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Generalized Risk Estimation for intelligent ADAS Functions

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Predictive Risk Estimation for Intelligent ADAS Functions

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***Abstract*— The estimation of risk is a central cornerstone in the evaluation of traffic scene situations for intelligent ADAS. This applies to all levels of functions ranging from simple advices and warning functions right down to the evaluation of possible behavior alternatives, planning and autonomous driving. Current risk handling includes probabilistic collision measures, behavior planning using spatial and temporal occupancy grids, and time metrics such as Time-To-X (e.g. TTC, Time-To-Collision). Especially TTX measures are often used to quantify the threat of a future event. In this paper, we develop a theory of the probability of critical future events such as collisions. We show that by introducing the concept of a "survival probability", we can derive previously heuristic risk measures like TTX, generalize them to temporally continous risk indicator functions and apply them to several different situations such as e.g. collision risk, risk of passing nearby without collision, risk at intersection entrance or risk in narrow curves. We show how the TTC-based risk can be described as a special case of a general distance-based risk, with the Time-To-Closest-Encounter (TTCE) and the Distance-of-Closest-Encounter (DCE) as modulating parameters. Furthermore, the introduced risk measures can be used for planning under general conditions, e.g. using "risk maps" to evaluate behavior options according to their risk.**

I. Introduction

Risk is by definition "the probability of something happening multiplied by the resulting cost or benefit if it does" ([1], see also the ISO 31000 definition [2]), implying that its estimation involves a prediction of future events and a probabilistic treatment. Specifically, such an estimation involves the computation of the expected (i.e., averaged) cost or benefit of future events, which makes the risk estimation useful for evaluating or planning future behavior.

For current research in Advanced Driver Assistance Systems (ADAS), *risk estimation* is used based on very different approaches. On one hand, spatial occupancy grids are used for constraining the drivable space to risk-free areas. They are frequently involved in path planning, as shown e.g. for regular lane-driving, parking or K-Turn maneuvers [3]. Such occupancy grids have also been extended to contexts with moving entities, incorporating object dynamics or using temporal safety margins when entering risk zones [4], however, their main focus is on the spatial aspects and their capabilities for expressing temporal aspects of risk are limited.

On the other hand, risk is used to quantify collision detection and probability of future collision. In [5], traffic participants receive a future probability distribution of their position that evolves over time, and an overlap of these distributions then indicates a collision probability. In [6], pairwise combinations of individual motion trajectories are used for a threat and situation assessment. In [7], collision risk is calculated by a combination of an impact severity factor with a ratio between the possible future locations of a car and hazarduous areas (e.g., areas predicted to be blocked by other cars). Since these collision probability calculations involve the temporal prediction of the road users, they are better suited than spatial occupancy grids for dealing with multiple behavior alternatives such as maneuver options at intersections or overtaking maneuvers.

An further often-used risk measure for ADAS are time metrics such as Time-To-X (TTX; also TTC, TTB, etc.), which are applied to measure the threat of a future event within a traffic situation [8], [9]; see also [10] for a TTC calculation in 2D under uncertainty. They are calculated by modeling the future behavior of the involved objects. It is checked whether a relevant event (like a collision) will occur and then the time is measured until the event occurs. TTX measures intuitively capture the probabilistic notion that, if an event lies further in the future, it gets more unlikely (for a variety of uncontrollable reasons). A problem with TTX measures is that a discrete decision is needed about whether the critical event will occur at all in the future; if it does not, TTX is set to ∞.

Although all these types of approaches consider different apects of risk (spatial risk, collision probability, probability of reacting to a critical event in a remaining time), it should be possible to derive them from a common underlying concept. What is missing are approaches that model the future risk in a more general way, for different critical factors (i.e., different types of risk), and continuously over time until a critical event occurs. In this paper, we will introduce such a model. We will show how it can be used to describe and systematically combine several risk factors, and derive the notion of a continuous future risk function over time that can be used for risk estimation, behavior evaluation and planning. Last but not least, we will show how the TTX-type metrics can be understood as a special case of our model.

II. Probabilistic Risk

Following the probabilistic risk definition from section I we introduce the risk of a traffic scene as the (ensemble)-averaged cost/benefit expectation. For the calculation of the ensemble-averaged expectation values a hypothetical statistical experiment is conducted by releasing similar traffic participants with slightly varied states and behavior and counting if and to which extent they get involved in a risky event. The slightly varied states account for the uncertainty involved in the experiments, including e.g. perturbations in

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the environment, the cars, the sensors, the drivers and their behaviors. The main purpose of risk estimation is therefore to calculate / predict / minimize the expectation value $< ... >$ of the damage [1]

$$\begin{aligned} &\text{risk} := < \text{damage} > \\ &:= \int_{\text{states}} \text{damage}\, P(\text{damage}|\text{states})\, P(\text{states})\, d\text{states} \ . \end{aligned} \tag{1}$$

It is useful to further split the first term on the right, since the damage does not directly depend on the states of involved traffic participants, but only if actually a critical event (like a crash) happens. We then get [2]

$$\begin{aligned} &P(\text{damage}|\text{states}) \\ &:= P(\text{damage}|\text{states}, \text{event} = \text{true})\, P(\text{event} = \text{true}|\text{states}) \end{aligned} \tag{2}$$

composed of the probability of an event like an accident or a collision given the states of the participants, and the probability of a damage given that such an event occurred (and given the states of all involved entities at the moment of the event). Typical states e.g. for collision events are the velocities and angles of the involved entities at the moment of an impact.

