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Preference-Based NSGA-II for Many-Objective Knapsack Problems

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***Abstract*—Many-objective optimization has attracted increaseing attention in the evolutionary multi-objective optimization (EMO) community. It has been repeatedly demonstrated that many-objective optimization problems with four or more objectives are very difficult to solve for EMO algorithms. Whereas a number of performance improvement attempts have been proposed, many-objective optimization is still difficult for EMO algorithms. In our previous study, we proposed a preference-based approach where Gaussian functions on a hyperplane in the objective space are used for preference representation. In this paper, we examine the behavior of our approach in the handling of combinatorial many-objective problems. Through computational experiments on multi-objective knapsack problems with 2-10 objectives, a set of well-distributed solutions over the preferred regions is obtained for each test problem. A trade-off relation between convergence and diversity for the preferred regions is also observed through computational experiments.**

Keywords—evolutionary multi-objective optimization (EMO); many-objective optimization; preference incorporation

I. INTRODUCTION

Many-objective optimization is a hot issue in the research field of evolutionary multi-objective optimization (EMO). This is because well-known EMO algorithms such as NSGA-II [2] and SPEA2 [3] do not work well on multi-objective problems with four or more objectives, as pointed out in the literature [4]-[7]. Many-objective problems are difficult for EMO algorithms because; (i) almost all solutions in the current population become non-dominated in early generations, and (ii) the number of solutions needed to approximate the entire Pareto front increases exponentially with an increase in the number of objectives.

Difficulties belonging to class (i) are related to a Pareto dominance-based fitness evaluation. When an EMO algorithm is applied to a multi-objective problem with two or three objectives, solutions in a population are usually classified into different ranks by a Pareto dominance relation. Thus, selection pressure toward the Pareto front is generated. However, when almost all solutions in a population are non-dominated, the Pareto dominance-based fitness evaluation cannot generate strong selection pressure even if the population is not close to the Pareto front in early generations.

Difficulties in class (ii) are related to the dimensionality of the Pareto front. In EMO algorithms, the Pareto front is approximated by a limited number of solutions. Thus, when the dimensionality of the Pareto front is large, it is very difficult to approximate the entire Pareto front.

One general idea to cope with these difficulties is to focus the multi-objective search by an EMO algorithm on a particular region in the objective space. This can be done by various methods such as a weighted indicator function [8] and a modified Pareto dominance relation [9]. Dimensionality reduction of the objective space (i.e., decreasing the number of objectives [10], [11]) can be viewed as focusing the multi-objective search on an objective sub-space. The most direct approach to the focused-search in the objective space is a preference incorporation into EMO algorithms (e.g., [12], [13]). The preference of a decision maker is used to drive a population toward his/her preferred region. In this case, the goal of EMO algorithms is not to search for the entire Pareto front but to find a set of non-dominated solutions which are well-distributed over his/her preferred region.

One important issue in the design of a preference-based EMO algorithm is how to represent a decision maker's preference. In the previous study [1], we proposed a new preference-based approach. In our approach, a decision maker's preference is represented by a single or multiple Gaussian functions on a hyperplane in the objective space. The center (i.e., the peak) of the Gaussian function shows the most preferred solution while its spread shows the width of the preferred region. Another important issue is how to use the preference in the multi-objective search. In our approach, the value of the Gaussian function at each solution in the objective space represents the degree of the decision maker's preference, which is used as the fitness of the solution.

In our former study [1], our preference-based approach was combined with NSGA-II and examined through computational experiments on DTLZ problems. In this paper, we use multi-objective knapsack problems and examine the scalability of our approach on the number of objectives.

This paper is organized as follows. First we explain our idea of preference-based multi-objective search in Section II. In Section III, we explain how our idea can be implemented in NSGA-II. Then, in Section IV, the effect of our approach is examined through computational experiments on multi-objective knapsack problems with 2-10 objectives. Finally, we conclude this paper in Section V.

II. Preference-Based Search

Usually EMO algorithms have been designed to approximate the entire Pareto front with a finite number of solutions. In the case of many-objective optimization, however, it is almost impossible to obtain the well-distributed non-dominated solutions over the entire Pareto front. This is because a huge number of solutions are needed for a good approximation of a high-dimensional Pareto front. One advantage of using preference-based EMO algorithms is that a decision maker can obtain more accurate information about the preferred region of the Pareto front (e.g., a local trade-off relation between each objective). In other words, preference-based EMO algorithms are supposed to find a set of solutions with the following two properties:

(1) Good convergence to the preferred region,
(2) Good distribution over the preferred region.

