descent like behavior selection approach to several different traffic scenarios types to show to generality of our approach.

Intersection Scenario (Speeding up): The first traffic scenario is an intersection scenario, where the ego-car is approaching an intersection on a minor road and another car is on the main road. The predicted trajectory of the other car on the main road is, that it continues driving with the constant velocity straight across the intersection.

The ego-car plans to drive straight over the intersection. Building a trajectory variation for the ego-car defined by the ego-car velocity as explained before, we gather a risk map as shown in Fig. 4 (top). This risk map shows a spot of high risk with the peak at the future ego-car position and velocity where the collision probability would be maximal.

The risk diffusion property can be seen by advancing from Fig. 4 (top) to (middle). The risk peak gets higher and sharper as it gets closer in time.

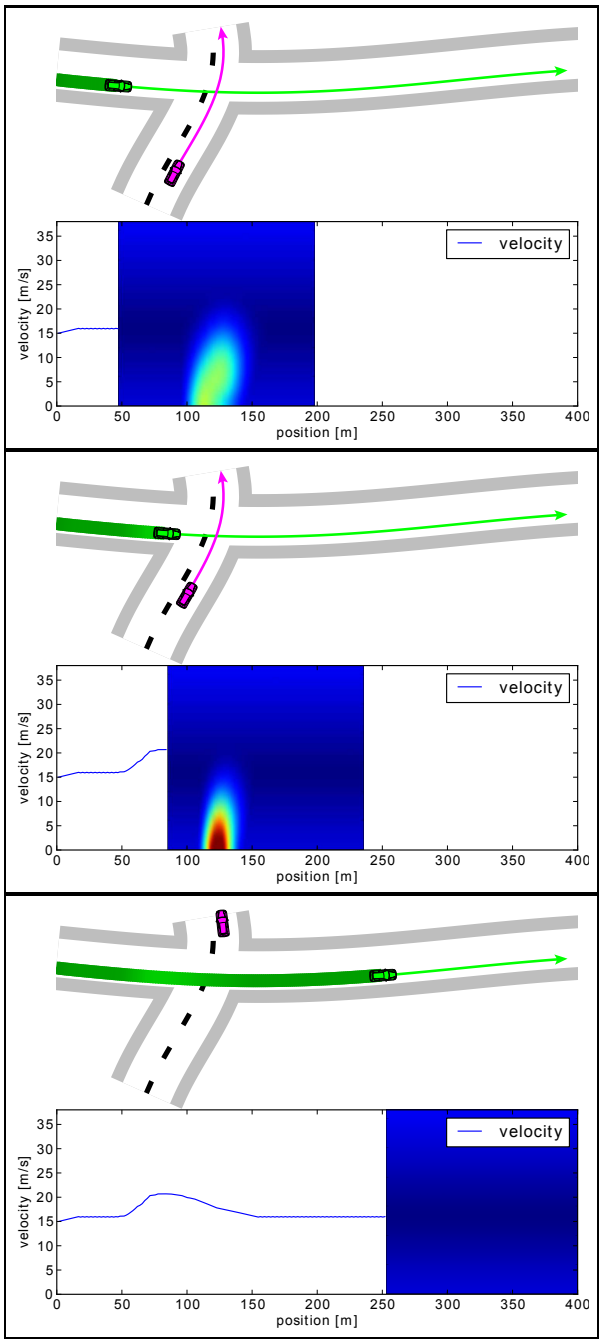

Fig. 4. Intersection Scenario I (Speeding up): Ego-car velocity v_e profiles (blue) at different points in time $t_1 < t_2 < t_3$ (top to bottom) of the ego-car (green) along the longitudinal ego-car path x_e. Each driven velocity course connects to a predictive risk map combined with the efficiency measure described in section V-A for evaluating future behavior. Using the predictive risk map for behavior selection results in speeding up to pass in front of the other car and then slowing down back to the desired travel velocity. The model parameters are set to $tc_0 = 0$, $m = 0.005$, $v_{e,des} = 15$ and $b = 5$.