However, for probabilistic risk estimation we are dealing with *future* critical events. Adding temporal indices to (2), we get

$$\begin{aligned} &P(\text{damage}_{t+s}|\text{states}_t) := \\ &\quad P(\text{damage}_{t+s}|\text{states}_{t+s}, \text{first_event}_{t+s} = \text{true}) \\ &\quad P(\text{first_event}_{t+s} = \text{true}|\text{states}_t) \ , \end{aligned} \tag{3}$$

for the predicted risk, where we have added "first" to the event to indicate that no event of that type should have occurred previously during the prediction interval $[t, t+s[$. The first factor quantifies again the damage probability, but now at the future time $t+s$ when the critical event is hypothesized to occur, whereas the second factor quantifies the probability of the critical event occuring at $t+s$. Since the estimation of the damage probability involves the future states at the event at $t+s$, we additionally need a way to calculate them from the states at t, which is the case e.g. if we have a deterministic state prediction model.

For a probabilistic risk estimation, we therefore have to model the

1) prediction of the future states of the involved entities
2) damage probability given the states at the critical event
3) probability of a future event given the current states.

Points 1) and 2) need a proper choice of prediction and damage approximation models, however once the choice is clear their usage is mostly straightforward. For 1), standard available prediction models can be used, e.g. kinematic models with longitudinal and transversal components for vehicles (a choice would e.g. be a constant acceleration model in longitudinal and lateral directions as used in [7]). For 2), e.g. several physical collision models can be used, like the forces and kinetic energies involved in an 2D inelastic impact, see e.g. [11], [12] [3]. Finally, for 3) we have to estimate the influence that the current and the predicted states will have on a future critical event, which will be elaborated in the next section. Examples of risk-related events (see section III-A) are collision resp. near-collision events, getting-off-the-road events in curves, and control loss events due to strong braking, but other critical event types can be modeled analogously in a straightforward manner.

III. Future Probability of Critical Events

In this section, the current and future probability of a critical event given the states of the participating entities (e.g. road elements, ego-car and other cars) are introduced and derived. In section III-A an *instantaneous event probability* P_e is introduced, corresponding to the last factor $P(\text{event} = \text{true}|\text{states})$ from (2). Using P_e, in section III-B the concept of a *survival function* is derived which allows to quantify the future time course of of P_e. Finally, in section III-C, the *future event probability* P_E is derived, which is the formal expression of the last factor $P(\text{first_event}_{t+s} = \text{true}|\text{states}_t)$ from (3), i.e., the probability that a critical event like an accident happens at a time $t+s$ in the future.

A. Event Rates

A popular choice for describing rare critical events in the technical and financial domain like a vehicle-to-vehicle collision are Poisson processes. A Poisson process is a stochastic process that counts the number of events that occur in a given time interval. It is characterized by the mean time τ (or its reciprocal *event rate* $\lambda := \tau^{-1}$) between events as the only parameter.

To allow for the modeling of time-continuous, state-dependent risks, we assume the event rate to be time-dependent, similar to the case of inhomogeneous Poisson processes. We then have an instantaneous event rate $\tau^{-1}[\text{states}(t)]$ and a probability of events for small time intervals $[t, t+\delta t]$

$$P_e(t, \delta t) = \tau^{-1}[\text{states}(t)]\, \delta t. \tag{4}$$

The event rate serves to model the instantaneous event probability if we have knowledge of the states of the involved entities. For example, a simple model for a distance-based collision event between two traffic participants at positions $\mathbf{x}_1$ and $\mathbf{x}_2$ and their distance $d := ||\mathbf{x}_1 - \mathbf{x}_2||$ can be given by

$$\tau_d^{-1} = \tau_{d,0}^{-1} \exp[-\beta_d\, (d - d_{\min})] \ , \tag{5}$$

[1] In the following, we will always write the term "damage" instead of cost/benefit, to underpin the relation with critical events as e.g. in critical traffic situations, but we will return to the cost/benefit idea later on for planning. For concrete critical events, the damage is usually quantified by an estimation of the cost of an accident.

[2] This approach can be easily generalized to include several types of events. Applicable events are e.g. collisions between road users, collisions with fixed structure, accidents by getting off the road in curves due to high velocity, accidents caused by control loss due to strong braking, but also less severe events like red light running.

[3] This is only a first approximation to a damage probability calculation; in more elaborated models further collision parameters can be taken into account like an orientation-dependent damage probability, since e.g. cars exhibit different damages for frontal, side or rear-end collision.

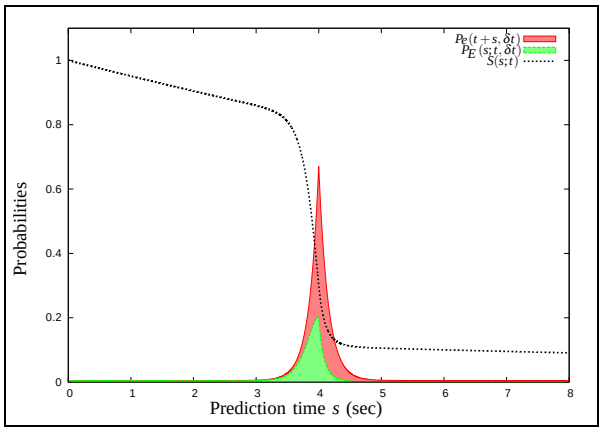

Fig. 1. Future probability of critical events, shown here for the example of a car to car approach with closest distance predicted at $s = 4$ sec in the future. The instantaneous event probability $P_e(t,\delta t)$ (4) (red large peak) is modeled via the event rate function $\tau^{-1}[\text{states}(t+s)]$ which depends on the predicted states of involved scene entities. The "survival function" $S(s;t)$ (8) (black dotted line) indicates the proportion of cars that started at t and which have not been involved in an event yet until $t+s$. It can be noticed that it decays faster for higher event rates. The event probability $P_E(s;t,\delta t)$ (green small peak) is a combination of the survival function and the instantaneous event rate and quantifies the proportion of "surviving" cars at a future time $t+s$ that get engaged in a critical event.

