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2014

Preprint:

This is an accepted article published in Engineering Optimization 2014. The final authenticated version is available online at: https://doi.org/[DOI not available]

Engineering Optimization IV – Rodrigues et al. (Eds)
© 2015 Taylor & Francis Group, London, ISBN 978-1-138-02725-1

Towards multi-objective topology optimization of structures subject to crash and static load cases

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ABSTRACT: In the optimization of automotive structures, typical requirements include stiffness and crashworthiness. The hybrid-cellular automata algorithm is a heuristic topology optimization approach, which targets a uniform distribution of the energy-based field variables in the structure. It has been applied to the topology optimization of structures subject to crash or static loads. Multiple load cases can be considered by a linear weighting of the field variables, however so far no approaches exist on combining the multi-disciplinary case of crash and static loadings in a concurrent topology optimization. Difficulties are the very different energy scales of the two disciplines and the potentially conflicting nature of the objectives. As first step, we propose the concurrent topology optimization of structures subject to crash and static loads by using a linear weighting of the internal energy densities absorbed during the crash load and the elemental strain energy densities caused by the static load. For a set of different weightings the solutions obtained by the algorithm can be evaluated in a multi-objective context. We demonstrate on a simply supported beam example, that for the separated disciplines the approach yields a set of non-dominated trade-off solutions. Furthermore, we show that such a set of solutions can also be obtained when combining crash and static load cases by carefully tuning the applied weights to the energy levels of the load cases.

1 INTRODUCTION

Key factors of future success for global automakers will depend on the application of light, smart, innovative materials. Consequently the development of design criteria for weight reduction plays a major role for further improvements in efficiency. In the future, production vehicles will require significantly lighter structural solutions which offer the best balance in terms of performance, cost, weight, volume and functionality criteria.

Advanced construction approaches must be investigated to improve crashworthiness including management of impact loads, reduce structure-borne sound propagation, and maintain high structural reliability in future lightweight vehicles. Concretely, a structure may be required which is sufficiently rigid under normal working conditions but in addition provides the capability to absorb a certain amount of energy in an impact scenario.

In this case, it is necessary to consider different types of loads in order to obtain a compromise solution in a multi-disciplinary optimization (MDO) (Duddeck 2008). To enable these future concept investigations we propose the development of computer driven multi-objective topology optimization approaches.

Considering the task of providing a multi-disciplinary concept design by topology optimization, the MDO problem is especially difficult when crash objectives are involved, since in most cases analytical sensitivities of the design variables are not available and heuristic methods need to be considered, even in the single objective case. Furthermore, due to the high number of design variables in typical topology optimization problems, existing multi-objective optimization algorithms, e.g. meta-heuristics like evolutionary algorithms (Coello, Lamont, & Veldhuizen 2006) cannot be applied directly.

The Hybrid Cellular Automata (HCA) method is a biologically-inspired approach for topology optimization motivated by the remodeling of bone growth (Tovar 2004) which has the capability to provide stiff structures (Tovar, Patel, Niebur, Sen, & Renaud 2006). In (Patel 2007), (Patel, Kang, Renaud, & Tovar 2009), the HCA approach is extended to the topology optimization of crashworthiness structures. The HCA approach targets a uniform distribution of a field variable within the structure. This is similar to the idea of fully stressed design (Patnaik & Hopkins 1998), which aims for an optimal solution by a full utilization of the available material. Although the HCA has been shown applicable to both disciplines separately, no literature

is known to the authors, in which it has been applied in a concurrent MDO for crashworthiness and stiffness.

Multiple load cases are treated by the HCA methodology by a linear weighting of the field variables, similar as in the general density-based topology optimization (Bendsøe & Sigmund 2004), in which the objectives for the load cases are often combined by a linear weighting i.e. the multi-objective problem is addressed by a scalarization of the multi-dimensional objective function.