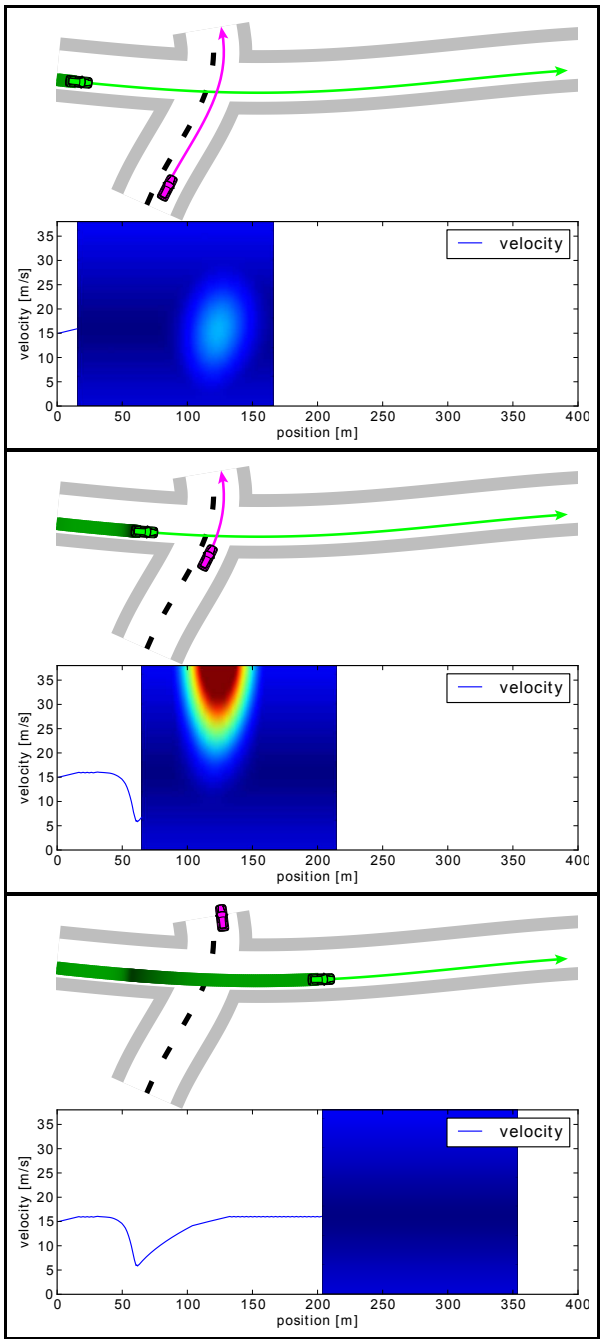

Fig. 5. Intersection Scenario II (Slowing down): Same scenario as in Fig. 4 but with a faster crossing car. Using the predictive risk map for behavior selection results in braking to let the other car pass.

Applying the gradient-descent-like behavior selection approach to the predictive risk map results in a selected trajectory with higher ego-car velocity in order to reduce the maximum predicted risk. Thus as shown in the time line in Fig. 4 (top to bottom) the ego-car accelerates first to pass the intersection with minimized cost (minimal risk with maximal efficiency) and then reduces the speed back to the desired travel velocity.

Intersection Scenario II (Slowing down): Here we investigate the same intersection scenario again. This time the other car on the main road is slightly faster. Thus the resulting risk map in Fig. 5 (top) shows the risk peak shifted upwards, because the collision probability is higher for ego-car trajectories with higher velocities.

As shown in Fig. 5 (top to bottom), applying the gradient-descent-like behavior selection approach results in slowing down in order to reduce the costs and thus also risk. The

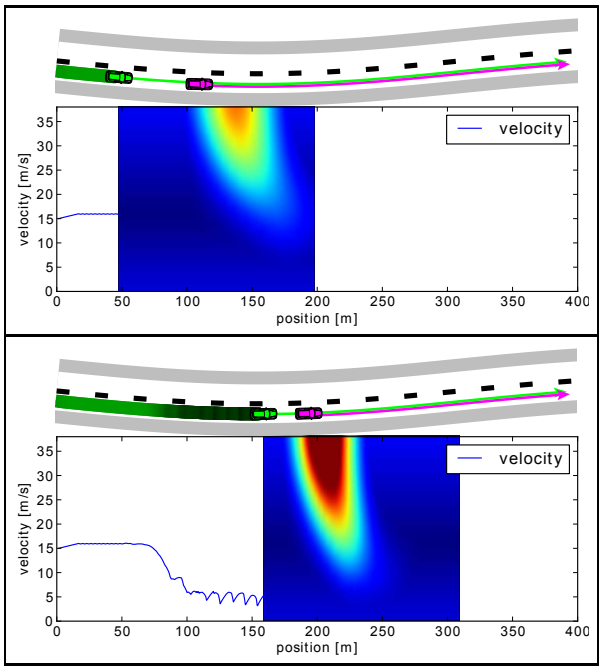

Fig. 6. Approaching Leading Car Scenario: Using the predictive risk map for behavior selection results in braking to follow the leading car at same speed.

ego-car lets the other car pass and then speeds up back to the desired travel velocity.