with the minimal geometrical collision distance $d_{\min}$ and the parameter $\tau_{d,0}^{-1}$ chosen in such a way that $P_e = 1$ for $d = d_{\min}$. Other critical events can be modeled similarly, e.g., the event rate of accidenting in a curve can be modeled using

$$\tau_c^{-1} = \tau_{c,0}^{-1} \exp[\beta_c\,(a_c - a_{c,\max})] \tag{6}$$

with the centrifugal acceleration $a_c := v^2/R$ (longitudinal velocity v and curve radius R) that should not surpass a maximal value $a_{c,max}$. Interesting is also an event rate that contributes to modeling a risk involved in sudden braking, e.g. by constraining the deceleration b to some maximal value $b_{\max}$ with

$$\tau_b^{-1} = \tau_{b,0}^{-1} \exp[\beta_b\,(b - b_{\max})]\ , \tag{7}$$

following the idea that too strong braking may lead to a loss of control. An analogous term for acceleration could be used to penalize sudden velocity changes, leading to smoother and more continuous trajectories.

Further risks, also related to less critical events like passing a traffic light at red, lead to similar additional event rates τ^{-1}. It should also be noted, that the proposed exponential function for the event rates can be exchanged by other function forms like sigmoidal ones, since their main porpose is to model the qualitative risk tendency given the parameters (e.g. a monotonously increasing risk for decreasing distances). In the exponential model, the parameters $\beta_b, \beta_c, \beta_d$ are used to quantify the steepness of the event probability increase with braking deceleration, centrifugal acceleration or distance, and similar parameters apply to sigmoidal event rate models. Their values will have to be determined empirically by comparison with accident statistics. In some cases they can be determined indirectly from the parameters of other popular risk measures if they are available, like it is shown for the case of collision risk in section IV.

B. Survival Probability

Starting from (4) at an arbitary time t, if we look at a statistical ensemble of equivalent cars, the future proportion of cars which will not have been involved in an event like e.g. an accident during the time interval $[t, t+s]$, with $0 \le s \le \delta t$ decreases with the accident event rate, so that for constant τ^{-1} we get a so-called *survival function for small time intervals* $S(s) = \exp\{-\tau^{-1} s\}$ which decreases monotonously from 1 to 0 when s goes from 0 to ∞. For time-varying τ^{-1} (depending on the state variables), we get

$$S(s;t) = \exp\{-\int_0^s \tau^{-1}[\text{states}(t+s')]\,ds'\} \tag{8}$$

as the *inhomogenous survival function* [4]. This function also decreases from 1 to 0, faster if the states indicate a higher event rate τ^{-1} (e.g. if two cars get close to each other and a collision becomes probable) and slower if the states are such that the traffic participants don't influence each other (e.g. if they are very far away from each other, so that there is no probability of collision).

C. Future Event Probability

How many cars will then engage in an accident at any time interval? The inhomogeneous survival function (8) tells us the probability that a car that started at t still remains accident-free at time $t+s$. The cars that have an accident are just those that cease to survive. This means that we get an event engagement probability density (e.g. an accident frequency)

$$E(s;t) = -\frac{d}{ds} S(s;t) \tag{9}$$

or,

$$E(s;t) = \tau^{-1}[\text{states}(t+s)] \exp\{-\int_0^s \tau^{-1}[\text{states}(t+s')]\,ds'\} \tag{10}$$

with the probability that an event occurs for the first time at a future time $t+s$, in the interval $[t+s, t+s+\delta t]$ (the last term from (3) and point 3 in the enumeration of section II) given by $P_E(s;t,\delta t) = E(s;t)\,\delta t$.

At any time step $t+s$, the number of cars is calculated that have not been engaged yet in any event during $[t, t+s[$, using the inhomogenous survival function from (8). At $t+s$ (after a time interval of length s), the remaining set of cars (the "surviving" cars) can be engaged in an event, and this occurs with the instantaneous event probability from (4) which depends on the current states from the environment situation at time $t+s$. Including several risk factors we then get for the probability of future critical events

$$P_E(s;t,\delta t) = \{\tau^{-1}[\text{states}(t+s)]\,\delta t\}\, S(s;t)\ , \tag{11}$$

[4] See [13], [14] for the background of survival functions in spiking neural networks, an area of research also dealing with rare events triggered by continuous event rates.

with the total survival function

$$S(s;t) = \exp\{-\int_0^s \tau^{-1}[\text{states}(t+s')]\,ds'\} \qquad (12)$$

and the total critical event rate (sum of the different threats)

$$\tau^{-1}[\text{states}(t+s)] = \sum_i \tau_i^{-1}[\text{states}_i(t+s)] \ , \qquad (13)$$

with each single event rate τ_i^{-1} given by the modeled risk factors (with examples for collision and centrifugal risks as described in section III-A). The term states_i indicates that different risk contributions may depend on scene state subsets contributed by different scene entities. Eqs. (11), (12) and (13) constitute, together with the different risk factors, the complete probabilistic model of a generalized, time-continuous predictive risk measure.