These two properties are explained in this section.

A. Convergence toward the Pareto front

In the case of many-objective optimization, the search ability of Pareto dominance-based EMO algorithms is severely deteriorated. Preference-based EMO algorithms can maintain selection pressure to the Pareto front. This is because the search space will be narrowed by considering the preference information. A variety of approaches have been proposed to represent the preference of a decision maker (e.g., use of a weighted hypervolume [14], use of a weighted vector [15], giving a new definition of dominance relation of each pair of objectives [16], [17], choice of only a few objectives [10], and specification of reference points [18]-[20]).

In our preference representation method [1], a set of Gaussian functions on a hyperplane is used to represent the preference of a decision maker. The decision maker is asked to specify his/her preference using an arbitrary number of Gaussian functions on a hyperplane, which is assumed to be an approximate Pareto front.

For example, if the decision maker is interested in the center of the Pareto front, a single Gaussian function will be placed on a center of the hyperplane. If the decision maker wants to obtain some solutions around the best solution with respect to each objective as well as the center of the Pareto front for an m-objective problem, $(m + 1)$ Gaussian functions will be used. In this manner, the decision maker can easily specify his/her preference even when he/she has no information about the true Pareto front. In our approach, the location of the hyperplane is updated at each generation using a set of solutions from parent and offspring populations. That is, the hyperplane is moved towards the Pareto front together with the current population. As a result, the preferred region on the hyperplane is also moved toward the Pareto front during the execution of our approach.

B. Diversity maintenance

It is usually important for EMO algorithms to show multiple non-dominated solutions to the decision maker. This is also the case in preference-based EMO algorithms. The decision maker usually wants to see several alternatives around the preferred region. Thus diversity maintenance is important even when the preferred region is specified. In R-NSGA-II [18], the minimum distance between solutions in the normalized objective space is preserved (i.e., ε-clearing). In our approach, we use the same idea on a hyperplane as follows. First, solutions are projected on a hyperplane. Next, a solution is randomly picked up from a population. Then, all solutions within a pre-specified minimum distance from the selected solution are removed from the population. The random selection of a solution and the removal of its neighbors within the pre-specified minimum distance are repeated until the distance between any pair of solutions in the current population becomes larger than or equal to the minimum distance.

III. Preference on a Hyperplane

In our approach, the decision maker is supposed to represent his/her preference using one or more Gaussian functions on a hyperplane in the objective space. In this section, we first explain how the hyperplane is specified in the objective space. Then we explain how Gaussian functions are used to represent preference of the decision maker.

A. Specification of Hyperplane

An m-objective maximization problem can be written as

$$\text{Maximize} \quad \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_m(\mathbf{x})), \tag{1}$$

where $\mathbf{f}(\mathbf{x})$ is an m-dimensional objective vector, $f_i(\mathbf{x})$ is the i-th objective to be maximized, and $\mathbf{x}$ is a decision vector. First, let us calculate the normalized m-objective vector $\mathbf{f}'(\mathbf{x}) = (f_1'(\mathbf{x}), f_2'(\mathbf{x}), ..., f_m'(\mathbf{x}))$, as follows:

$$f_k'(\mathbf{x}) = \frac{f_k^{\max} - f_k(\mathbf{x})}{f_k^{\max} - f_k^{\text{ext}}}, \text{ for } k = 1, 2, \ldots, m, \tag{2}$$

where $f_k^{\max}$ is the maximum value for the k-th objective, f_k^{ext} is the k-th objective value of an extreme vector $\mathbf{x}_k^{\text{ext}}$ for the k-th objective that can minimize the following achievement scalarizing function ($k = 1, 2, ..., m$):

$$ASF(\mathbf{x}, \mathbf{d}_k) = \max_{i=1}^{m} \frac{f_i^{\text{t}}(\mathbf{x})}{d_i}, \tag{3}$$

$$f_k^{\text{t}}(\mathbf{x}) = f_k^{\max} - f_k(\mathbf{x}) \ , \tag{4}$$

where $\mathbf{d}_k$ is a weight vector indicating the k-th-axis direction. For avoiding divide-by-zero, $d_i = 0$ is replaced with $d_i = 10^{-6}$. For example, the third-axis direction $\mathbf{d}_3$ (i.e., $k = 3$) is defined for a six-objective problem (i.e., $m = 6$) as $\mathbf{d}_3 = (10^{-6}, 10^{-6}, 1, 10^{-6}, 10^{-6}, 10^{-6})$. These maximum and extreme values are updated at every generation using the current population. From