Approaching Leading Car Scenario: Here we analyze a scenario where the ego-car is driving along a road and approaching a slower car in front on the same lane.

The predictive risk map illustrated in Fig. 6 shows a curved area of high risk. For ego-car trajectories with velocities greater than the velocity of the leading car we see that higher ego-car velocities lead to higher risk, since the time to potential collision is shorter than for lower velocities. Additionally at higher velocities the risk peak is closer to the ego-car position, as the point of closest encounter (PCE) moves towards the ego-car for increasing velocities. Reducing the ego-car velocity to the velocity of the leading car lets the PCE move far away from the ego-car position and the time to closest encounter also increases to infinity, which both results is very far away, very low risk. For velocities lower than the leading cars velocity there is now risk at all anymore.

Applying the gradient-descent-like behavior selection approach reduces the ego-car velocity down to the same velocity as the leading car ($v_e = v_o$) and lets the ego-car follow behind [4].

Superposition of Multiple Risk Factors at an Intersection Scenario: As it is possible to superpose several risk factors in one predictive risk map we show a scenario where the ego-car is approaching an intersection on a minor road, while a second car wants to cross on the main road and a third car is driving in front of the ego-car. Each of the other

[4]The oscillations in Fig. 6 are a result of the recalculation step width and the risk map discretization.

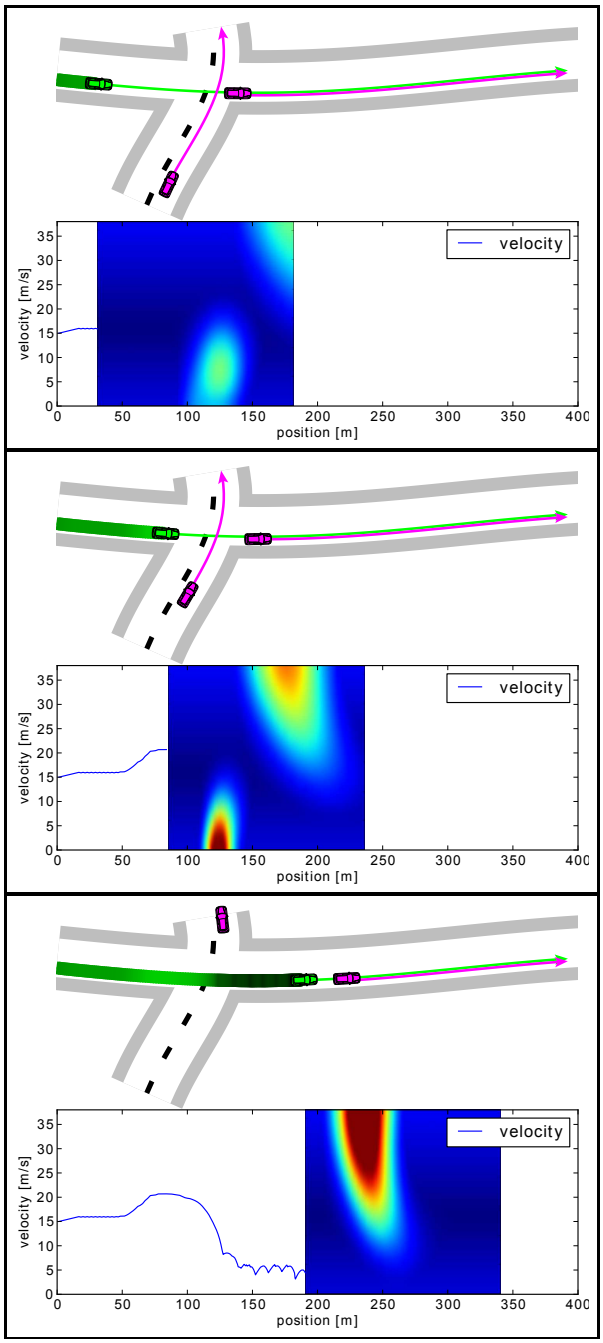

Fig. 7. Superposition of Multiple Risk Factors at an Intersection: Using the predictive risk map for behavior selection results in speeding up to pass in front of the crossing car followed by braking to follow behind the leading car at same speed.