Fig. 1 shows typical curves for the critical event rate, the survival function and the probability of future critical events for a car-to-car closeup scenario with the closest distance predicted at $s = 4$ sec. The survival function decays steadily from 1 to 0, with a pronounced dip when the risk indicators given by the critical event rate increase. The predicted probability of future critical events (green small peak) is then a combination of the survival probability (the probability that the ego-car has "survived" from t, i.e., now, until $t+s$ without getting involved in a critical event) and the critical event rate at $t+s$. The survival function thus comprises the information about the accumulated vehicle risk history for the prediction time, and the effect is that the probability of critical events that are predicted to occur further in the future at $t+s$ is lowered because the vehicle may have been engaged e.g. already in an accident during $[t, t+s]$.

IV. TTX-Type Criticality Indicators and Event Probabilities

The **T**ime-**T**o-**C**ollision (**TTC**) and generally the **T**ime-**T**o-**X** (**TTX**) family of indicators (e.g. Time-To-Brake, Time-To-React) are calculated by extrapolating the current states and characterizing the criticality of a future singular event, as e.g. the collision within a particular traffic situation, by the time until that event occurs [5].

The event probability is usually chosen to be a monotonously decreasing function with TTC so that a collision event becomes more probable and even certain for $\text{TTC} \to 0$, e.g.

$$P_{\text{TTC}}(\text{TTC};t,\delta t) \sim \text{TTC}^{-1}(\text{states}_t)\,\delta t \qquad (14)$$

(we use $P_{\text{TTC}}(\text{TTC};t,\delta t) = [\text{TTC}(\text{states}_t) + \delta t]^{-1}\,\delta t$ so that the probability is in the interval $[0,1]$). The corresponding curve is shown in Fig. 2, black/thin line.

TTC relies on a hypothetical collision to occur with certainty 1 at $t+\text{TTC}$. If the states are such that a collision will not occur, then TTC does not exist (it is $\text{TTC} = \infty$) and a risk formulation is not possible.

[5] In the following, we will restrict to TTC for our argumentation.

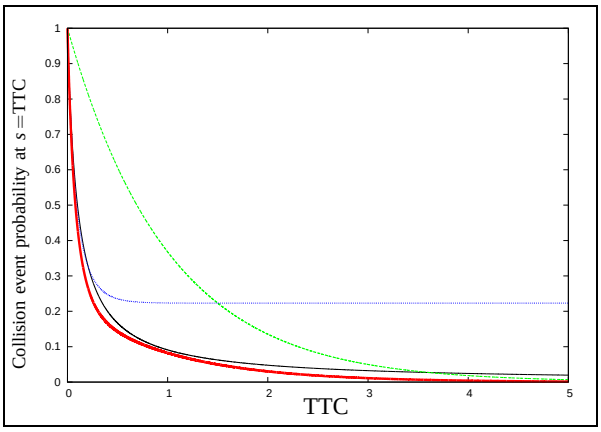

Fig. 2. Probability of a predicted collision event plotted over the remaining **T**ime-**T**o-**C**ontact (**TTC**) for a general collision risk. Black thin line: Typical TTC^{-1}-curve for the heuristic description of a risk that depends on the time left until the collision occurs. All other lines: Collision probability according to the predicted risk estimation. Green dashed line: Collision event probability (16) based on a constant collision "escape" rate during the time prior to the collision. Blue dotted line: Collision event probability (17) with distance dependent event rate. Red thick line: Collision event probability (18) according to the mixed model ("collision escape" rate + distance-dependent event rate) which was numerically fitted to match the TTC^{-1}-curve. Parameters: $\delta t = 0.1$sec, $\tau_0 = 1$, $\delta v = 10$ m/sec (36 km/h), $\beta_d = 1/1.5$, $d_{\min} = 4$ m, $\tau_{d,0} = \delta t = 0.1$. The collision event probability is normalized to $[0,1]$, however for the distance-based risk it remains constant > 0 for large TTC because a large TTC means large initial distances for which the distance-based risk is negligible so that it only becomes noticeable when the distance gets small.

However, with the help of the proposed risk estimation approach from section III-C, we can formulate the TTC-based risk using the probability of future events. For a finite $\text{TTC} < \infty$, the state prediction indicates that a collision event is certain in a TTC time in the future, meaning that the event rate (first factor from right side of (11)) at that time is such that $\tau^{-1}[\text{states}(t+\text{TTC})]\,\delta t = 1$. This leads to

$$P_E(\text{TTC};t,\delta t) = S(\text{TTC};t) \ , \qquad (15)$$

i.e., the TTC event probability is equal to the survival probability at $s = \text{TTC}$. This means that a TTC-based risk can be estimated by calculating the probability that a car will actually reach the point where the collision will occur according to the state prediction models.

As the easiest case, let us first consider a constant event rate $\tau^{-1}(t) := \tau_0^{-1}$. This is a reasonable assumption if we consider not accidents or collisions but all types of events that may deviate a driver from its current path so that a collision cannot occur any more at $t+\text{TTC}$ (i.e., he manages to "escape" from the collision with a certain probabilty by e.g. braking, steering or any other world state change that results in an avoidance of the collision). The predicted probability of getting engaged in the collision decreases steadily when there is more time at the drivers' disposal, because of uncertain influences of the environment or because the driver may act in a way that e.g. avoids a collision.

From (15), we then get that

$$P_{E,0}(\text{TTC};t,\delta t) = \exp\{-\tau_0^{-1}\,\text{TTC}\} \ . \qquad (16)$$

However, an exponential decay is much slower than the assumed TTC^{-1} dependency, so that a constant event rate is not sufficient to explain the usual TTC assumption from (14), see green/dashed curve in Fig. 2 vs. black/thin curve.