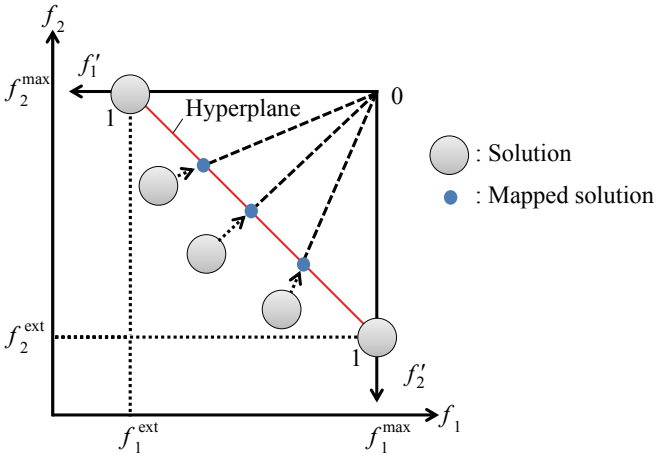


Fig. 1. Mapping of solutions to a hyperplane.

a normalized m-objective vector, the hyperplane is computed as in Fig. 1.

B. Projection of Solutions

In our approach, how much the decision maker prefers a solution is calculated as a *preference grade*. For the calculation of the preference grade of each solution, solutions in the population are mapped onto the hyperplane. On the hyperplane, the preference grade of each solution is calculated as a value of Gaussian functions. From the normalized vector $\mathbf{f}'(\mathbf{x})$ in the previous subsection, the solution $\mathbf{x}$ is normalized as follows:

$$h_j(\mathbf{x}) = \frac{f_j'(\mathbf{x})}{f_1'(\mathbf{x}) + f_2'(\mathbf{x}) + \ldots + f_m'(\mathbf{x})}, \text{ for } j = 1, 2, \ldots, m. \quad (5)$$

This vector $\mathbf{h}(\mathbf{x}) = (h_1(\mathbf{x}), h_2(\mathbf{x}), \ldots, h_m(\mathbf{x}))$ represents the location of the mapped solution $\mathbf{x}$ on the hyperplane. From (5), it is clear that the following relation always holds: $h_1(\mathbf{x}) + h_2(\mathbf{x}) + \ldots + h_m(\mathbf{x}) = 1$.

C. Calculation of Preference Grade

Let us assume that q Gaussian functions are given on the hyperplane. The preference grade $p(\mathbf{x})$ of each solution $\mathbf{x}$ in the current population is calculated as follows:

$$p(\mathbf{x}) = \max\{p_1(\mathbf{x}), p_2(\mathbf{x}), \ldots, p_q(\mathbf{x})\},$$
$$p_i(\mathbf{x}) = \exp\left\{-\sum_{j=1}^{m} \frac{\left(h_i(\mathbf{x}) - w_j^i\right)^2}{\left(s_j^i\right)^2}\right\}, \; i = 1, 2, \ldots, q, \quad (6)$$

where w_j^i is the j-th value of a peak control vector and s_j^i is the j-th value of a spread control vector of the i-th Gaussian function. These vectors are assumed to be specified by the decision maker under the following conditions:

$$\mathbf{w}^i = (w_1^i, w_2^i, \ldots, w_m^i), \; w_1^i + w_2^i + \ldots + w_m^i = 1, \; w_j^i \geq 0,$$
$$\mathbf{s}^i = (s_1^i, s_2^i, \ldots, s_m^i), \; s_j^i \geq 0, \text{ for } j = 1, 2, \ldots, m. \quad (7)$$

The peak control vector $\mathbf{w}^i$ defines the most preferred point while the spread control vector $\mathbf{s}^i$ defines the distribution of the preferred region around the most preferred point on the hyperplane.