An example for multi-objective topology optimization is the optimization of compliant mechanisms for combining stiffness and flexibility requirements in (Luo, Chen, Yang, Zhang, & Abdel-Malek 2005). A related approach was also applied in (Shinde, Detwiler, & Tovar 2013), in which a thin-walled tubular structure is optimized for a controlled buckling behavior by a weighting approach based on the extension of the HCA for controlled crash behavior (Bandi 2012). They apply a weighting of the mutual potential energy and the strain energy, aiming for the optimization of crashworthiness objectives. The weighting approach in multi-objective topology optimization has also been applied in more clearly multi-disciplinary contexts e.g. for designing microstructures combining stiffness and thermo elasticity (Deng, Yan, & Cheng 2013) or stiffness and fluid permeability (Guest & Prévost 2006). These approaches are mentioned to highlight that a multi-objective topology optimization can also be achieved when different disciplines are involved.

In general multi-objective optimization, a critical aspect of the weighting approach is the choice of the appropriate weights, since besides the preference for the objectives the weights also need to take account for the different scales (Marler & Arora 2010). In case of a combination of static and crash loads, the energy occurring in the two disciplines can be magnitudes apart. Although in (Bandi 2012) an adaptive weighting has been proposed in order to deal with different magnitudes of crash load cases, we show that multi-objective trade-off solutions when combining crash and static load cases can be obtained relatively straight forward by a linear weighting of the energy densities when applying a careful scaling.

In the next section, we provide the optimization problem formulation and describe the multi-objective optimization approach. Section 3 presents the design space and load cases utilized for testing. The method is evaluated by conducting the optimization for a set of weight vectors and the results are presented in Sec. 4. The paper is concluded in Sec. 5.

2 TOPOLOGY OPTIMIZATION METHODOLOGY

Topology optimization algorithms commonly operate on the finite element mesh representation of the design space available for the structure, in which each element is a binary decision variable. In density-based topology optimization methods (Bendsøe & Sigmund 2004), the binary problem is relaxed and each element is assigned a continuous density variable. A frequently used material interpolation scheme is the solid isotropic material with penalization (SIMP) material model (Bendsøe 1989) according to which the Young's modulus of the material within an element is determined as $E_i(\rho_i) = \rho_i^p E_0$, with density ρ_i corresponding to element i, Young's modulus of the full material E_0 and penalization p.

The minimum compliance topology optimization problem can be stated as:

$$\begin{aligned} &\min_{\boldsymbol{\rho}} c(\boldsymbol{\rho}) = \mathbf{u}^T\mathbf{f} \\ &\text{subject to : } \mathbf{K}(\boldsymbol{\rho})\mathbf{u} = \mathbf{f}, \\ &\qquad V(\boldsymbol{\rho}) = V_f, \\ &\qquad 0 < \rho_{\min} \leq \rho_i \leq 1,\ i = 1,\ldots,N\ , \end{aligned} \tag{1}$$

with the compliance c, the displacement vector $\mathbf{u}$ and the load vector $\mathbf{f}$. In the equilibrium equation, $\mathbf{K}$ is the stiffness matrix. A constraint V_f is imposed on the volume fraction $V(\boldsymbol{\rho})$ of the structure. In order to avoid numerical difficulties, a minimum density $\rho_{\min}$ is defined.

For crash loads the linear elastic material model is not applicable since the structure can be subject to plastic deformation. In this case, the SIMP interpolation can be extended to the yield stress σ_{Y_i} and the strain-hardening modulus E_{h_i} (Patel 2007):

$$\sigma_{\mathrm{Y}_i} = \rho_i^p \sigma_{\mathrm{Y}_0},\quad E_{\mathrm{h}_i} = \rho_i^p E_{\mathrm{h}_0}\ , \tag{2}$$

with the yield stress σ_{Y_0} and the strain hardening modulus E_{h_0} of the full material.

The maximization of the energy absorption for the crash can be formulated as:

$$\begin{aligned} &\max_{\boldsymbol{\rho}} \mathrm{EA}(\boldsymbol{\rho}) \\ &\text{subject to : } \mathbf{r}(\boldsymbol{\rho}) = \mathbf{0}, \\ &\qquad V(\boldsymbol{\rho}) = V_f, \\ &\qquad 0 < \rho_{\min} \leq \rho_i \leq 1,\ i = 1,\ldots,N\ , \end{aligned} \tag{3}$$

with the total energy EA absorbed by the structure and the residual $\mathbf{r}(\boldsymbol{\rho})$ of the finite element analysis.