Now let's additionally assume for TTC that the two cars are approaching each other with constant velocity difference Δv so that the distance over time is $d(t+s) := d_t - \Delta v s$. With an initial distance d_t at time t that fulfills e.g. the TTC conditions $d(t+\mathrm{TTC}) = d_{\min}$ [6], we get $d(t+s) := d_{\min} + \Delta v\,\mathrm{TTC} - \Delta v s$ and with the distance-dependent event rate from (5), this leads to

$$\begin{aligned} &P_{E,d}(\mathrm{TTC};t,\delta t) \qquad (17)\\ &= \exp\{-\int_0^{\mathrm{TTC}} \tau_{d,0}^{-1} \exp[-\beta_d \Delta v\,(\mathrm{TTC}-s')]\,ds'\}\\ &= \exp\{-(\beta_d \Delta v\,\tau_{d,0})^{-1}\,[1-\exp(-\beta_d \Delta v\,\mathrm{TTC}]\}\ . \end{aligned}$$

In this case, the decay of $P_{E,d}$ with TTC is much faster than TTC^{-1} for very small TTC, but saturates above zero for large TTC.

Combining both event rates, however, we get a mixed model (we can use as an additional condition on the constants $\tau_{d,0}^{-1} = \delta t$ if we assume that the collision is caused by spatial proximity only so that the distance-based event rate fulfills $\tau_d^{-1}[\mathrm{states}(t+\mathrm{TTC})]\,\delta t = 1$)

$$\begin{aligned} &P_{E,m}(\mathrm{TTC};t,\delta t) = S_m(TTC,t) =\\ &\exp\{-\tau_0^{-1}\mathrm{TTC}\} \qquad (18)\\ &\exp\{-(\beta_d \Delta v\,\tau_{d,0})^{-1}\,[1-\exp(-\beta_d \Delta v\,\mathrm{TTC})]\}\ . \end{aligned}$$

The resulting future collision probability as a function of TTC can be fitted to approximate the TTC^{-1} decay to a very good extent, as can be seen by comparing the black/thin line and the red/thick line in Fig. 2. We have therefore derived an analytical expression of the TTC-dependent risk based on physically interpretable parameters, like the velocity difference Δv and the event rates τ_0^{-1} and $\tau_{d,0}^{-1}$. But more importantly, once we have set these parameters to approximate a desired TTC-dependent risk function, we can apply the same function also for calculating the future collision event probability $P_{E,m}(s;t,\delta t)$ for $s \neq TTC$, or for situations where no TTC exists! As a consequence, we have gained an estimation of future risk that is continuous in space and time, that applies to arbitrary prediction times and to a variety of different risks and driving situations, including the no collision cases like two cars passing by to each other without colliding.

In Fig. 2, we show a TTC^{-1} curve along with its approximation by the predictive risk model presented in this paper. Fig. 2 shows corresponding functions for the future event probability over time, for different TTC times.

V. Continuous Event Probabilities

In the last section IV, we have gained an analytical expression for the TTC-dependent probability of a collision risk at the particular time of collision (18), which is consistent with the heuristic TTC^{-1} curve. Using the survival function (12) we have, however, derived this expression from instantaneous risk parameters that continuously apply throughout the time interval until a risky event occurs.

This means that we can now calculate the risk for times $s \in [0, \mathrm{TTC}]$, or even for cases $s > 0$ where no TTC can be calculated ($\mathrm{TTC} = \infty$). The probability of future critical events according to Eqs. (11), (12) and (13) applies to any times, so that the notion of a TTC is not needed any more. We have therefore derived expressions for the risk event probabilities which can be applied to arbitrary driving conditions with a variety of risks, such as collision risks, curvature and deceleration risks, but also traffic light surpassing or rule violation risks.

Usually, the risks will be modeled by an appropriate combination of event rates as described in section III-A. The survival function (12) then has to be calculated based on the assumed predicted states of the entities in the scene. This is often done numerically, with the integration of III-A being straightforward. In addition, independently predicted entities (like multiple vehicles) can be incorporated by adding the corresponding event rates for according to (13).

To explain the concept of continuous event probabilities, we will now show as an example the continuous event probabilities for the mixed event rate model from section IV. To enable times $s > \mathrm{TTC}$, we set for the distance (here we chose $d_{\min} = 0$ for simplicity, but the calculation can be performed in the same way for $d_{\min} > 0$)

$$d(t+s) := \Delta v\,\mathrm{abs}(\mathrm{TTC} - s)\ . \qquad (19)$$

From Eqs. (11), (12), the survival function can be expressed

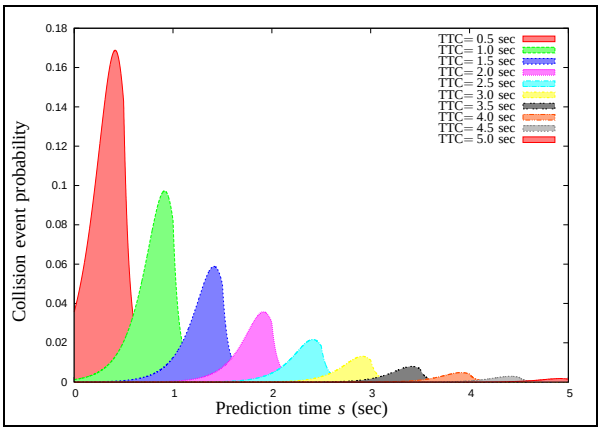

Fig. 3. Probability distributions $P_{E,d}(s;t,\delta t)$ (starting time t, prediction time difference s / future time $t+s$, probability time interval δt) according to (20),(21) and (22) for a longitudinal collision event in the car approaching scenario (two cars in the same lane), for different TTC and mixed event rates (constant "escape" rate and distance-based collision risk event rate). For a fixed TTC, the maximal collision probability is at the prediction time close to $s = \mathrm{TTC}$, rising steadily before and decaying abruptly after the predicted collision time. Parameters: $\delta t = 0.1$ sec, $\tau_0 = 1$, $\delta v = 10$ m/sec (36 km/h), $\beta_d = 1/1.5$, $d_{\min} = 4$ m, $\tau_{d,0} = \delta t = 0.1$, TTC=$\{0.5,...,5.0\}$ sec (corresponding to distances of $\{5,...,50\}$ m). The total event probabilities for the mixed event rate model are normalized to 1, whereas the integrals of the collision probability distributions are < 1, since collision avoidance reactions are taken into account by the escape rate in the mixed model.