cars result in a risk spot similar to the risk spots shown in the previous scenarios. We calculate the contributions for each traffic scene entity and superpose them in one risk map. We apply the gradient-descent-like behavior selection approach to the combined risk map in order to determine a cost minimizing behavior. As shown in Fig. 7 (top to bottom) the ego-car accelerates first to pass in front of the crossing car and then slows down to follow the car driving in front on the same lane.

Passing on Narrow Road Scenario: In this scenario two

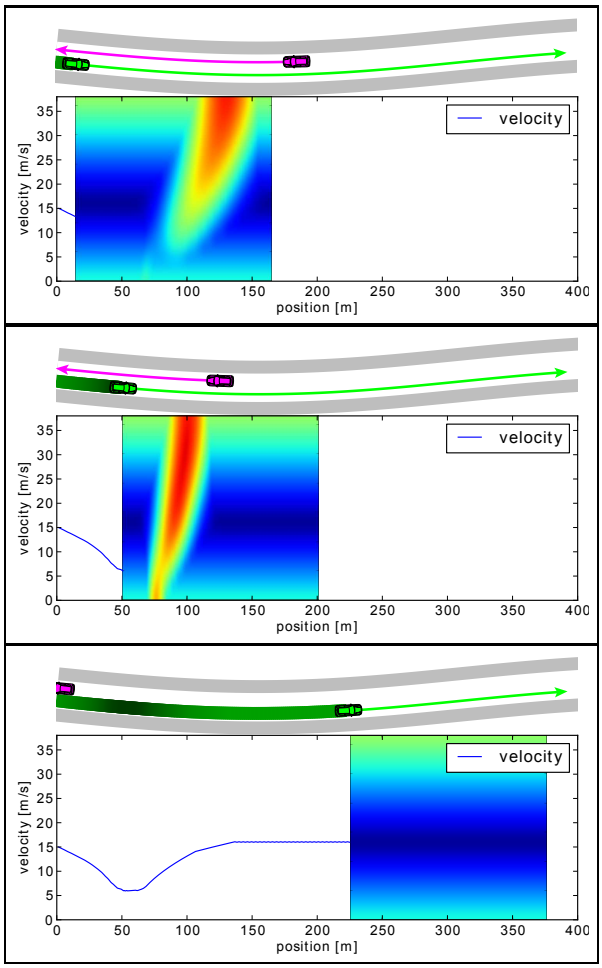

Fig. 8. Passing on Narrow Road Scenario: Using the predictive risk map for behavior selection results in slowing down until certain of safe passing and then speeding up again.