The idea of using a hyperplane is inspired from MO-NSGA-II [21] where well-distributed reference points on the hyperplane are used in order to obtain well-distributed and well-converged solutions. However, our approach uses the hyperplane to represent the decision maker's preference through Gaussian functions. The Gaussian functions are used to drive the multi-objective search by an EMO algorithm to the preferred region with large values of the Gaussian functions. In Fig. 2, we illustrate an example of the preferred regions specified by Gaussian functions for the case of three-objective optimization. Fig. 2 illustrates four preferred regions on a hyperplane using Gaussian functions.

To implement the procedure, we use the framework of NSGA-II. We made two modifications to NSGA-II. One is solution selection for crossover. In the original NSGA-II, solutions are selected by using binary tournament selection based on its rank assignment mechanism called "non-dominated sorting" and a secondary measure called "crowding distance". However, we use the preference grade $p_i(\mathbf{x})$ from i-th Gaussian function instead of a crowding distance in our method. That is, parent solutions are selected by binary tournament selection based on their non-domination rank and the preference grade. This is because our interest is focused on the preferred regions rather than the entire Pareto front in our preference method.

The other modification is population update. In our approach, we replace the crowding distance with the preference grade $p(\mathbf{x})$. That is, the next population is constructed from the current population and the offspring population based on their rank and the preference grade $p(\mathbf{x})$. By considering the preference grade instead of the crowding distance, we can expect to keep high selection pressure toward the preferred region even if almost all solutions become non-dominated in many-objective optimization.

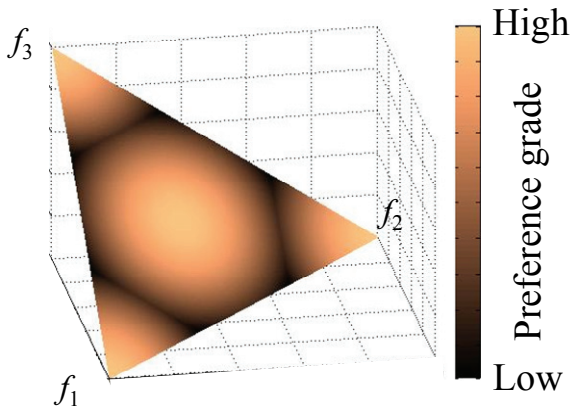


Fig. 2. An example of Gaussian functions on a hyperplane. The preferences around each corner and center part of the hyperplane are represented by four Gaussian functions.

For the diversity maintenance, we also consider the minimum distance $d_{\min}$ on the hyperplane in the above population update procedure. Solutions having a distance of $d_{\min}$ or less to their nearest neighbor are assigned the smallest preference grade in order to discourage them to remain in the race. As the result, a Euclidean distance between any solutions in a population is more than $d_{\min}$ on a hyperplane. Additionally, the extreme solutions selected by using (3) are assigned the largest preference grade. This is because we want to keep the extreme solutions that are needed for constructing a widespread hyperplane. These mechanisms are used in the calculation of the preference grade $p(\mathbf{x})$ in the population update. In the next section, we show the performance of our preference-based NSGA-II, namely, NSGA-II using preferences on a hyperplane (NSGA-II-PH).

IV. Computational Experiments

In this section, we demonstrate the performance of NSGA-II-PH through computational experiments on multi-objective 500-item 0/1 knapsack problems of Zitzler & Thiele [22] with two to ten objectives. In this paper, we denote the m-objective n-item 0/1 knapsack problem as m-n problem (i.e., 2-500, 4-500, 6-500, 8-500, and 10-500). In our computational experiments, solutions of each test problem were coded as binary strings of length 500. We applied the same greedy repair method as in Zitzler & Thiele [22] to infeasible solutions. The following setting was used in NSGA-II-PH:

Crossover probability: 0.8 (Uniform crossover),
Mutation probability: 0.002 (Bit-flip mutation),
Stopping condition: 40,000 solution evaluations.

Fig. 3 shows all solutions in the final generation in a single run on the 2-500 problem. We use a population size of 50, $d_{\min}$ = 0.005, and the following sets of Gaussian functions.

Center preference

In this specification, a center region on the hyperplane is preferred.

$\mathbf{w}^1 = (0.5, 0.5)$, $\mathbf{s}^1 = (1.0, 1.0)$.

Uniform hybrid preferences

This specification represents the preference of a center and each end on the hyperplane.

$\mathbf{w}^1 = (1.0, 0.0)$, $\mathbf{s}^1 = (1.0, 1.0)$,
$\mathbf{w}^2 = (0.0, 1.0)$, $\mathbf{s}^2 = (1.0, 1.0)$,
$\mathbf{w}^3 = (0.5, 0.5)$, $\mathbf{s}^3 = (1.0, 1.0)$.