[6] $d_{\min}$ being the collision distance that in a more realistic model may also depend on geometry considerations, etc.

analytically as the multiplication of the individual survival functions for the constant and the distance-dependent event rates, respectively,

$$S_m(s;t) = S_0(s;t)S_d(s;t) = \exp\{-\tau_0^{-1}s\}\exp\{-(\beta_d\,\Delta v\,\tau_{d,0})^{-1} [\exp(-\beta_d\,\Delta v\,(\mathrm{TTC}-s)) - \exp(-\beta_d\,\Delta v\,\mathrm{TTC})]\} \quad (20)$$

for $s \leq \mathrm{TTC}$ and

$$S_m(s;t) = S_0(s;t)S_d(s;t) = \exp\{-\tau_0^{-1}s\}\exp\{-(\beta_d\,\Delta v\,\tau_{d,0})^{-1} [2 - \exp(-\beta_d\,\Delta v\,(s-\mathrm{TTC})) - \exp(-\beta_d\,\Delta v\,\mathrm{TTC})]\} \quad (21)$$

for $s > \mathrm{TTC}$.

The event probability of a collision is then given by the event probability for the distance dependent risk, which is gained by the combination of the mixed model survival function with the distance-dependent event rate, which results in

$$P_{E,d}(s;t,\delta t) := \tau_{d,0}^{-1}\,\delta t \exp[-\beta_d\,\Delta v\,\mathrm{abs}(\mathrm{TTC}-s)]\,S_m(s;t)\ . \quad (22)$$

In Fig. 3, we show the probability function for the longitudinal collision event (two cars in the same lane) for several different TTC. For a fixed TTC, $P_{E,d}$ resembles a skewed Gaussian around $s = \mathrm{TTC}$, which rises when the collision risk caused by short distances increases (with the distance being minimal at $s = \mathrm{TTC}$), and which for $s > \mathrm{TTC}$ decays to 0 after the involved car has been engaged in the collision to a large probability. For large TTC, the probability of getting involved in a collision accident at all is low, increasing steadily for $\mathrm{TTC} \to 0$.

VI. Generalization of TTC: Predictive Risk without Collision

Consider now an approach between two cars with a smallest approach distance **DCE** (the **D**istance-of-**C**losest-**E**ncounter) which however does not lead to a collision, as it is e.g. the case for a car passing another car on a parallel lane. For simplicity, we assume that the distance decreases linearly down to the minimal approaching distance DCE and afterwards increases again linearly [7]. In this case, the approach starting at t is described by

$$d(t+s) := \mathrm{DCE} + \Delta v\,\mathrm{abs}(\mathrm{TTCE}-s) \quad (23)$$

with the **T**ime-**T**o-**C**losest-**E**ncounter **TTCE**, so that $d(t+\mathrm{TTCE}) = \mathrm{DCE}$ and $d(t+s) > \mathrm{DCE}$ otherwise.

The distance-dependent event rate then becomes

$$\tau_d^{-1}[d(t+s)] = \tau_{d,0}^{-1}\exp[-\beta_d\,(\mathrm{DCE} + \Delta v\,\mathrm{abs}(\mathrm{TTCE}-s))] = \tilde{\tau}_{d,0}^{-1}\exp[-\beta_d\,\Delta v\,\mathrm{abs}(\mathrm{TTCE}-s)] \quad (24)$$

i.e., by modulation with the amplitude $\exp(-\beta_d\,\mathrm{DCE}) < 1$ we get a modified event rate parameter

$$\tilde{\tau}_{d,0}^{-1} := \tau_{d,0}^{-1}\exp(-\beta_d\,\mathrm{DCE}) \quad (25)$$

[7]This model serves as an approximation to the Euclidean distance between cars passing each other on parallel lanes; however, the fully correct derivation for 2D distances can be calculated with the same formalism.

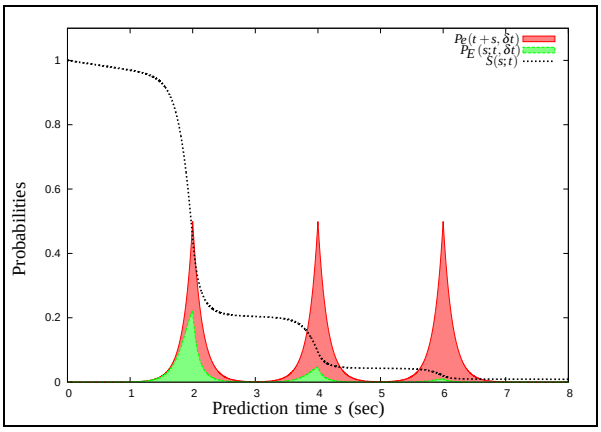

Fig. 4. Future event rate (red large peaks), future collision event probability (green small peaks) and survival function (dotted line) for the case of multiple risks induced by multiple other vehicles (vehicle passing scenario). The survival function decays in proportion to the collision event probabilities, so that earlier events have a larger influence on the predictive risk calculations.

which takes the distance of closest approach into account. We finally arrive at the analogous expressions as above in Eqs. (20), (21) and (22), with all $\tau_{d,0}^{-1}$ replaced by $\tilde{\tau}_{d,0}^{-1}$ and all TTC replaced by TTCE.