Both problems can be addressed by the HCA approach, in which each element is mapped to a cell representation, which in most cases uses the same discretization as the finite element mesh, since the largest part of the computational effort is required by the analysis. The state of each cell is defined by a field variable S_i and the material density ρ_i. The optimization is heuristically addressed by applying a control based update rule aiming for a uniform distribution of the field variable in order to achieve an optimized

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material utilization. While not strictly minimizing the objective function in a mathematical sense, this nevertheless leads to useful designs from an engineering perspective, based on the idea of fully stressed design. In the design of stiff structures, the elemental strain energy density (SED) is used as field variable (Tovar, Patel, Niebur, Sen, & Renaud 2006), whereas in the design of crashworthiness structures, the elemental internal energy density (IED) is used (Patel 2007). A design update, respectively an update of the cell states can be obtained by:

$$\rho_i^{\text{new}} = \rho_i + K_{\text{P}} \left(S_i - S^* \right) \quad , \tag{4}$$

where K_{P} is a scaling parameter and S^* is a set-point for the field variables, which is adapted in each iteration so that the desired volume constraint holds. In order to avoid numerical instabilities and to impose a minimum length scale, spatial and temporal filtering of the field variables is proposed in (Patel 2007).

In the optimization of multiple load cases, the field variables of the load cases are combined by a linear weighting:

$$S_i = \sum_{l=1}^{L} w_l S_{il} \quad , \tag{5}$$

with the number of load cases L, a weight w_l for each load case and the field variable S_{il} of element i for load case l.

So far the combination of concurrent static and crash load cases has not been considered yet. We propose to optimize a linear weighting of the objectives in (1) and (3) by applying the HCA methodology for the case of distinguished field variables:

$$S_{il} = \begin{cases} \text{IED}_{il} & \text{if } l \in \mathcal{L}_{\text{crash}} \\ \text{SED}_{il} & \text{if } l \in \mathcal{L}_{\text{static}} \end{cases} \quad , \tag{6}$$

with the set of crash and static load cases, $\mathcal{L}_{\text{crash}}$ and $\mathcal{L}_{\text{static}}$, respectively. Since both quantities are energies, the summation in (5) is physically meaningful.

An approximation of the Pareto front of the MDO problem can be obtained by several runs with different weights. A preference for a load case can be expressed by adapting the appropriate weight. However, besides expressing preferences, the weights in (5) need to account for different scales of the field variables. The energies occurring in a crash load case are usually orders of magnitudes higher than those occurring for a static load. In order to avoid a dominance of crash load cases, which leads to a neglect of stiffness requirements we propose to scale the field variables of the load cases by a scaling factor s_l:

$$S_i = \sum_{l=1}^{L} w_l \frac{1}{s_l} S_{il} \quad . \tag{7}$$

The idea of this formulation is to decouple the scaling of the load cases from the preference of the engineer, so that once a suitable s_l is determined, the choice of the weights can be based on the desired trade-off.

We propose to choose the scaling factor

$$s_l = \frac{\sum_{i=1}^{N} S_{il}^{\text{full}}}{\min_{l'} \sum_{i=1}^{N} S_{il'}^{\text{full}}} \quad , \tag{8}$$

with the field variables S_{il}^{full} obtained from an analysis of the full design, i.e. when $\rho_i = 1$ for all elements, and the indices for the load cases l, respectively l'. The values for S_{il}^{full} can be easily obtained a-priori to the optimization by one additional analysis per load case and are an indicator of the different magnitudes of energies involved in the load cases. Concretely, the elemental IED or SED values of load case l are scaled by the ratio of the total energy absorbed by load case l to the total energy absorbed by the load case which overall absorbs the smallest amount of energy.

3 TEST MODELS

As a benchmark problem, we consider the optimization of a simply supported aluminum beam. The length of the beam is 600 mm, with a square cross section, with 50 mm width and height. The geometry and the location of the supports are shown in Fig. 1. The beam is subject to two crash and two static load cases.

The crash load is inspired by a three pole bending test. A cylindrical pole impacts the beam on the top with a constant velocity of 1 m/s up to a total intrusion of 50 mm. For the first crash load case the pole is impacting in the center of the beam, while for the second load case an offset by 50 mm in x-direction is implemented. The crash load cases in deformed and undeformed states are depicted in Fig. 2. The crash load cases are identified as load case 1a) and load case 1b).