Modified hybrid preferences

In this specification, the center and end regions are also preferred. However, the decision maker has much more interest in the center region than the end region.

$\mathbf{w}^1 = (1.0, 0.0)$, $\mathbf{s}^1 = (0.4, 0.4)$,
$\mathbf{w}^2 = (0.0, 1.0)$, $\mathbf{s}^2 = (0.4, 0.4)$,
$\mathbf{w}^3 = (0.5, 0.5)$, $\mathbf{s}^3 = (1.0, 1.0)$.

The dotted line in the plot shows the true Pareto front of the 2-500 problem. Open circles represent the final solutions. We can observe that the final solutions converged to each preferred region. From a comparison between the results of using the uniform hybrid preferences (Fig. 3 (b)) and modified ones (Fig. 3 (c)), we can also see that much more solutions converged on the center region in the modified hybrid preferences. It means that the final solutions were distributed according to not only the peak control vector but also the spread control vector of Gaussian functions in NSGA-II-PH.

Next, we examine the effect of $d_{\min}$ on the performance of NSGA-II-PH. We tried two settings: $d_{\min}$ = 0.01 and $d_{\min}$ = 0.1. We also use a population size of 100. For examining convergence and diversity ability for each problem, the following two simple measures are used [5]:

Maximum sum of the objective values (MaxSum)

The MaxSum measure is the maximum sum of the m-objective values over all solutions at each generation. In every generation, we calculated the maximum sum of the objective values as follows:

$$\mathrm{MaxSum}(P_t) = \max_{\mathbf{x} \in P_t} \left\{ \sum_{i=1}^{m} f_i(\mathbf{x}) \right\}, \tag{8}$$

where P_t is a population at the t-th generation. This measure evaluates the search progress toward the center of the Pareto front.

Sum of the ranges of the objective values (Range)

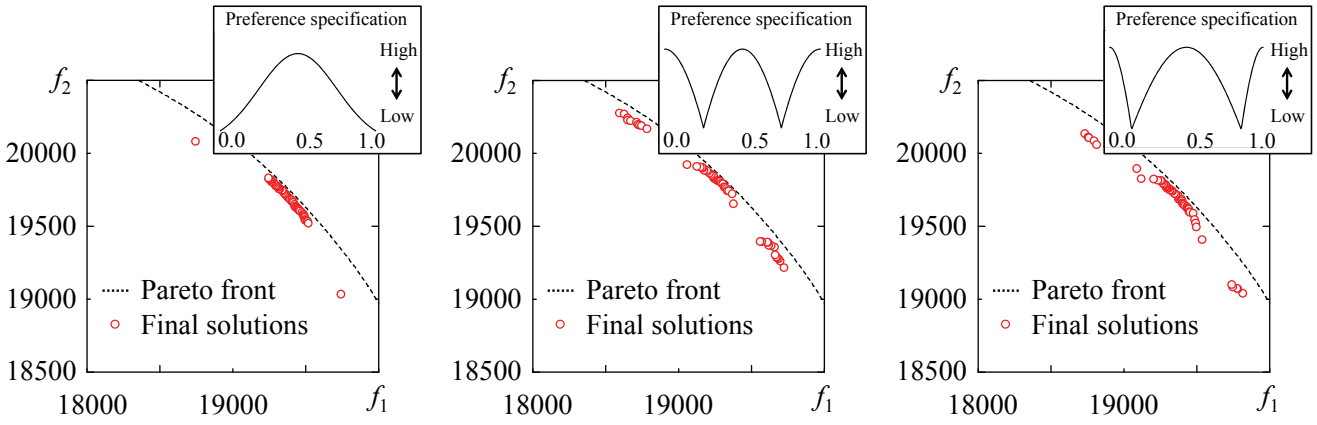
The Range measure shows the sum of the difference between maximum value and minimum value of each objective value. This measure evaluates the spread of solutions in the objective space in each generation.

$$\mathrm{Range}(P_t) = \sum_{i=1}^{m} \left[\max_{\mathbf{x} \in P_t} \{ f_i(\mathbf{x}) \} - \min_{\mathbf{x} \in P_t} \{ f_i(\mathbf{x}) \} \right]. \tag{9}$$