A larger car size or a narrower street on which the cars have to pass each other lead to a smaller **DCE** in (23), and, subsequently, to a larger event rate parameter (25) and a larger overall amplitude of the future event probability. If the passage gets so narrow that the cars will actually collide, **DCE** becomes 0 and $\tilde{\tau} \to \tau$, so that the **DCE** and **TTCE** based risk estimation smoothly approaches the standard, collision based risk estimation case. At the same time, if a collision becomes certain, the survival probability S_m after the collision approaches zero.

The consequence from the considerations from sections V and VI is that predictive risks can be calculated for arbitrary future time intervals and even if, for deterministic considerations, there is no evident collision identifiable along the predicted trajectories. This is due to the consideration that, with advancing prediction time, trajectories and distances can be subject to measurement errors or can be modified by yet unknown circumstances.

This means that the probability of a collision is non-zero even for the case where the deterministic prediction of trajectories would indicate no collision. However it depends on DCE and decreases rapidly with increasing DCE due to the double influence of the DCE-dependent modulation of $\tilde{\tau}_{d,0}^{-1}$ in (22): in the event rate term and in the survival function.

Fig. 1 shows the corresponding survival and collision event probability for the non-collision case, with TTCE= 4 sec. One can see that the event probability $P_{e,d}$ has a maximum at around $s =$TTCE and that afterwards, we have $S_m > 0$ since the instantaneous collision probability at the encounter with distance DCE$> d_{\min}$ was < 1.

The distance-dependent risk indicators TTCE and DCE generalize the concept of TTC and are valid for both the collision and the non-collision case. For the collision case,

we get DCE$\rightarrow$ 0 (or DCE$\rightarrow d_{\min}$) and TTCE$\rightarrow$TTC.

As explained before in section III-C, multiple risks lead to an addition of event rates. We can therefore investigate the case of the combined collision risk with several cars when a car is passing them on a neighbouring lane. Let's say that the cars are regularly spaced so that the distances to closest encounter obey $\text{DCE}_n = \text{DCE}$ and the times until these distances are reached e.g. at $\text{TCCE}_n = \{2,4,6\}$ sec.

Fig. 4 shows the future collision event probability for the multiple-car passing scenario, for the three different approach times TCCE_n and common DCE. It can be seen that the collision probability at the first encounter (first peak from the left) largely determines the next peak heights. The reason is that if the first encounter is already very likely to result in a collision, there is no probability that the car will even get to the next encounters (i.e., that it will "survive" the first encounter) and we can neglect them for further considerations. Nevertheless, if the first encounter is not very risky, the probability of collision for the following encounters remains high and should be considered for further planning, since the car is likely to survive until it gets to them.

Using the generalized risk considerations from the previous sections, we can now divise a strategy to trigger warnings or actions for a fixed prediction time horizon of, say, n seconds. If such a system should be triggered by any risk occurring in the next future time horizon $[0,n]$, then the main indicator is the survival probability drop around $s = n$. After a crash happening with certainty, the survival probability will become zero (this assumption can be relaxed towards the near-collision case by incorporating a desired minimal warning distance DCE as introduced previously in this section). By comparing, at $s = n$, the survival probability when no critical events happen (escape rate / avoidance reactions only) with the total escape rate including the critival events, we can set a trigger threshold choosing a value that lies within the two. However, the sensitivity of such a trigger threshold is best for shorter prediction intervals, since the uncertainty introduced by the escape rate reduces the sensible trigger interval to zero for $s \rightarrow \infty$.

VII. Evaluation of Trajectory Risk

The derived risk estimation scheme now allows to evaluate the risk of predicted trajectories of traffic participants, resp. the risk that one traffic participant undergoes on its (predicted or real) trajectory in relation to other traffic participants or world elements like road structures. The novelty is that it takes the full world dynamics into account, resulting in a function of risk over predicted time.

For real, already driven trajectories, there is no survival probability that has to be taken into accound, so that according to the continuous risk formulation the probability that a critical event could have happened is directly given by the total critical event rate according to (13). We then have that

$$P_E(0;t,\delta t) = \tau^{-1}[\text{states}(t)] = \sum_i \tau_i^{-1}[\text{states}_i(t)]\,\delta t \qquad (26)$$

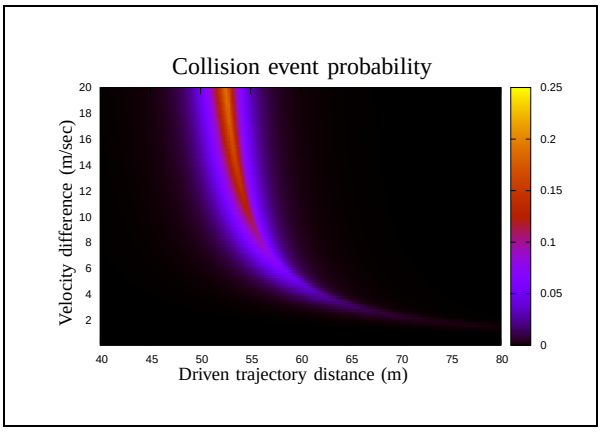

Fig. 5. Risk map for the future collision event probability for a car approaching another car on the same lane. Both cars are assumed to drive with constant velocities. The maxima of risk are higher and shifted to the left for higher velocities, which is a direct account of the lower TCE and closer PCE. For velocity differences $\Delta v \rightarrow 0$ (both cars driving with same constant velocity), the risk vanishes. A behavior planning based on this risk map would try to avoid high risk peaks and correspondingly reduce the velocity along the risk ridge until the velocity difference is zero. Parameters: Initial distance $d(0) = 50$ m, velocities difference $\Delta v \in [0,20]$ m/sec, $\delta t = 0.1$ sec, $\tau_0 = 10$, $\beta_d = 1/1.5$, $d_{\min} = 4$ m, $\tau_{d,0} = 3 * \delta t = 0.3$.