The static load cases are applied perpendicular to the crash load cases along the y-axis. The first load case is applied to the center of the front of the beam, while the second load case is applied from the back side with

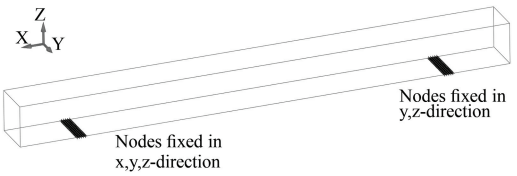

Figure 1. The supports of the beam model.

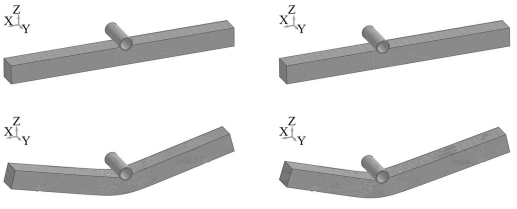

Figure 2. Crash load cases in initial (top) and deformed (bottom) state, left: 1a) center crash, right: 2a) offset crash.

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an offset of 70 mm along the x-direction. A distributed force of 10 kN is applied. The static load cases 2a) and 2b) are shown in Fig. 3. The orthogonality of the crash and static load cases is chosen in order to avoid a direct conflict between the objective of maximizing the energy absorption for the crash load and minimizing the energy absorption for the static loads.

The explicit non-linear finite element solver capabilities of the software LS-Dyna are used in order to carry out the simulation of the crash load cases, and its implicit linear solver is used for the static load case simulations. The material is modeled with a linear elastic model (MAT_001) in case of the static load cases and with a piecewise-linear elastic-plastic material model (MAT_024) in case of the crash loads. The material parameters are given in Tab. 1. The impactor is modeled as rigid. The static force is distributed to an area of 25 nodes.

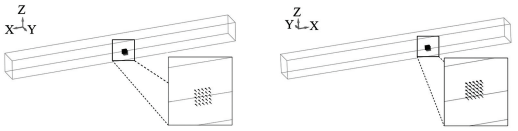


Figure 3. Static load cases, left: 2a) Center load in negative y-direction applied to front side, right: 2b) Offset load in positive y-direction applied to back side (axis rotated).

Table 1. Properties and values of aluminum material model.

Property	Value
Density	$2.7e^{-9}$ kg/mm^3
Poisson's ratio	0.33
Young's modulus	$7e^4$ MPa
Yield stress	180 MPa
Effective plastic strain	**Effective stress/MPa**
0.01	190.0
0.02	197.0
0.05	211.5
0.10	225.8
0.15	233.6
0.20	238.5
0.40	248.5

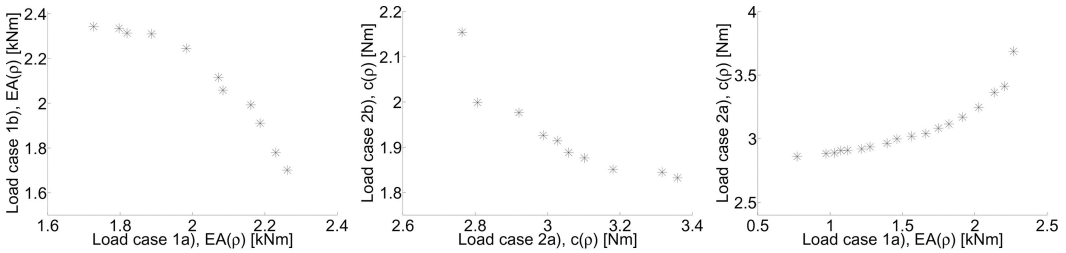
The beam is meshed with a hexahedral mesh with an element edge length of 2.5 mm, in total 96,000 elements.

4 EXPERIMENTS

In the first and second experiment, the multi-objective optimization subject to two load cases of the separated disciplines is carried out. The achieved objective values for a multi-objective optimization of the beam subject to crash load cases 1a) and 1b), followed by a multi-objective optimization of the beam subject to static load cases 2a) and 2b) are analyzed. In these two experiments it is verified that the HCA approach on the one hand provides optimized solutions for the defined objectives and load cases and on the other hand to demonstrate the feasibility of the multi-objective approach. In a third experiment, the MDO problem is addressed. Crash load case 1a) and static load case 2a) are combined in a multi-objective topology optimization.