We show the average of the MaxSum measure and the Range measure at each generation in Fig. 4 and Fig. 5, respectively. These results in Fig. 4 and Fig. 5 are normalized so that the average results of initial populations over 20 runs become 100. We use the center preference for each problem in NSGA-II-PH (e.g., $\mathbf{w}^1$ = (0.25, 0.25, 0.25, 0.25) and $\mathbf{s}^1$ = (1.0, 1.0, 1.0, 1.0) are used for the 4-500 problem). The MaxSum measure represents the convergence toward the preference and the Range measure represents the diversity of the population of each generation. For comparison, we also show the results of NSGA-II in Fig. 4 (c) and Fig. 5 (c).

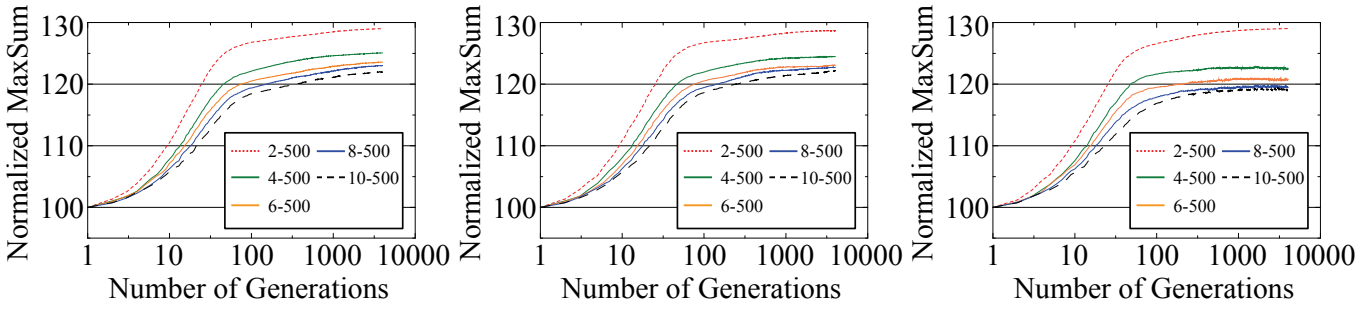
We can see from Fig. 4 that the convergence to the center region of the Pareto front became worse with the increase in the number of objectives in NSGA-II. In NSGA-II-PH, the convergence toward the preference region also became worse. However, NSGA-II-PH took larger values of the MaxSum than NSGA-II at the end of the generation.

From Fig. 5, we can see that the Range of obtained solutions in many-objective problems was small for NSGA-II-PH. On the other hand, the diversity of solutions increased with the increase in the number of objectives in NSGA-II. From the comparison of Fig. 5 (a) and Fig. 5 (b), we can also see that NSGA-II-PH with $d_{\min}$ = 0.1 takes larger values of the Range measure than that with $d_{\min}$ = 0.01 at the end of the generation.



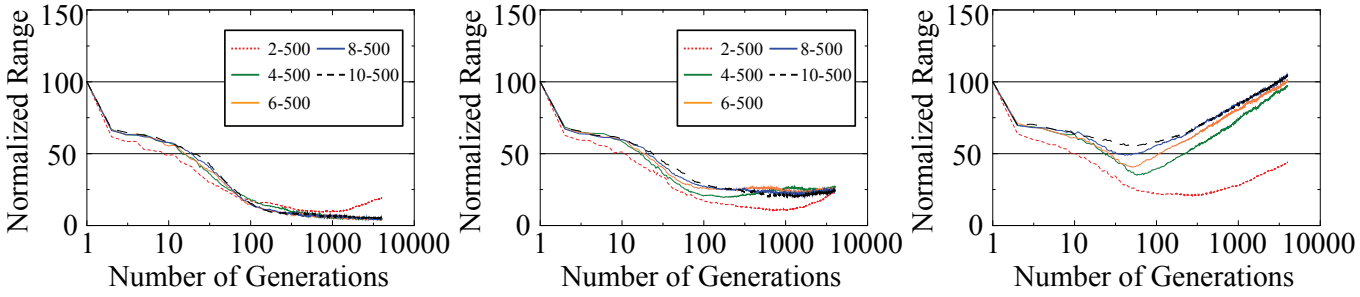
(a) Using the center preference (b) Using the uniform hybrid preferences (c) Using the modified hybrid preferences

Fig. 3. Results of a single run on the 2-500 problem for each preference.