and the corresponding damage probability function becomes (following (2))

$$P(\text{damage}_t|\text{states}_t) = P(\text{damage}_t|\text{states}_t, \text{event}_t = \text{true})\, P_E(0;t,\delta t)\ . \qquad (27)$$

We can use this function to either calculate the averaged damage over e.g. the time course of the trajectory according to

$$\text{risk} := <\text{damage}> := \int_t \text{damage}_t\, P(\text{damage}_t|\text{states}_t)\, dt \qquad (28)$$

or to analyze the statistics of the damage probability like e.g. the points of highest risk, etc. In this way, we can compare different trajectories by their overall risk, and evaluate the quality of different driving behaviors.

For predicted trajectories, instead of (26) we have to use the critical event rate discounted by the survival function according to (13). The survival function then serves to set the risk estimation time horizon that we need to consider; when the survival function gets close to 0 we can stop. The damage probability function over the predicted time s (starting at t) becomes

$$P(\text{damage}_{t+s}|\text{states}_{t+s}) = P(\text{damage}_{t+s}|\text{states}_{t+s}, \text{event}_{t+s} = \text{true})\, P_E(s;t,\delta t)\ . \qquad (29)$$

and we can calculate e.g. the averaged damage, now over the predicted trajectory, according to

$$\text{risk} := <\text{damage}> := \int_s \text{damage}_{t+s}\, P(\text{damage}_{t+s}|\text{states}_{t+s})\, ds\ . \qquad (30)$$

The critical event rate for the collision probability of two traffic participants was modelled depending on their distance.

This means that, for this special case, an additional event rate term has to be incorporated for each other traffic participant (or road structure amenable for a collision) that should be considered, calculating the pairwise distances between each of them and the considered traffic participant. We therefore get

$$\tau^{-1}[\text{states}(t)] = \tau^{-1}_{\text{other risks}} + \sum_i \tau_i^{-1}[d_i(t)] \tag{31}$$

with $d_i(t) := ||\mathbf{x}(t) - \mathbf{x}_i(t)||$ (if we want to consider spatially extended objects, we may use e.g. $d_i := \min_{\mathbf{x}(t),\mathbf{x}_i(t)} ||\mathbf{x}(t) - \mathbf{x}_i(t)||$, where $\mathbf{x}$ and $\mathbf{x}_i$ now represent all possible points covered by the extended object).

VIII. Summary, Conclusions and Outlook

We have presented a framework for the calculation of future risks for driving scenarios taking event uncertainties into account. The uncertainties and the risks accumulate over prediction time, which expresses in a survival function that comprises the proportion of cars which have not been engaged in a critical event up to a certain time point in the future. The risk estimation is straightforwardly applicable using Eqs. (11), (12) and (13) based on the predicted sequence of states of the relevant entities involved in a traffic situation. The proposed framework generalizes to several different risks, so that the main modeling choice is the decision on the types of risk that we want to include into our considerations, and their corresponding event rate functions as explained in section III-A. Once we are able to calculate the future risk function, we can use it to evaluate given trajectories or explore beneficial behavior (in terms of risk).

A consequence of the proposed framework is a generalization of the well-known TTC criticality indicator to arbitrary collision risk. Usually, TTC does not account for uncertainty in state information and relies on a collision to occur with certainty. Therefore it is currently not applicable to other situations than 1D, longitudinal behavior like vehicle following (see however [10]). Here we could show that TTC can be derived as a special case of a distance-dependent probabilistic risk, which in its full formulation can now be used for arbitrary situations. We have also shown that this generalization can be undertood in terms of criticality indicators like the Time-To-Closest-Encounter TTCE and Distance-of-Closest-Encounter DCE, which open the way for a heuristic description of general collision risks that extends the TTC concept.

A promising application direction of the predictive risk estimation is in behavior planning. We are currently considering the risk calculation of selected variations of the ego-car behavior. Consider e.g. the longitudinal velocity adjustment in a car following scenario. We assume the ego car to drive with constant velocity, but other choices for trajectories can be treated similarly. Combined with the leading (i.e., other) car trajectory, we can calculate the predicted future risk function for each of the ego-car trajectories. If we combine these risk functions over the travelled ego-car trajectory distance in a plot over the ego-car trajectory parameters as additional axis (or axes, if multiple), we get a *risk map*, which indicates the critical zones in terms of future time / points of the driven trajectory and trajectory parameters.

Fig. 5 shows such a risk map for a car following scenario. We can see a risk ridge decreasing downwards and rightwards. From the map we can see explicitly that the best longitudinal strategy would be to reduce the velocity difference in order to circumvent the ridge in the bottom part of the map. However, there are other ways to reach the safe zone in the upper right of the map, such as e.g. an overtaking maneuver. This implies a risk estimation using an alternative predicted trajectory, which also leads to an alternative predictive risk estimation and consequently, an alternative risk map. In this case, we have to consider several risk maps for the different maneuver options, and select the behavior based on these options.

In future work, we will apply the predictive risk estimation in combination with risk maps to trajectory evaluation, selection and planning of cars in dynamic environents.

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