The multi-objective optimization is realized by choosing $w_{1j} = 0.05 \cdot j$, with $j = 0, \ldots, 20$, i.e. 21 different weight vectors in total, and $w_{2j} = 1 - w_{1j}$. Weight w_{1j} equal to one or zero corresponds to a single load case optimization. The whole beam defines the design space, and a target volume fraction of $V_f = 0.25$ is chosen. For each weight combination, a topology optimization is run using the implementation of the HCA method available through the commercial software tool LS-TaSC (Roux 2011). For the optimization algorithm the only difference between the static and the crash load cases are the analysis and the field variables. As explained in Sec. 2 the target of the HCA is a uniform distribution of the field variables in either case. A filter radius of 8.8 mm is chosen to ensure a minimum length scale of the optimized design and each topology optimization is run for 50 iterations.

The non-dominated solutions in terms of energy absorption for the multi-objective optimization of the crash load cases are plotted in Fig. 4a. Similarly, the solutions for the multi-objective optimization of the static load cases in terms of compliance are plotted in Fig. 4b. The values of $EA(\boldsymbol{\rho})$ and $c(\boldsymbol{\rho})$ are obtained from the final value of the LS-Dyna glstat database.



(a) Two crash load cases. (b) Two static load cases. (c) A crash and a static load case.

Figure 4. The non-dominated solutions found for multi-objective optimization of the beam with different weight vectors, subject to two load cases. For crash load cases the energy absorption is maximized, for static load cases the compliance is minimized. Note the different units for crash and static load cases.

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No scaling factor was applied since the energies of the two load cases are within the same order of magnitude. Since the methodology is a heuristic approach and the number of design variables is very high it cannot be expected to find a global optimum of the scalarized objective function, resulting in a subset of dominated solutions. Nevertheless, a set of non-dominated solutions is found, demonstrating that the method of multi-objective topology optimization with HCA by different weightings is able to provide the designer with a choice depending on the requirements for each load case with regard to the objectives in (1) and (3).

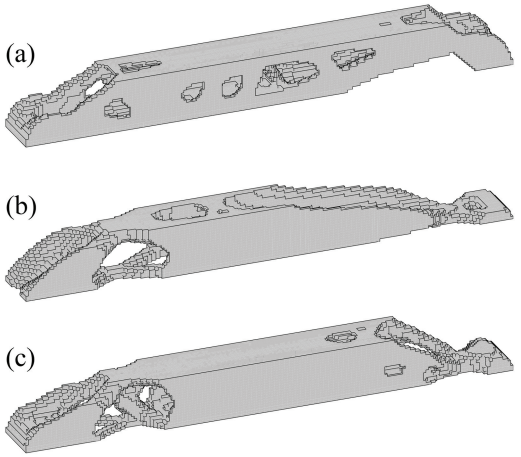

Figure 5. Trade-off solutions that for the respective multi-objective optimization maximize the minimum of the two performance values: Two crash loads (a), two static loads (b), crash and static load (c).

In the case of multi-objective optimization of crash and static load cases, following (8) the scaling factors are determined to $s_1 \approx 2250$ and $s_2 = 1$ and the weighting in (7) is applied. The resulting non-dominated solutions in objective space are plotted in Fig. 4c.

It can be seen that also in this case an approximation of the Pareto front is obtained. Only three solutions are dominated, resulting in a smooth Pareto-front approximation with evenly distributed solutions (relative to the preceding experiments). The overall approximation of the Pareto front is rather convex, showing that for the considered trade-off solutions exist which concurrently satisfy energy absorption and stiffness requirements. This shows that the scaling method proposed in (8) is efficient in providing solutions to the problem of multi-objective topology optimization of crash and static load cases.

For the separated experiments, we define the best design obtained for one load case as 100% performance. This facilitates to compare all solutions obtained by the multi-objective approach with respect to these baselines by using the performance measures

$$p_{c_j} = \frac{\min_{j'}(c_{j'})}{c_j}, \quad p_{EA_j} = \frac{EA_j}{\max_{j'}(EA_{j'})} , \tag{9}$$

with the compliance c_j and the absorbed energy EA_j of the optimized solution for the load case. Table 2 provides the performance values for all experiments.