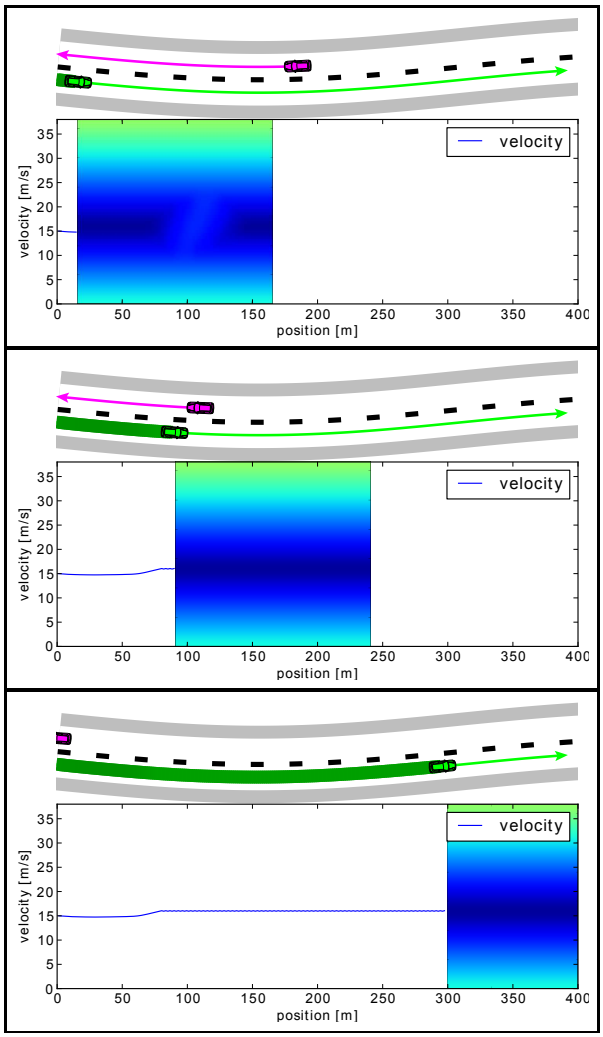

Fig. 9. Passing on Broad Road Scenario: Same scenario as in Fig. 8 but with a broader road and higher spatial distance between the future trajectories of both cars. Due to the broad road the risk is lower and the ego-car is earlier certain of safe passing. Thus it only slows down by a small amount until the other car has passed.

cars, the ego-car and another traffic participant want to pass each other on a narrow road, as illustrated in Fig. 8. The calculated risk map shows high risk at the position of closest encounter when the cars are still far away from each other. So the ego-car starts braking in order to reduce future risk. At a certain time the diffusion term is small enough and the risk decreases again. This can be understood as being certain of safe passing. The ego-car speeds back up to the desired travel velocity. This is the same behavior as we would expect from a human, who would brake until he is certain of safe passing and would then speed up again.

Passing on Broad Road Scenario: If we now evaluate this behavior on a broader road as illustrated in Fig. 9 the ego-car only slows down a small amount, because the certainty of safe passing is higher for broader roads.

VI. Outlook

The basic idea of predictive risk maps can be easily extended for behavior selection, by incorporating other classes of traffic scene entities like stationary objects or pedestrians. Additional interesting extensions include the application to scenarios with different types or risk as e.g. for centrifugal force risk, surface dependent risk, or traffic red light running risk.

As traffic rules target to minimize risk in traffic in general, the understanding of violation of traffic rules as risk indicators and the integration of traffic rules in predictive risk maps is a straightforward idea. We plan to apply the approach to scenarios with more traffic scene entities to show the scalability of our approach. In order to enable multiple behavior alternatives for the ego-car we strive towards a system with a situation analysis step taking risk evaluation into account.

Predictive risk maps allow the planning of new trajectories out of the given variation of predicted trajectories by combining trajectory transitions. Knowing that the gradient based approach for behavior selection possibly has problems with local minima, a further step will be the usage of a full motion planner like RRT* for behavior planning instead of the current gradient-based approach.

VII. Conclusion

This paper focused on the general problem of risk assessment in dynamic traffic environments for future behavior evaluation and planning. We introduced a method to derive predictive risk maps, which are gained from a variation of predicted spatio temporal ego-car trajectories and predicted spatio temporal trajectories of other traffic scene entities, and the calculation of risk for every point in time from the ego car and the other entity trajectories.

A predictive risk map indicates how risky a certain behavior will be in the future and enables the evaluation of risk-minimizing behavior. For the risk map of collision scenarios we generalized the well-known TTC risk indicator to the concepts of Time-to-Closest-Encounter TTCE and Distance-of-Closest-Encounter DCE, which allow to model a continuous collision risk that is valid even for non-colliding trajectories and generally for distance-based cases (the TTC risk indicator is currently only sensible for the collision case and in 1D).

Using predictive risk maps, behavior planning is achieved by searching for cost-optimized paths across the map. The resulting target behavior represented by the path is beneficial in terms of risk and cost as long as the map itself is not modified considerably, which is the case if additional time delays remain small and the other entities continue moving as originally assumed for their prediction.

In this work we consider a certain state uncertainty by the risk estimation. Additional uncertainties during the trajectory prediction can be incorporated by the superposition of their corresponding risk maps.

The cost-optimized paths across the risk maps can in principle be gained with a variety of search or planning algorithms; here we used a simple gradient-descent-like method for behavior selection. To show the benefits of the risk maps, we then applied our approach to several different intersection scenarios including car following, car crossing and mixed traffic situations. In all cases, the presented predictive risk map approach converged to viable solutions for the ego-car behavior planning, braking or accelerating depending on the future risks and finding the right time gaps to pass an intersection.

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