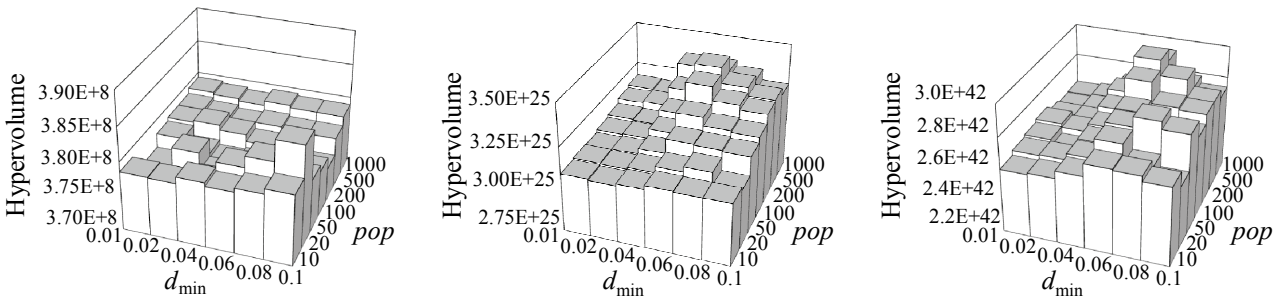
(a) NSGA-II-PH ($d_{\min} = 0.01$) (b) NSGA-II-PH ($d_{\min} = 0.1$) (c) NSGA-II

Fig. 4. Results of MaxSum of NSGA-II-PH with the center preference and NSGA-II.



(a) NSGA-II-PH ($d_{\min} = 0.01$) (b) NSGA-II-PH ($d_{\min} = 0.1$) (c) NSGA-II

Fig. 5. Results of Range of NSGA-II-PH with the center preference and NSGA-II.



(a) Hypervolume for the 2-500 problem (b) Hypervolume for the 6-500 problem (c) Hypervolume for the 10-500 problem

Fig. 6. Results of mean hypervolume of NSGA-II-PH with the center preference.

We used the fixed population size in the above computational experiments. Next, we used the combination of the diversity maintenance parameter $d_{\min}$ = {0.01, 0.02, 0.04, 0.06, 0.08, 0.1} and population size *pop* = {10, 20, 50, 100, 200, 500, 1000} by using the hypervolume in order to examine the relation of $d_{\min}$ and *pop*. For each combination, the average value of the hypervolume measure was calculated over 20 runs for each test problem. Experimental results on the 2-500, 6-500, and 10-500 test problems are summarized in Fig. 6. As the reference point for the hypervolume calculation, we used the origin point (0, 0, ..., 0) of the objective space.

From Fig. 6, we can observe that the best result was obtained in *pop* = 20 on the 2-500 problem whereas *pop* = 500 worked well on the 6-500 and 10-500 problems. This means a number of solutions are still needed for a good distribution over the preferred region on many-objective problems. It is also observed that good results were not obtained at the left side (i.e., $d_{\min}$ = 0.01, 0.02) in 6-500 and 10-500 problems. It means small $d_{\min}$ cannot maintain the spread of solutions (see Fig. 5 (a)). These results show that $d_{\min}$ should be carefully chosen according to the population size.

V. CONCLUSIONS

In this paper, we examined the performance of our preference-based EMO algorithm for combinatorial multi-objective problems. In our approach, the preference is represented by Gaussian functions on a hyperplane in the objective space. Our idea of using the preference is combined with a framework of NSGA-II (NSGA-II-PH). In NSGA-II-PH, solutions in a population are normalized and mapped on a hyperplane. The preference grade of solutions (how much a decision maker prefers a solution) is calculated by Gaussian functions. In the same front, solutions with larger preference grades are favored in the search process. Diversity around preferred regions on the hyperplane is controlled by considering the distance between each solution on a hyperplane. The effectiveness of these implementation schemes was examined through computational experiments on multi-objective 0/1 knapsack problems. Experimental results revealed the followings; (i) the distribution of the final solutions can be controlled by Gaussian functions, and (ii) there is a trade-off relation between convergence and diversity ability concerning the $d_{\min}$ parameter. It would be interesting to examine the performance of the combination of different $d_{\min}$ values or design of an on-line adaptation mechanism of $d_{\min}$. These interesting issues are left for future research.

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