Table 2. Performance of the optimized designs in terms of compliance and crash energy absorption related to the best design as baseline. Dominated solutions are in parenthesis. The bold values are the trade-offs that maximize the minimum of the two performance measures.

	Crash Loads		Static Loads		Crash and Static Load	
w_{1j}	$p_{EA_j}^{LC1a)}$ [%]	$p_{EA_j}^{LC1b)}$ [%]	$p_{c_j}^{LC2a)}$ [%]	$p_{c_j}^{LC2b)}$ [%]	$p_{EA_j}^{LC1a)}$ [%]	$p_{c_j}^{LC2a)}$[%]
0.00	(50.99)	(96.75)	(60.36)	(98.68)	34.06	100.00
0.05	(61.79)	(97.35)	(70.50)	(98.19)	42.87	99.21
0.10	(74.49)	(97.26)	82.28	100.00	45.40	99.01
0.15	76.34	100.00	83.33	99.32	47.38	98.40
0.20	(75.15)	(99.38)	86.88	99.00	49.47	98.31
0.25	(76.47)	(99.39)	(86.50)	(98.25)	(51.32)	(97.89)
0.30	(75.83)	(97.73)	89.10	97.65	53.72	97.97
0.35	(74.12)	(99.24)	90.37	97.04	56.26	97.41
0.40	79.46	99.67	91.30	95.72	(58.57)	(96.20)
0.45	80.43	98.76	92.49	95.11	61.47	96.49
0.50	83.42	98.60	(94.16)	(92.01)	64.47	95.39
0.55	(79.60)	(97.05)	**94.62**	**92.70**	68.96	94.69
0.60	87.66	95.85	98.48	91.67	73.28	94.10
0.65	**91.60**	**90.30**	(96.60)	(89.34)	77.10	92.76
0.70	92.14	87.86	(97.55)	(86.80)	80.32	91.81
0.75	95.55	85.07	100.00	85.06	84.47	90.23
0.80	96.69	81.57	(99.35)	(79.83)	**89.40**	**88.10**
0.85	(98.34)	(75.95)	(99.60)	(78.04)	94.13	85.03
0.90	(97.37)	(69.02)	(99.23)	(73.32)	97.27	83.81
0.95	98.60	75.96	(99.52)	(64.94)	100.00	77.58
1.00	100.00	72.63	(99.56)	(75.13)	(99.77)	(0.00)

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Dominated solutions are in parenthesis. The table is a proposal how to process the results in order to facilitate the choice of a trade-off solution practically.

Surprisingly, all except two of the single load case optimizations are dominated. This possibly can be attributed to the local and heuristic nature of the optimization. An exception is the multi-disciplinary case, in which the algorithm removed the material in the region of the static load in the single load case optimization of the crash. In all experiments trade-off solutions are found. A possible choice for the final design is to chose the trade-off that maximizes the minimum performance measure. Those solutions are highlighted in Tab. 2. The corresponding structures are shown in Fig. 5. Intermediate density elements are plotted in the same color as full density elements. In the multi-disciplinary case a design is found that maintains more than 88% of the baseline performance for both disciplines.

5 CONCLUSION

This paper addresses the multi-disciplinary topology optimization of structures subject to crash and static load cases. A multi-objective optimization is performed by using a set of different weightings in a multi-load case optimization with the hybrid cellular automata approach. For crash load cases, energy maximization and for static load cases, compliance minimization is considered as objective, respectively. As a first step, in experiments on a beam example, it is verified that the approach is able to provide a set of non-dominated trade-off solutions in the case of applying either two static load cases or two crash load cases in the optimization. In the multi-disciplinary case, the structure is optimized concurrently for a crash and a static load case. We propose a linear weighting of the internal energy densities caused by the impact and the strain energy densities caused by the static load. In order to avoid dominance of crash load cases, the energy densities are scaled based on the ratios of the total energy absorbed by the full design. An approximation of the Pareto front is obtained consisting of a set of trade-off solutions which satisfy crash and and stiffness requirements. The results demonstrate for the considered problem, that within the HCA approach, crash and static load cases can be treated alike assuming that a suitable scaling of the load cases is applied.

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