# References or Preferences – Rethinking Many-objective Evolutionary Optimization

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***Abstract*—Past decades have witnessed a rapid development in research on multi- and many-objective evolutionary optimization. Reference-based and preference-based strategies are both widely used in dealing with the multi- and many-objective optimization problems. However, little effort has been devoted to a critical analysis of similarities and differences between the two approaches.**

**This paper revisits the methodologies, compares the similarities and differences, and discusses the limitations of reference-based and preference-based many-objective evolutionary algorithms. Our analyses reveal that preference information may be embedded into reference-based methods in dealing with irregular problems so that the objective space can be better explored and a solution set of interest to the user will be obtained. Meanwhile, it is far from trivial for a decision-maker to provide informed preferences without sufficient *a priori* knowledge of the problem in the preference-based optimization. Therefore, this paper suggests a new approach to many-objective optimization problems that integrates preference-based and reference-based methodologies, where the solutions of natural interest such as the knee regions are identified at first and then the acquired knowledge of the knee regions can be used in reference-based methods. This way, accurate, diverse and preferred solutions can be obtained, and a deeper insight into the problem can be gained.**

***Index Terms*—component, formatting, style, styling, insert**

## I. Introduction

In the real world, a large number of optimization problems often involve multiple or many objectives, and the objectives commonly conflict with each other. Thus, no single solution can satisfy all objectives but a set of trade-off solutions can be found in the multi- or many-objective optimization [1], [2].

Without loss of generality, an optimization problem can be described as the minimization of $m$ objectives:

$$\text{minimize} \quad F(\vec{x}) = (f_1(\vec{x}), \cdots, f_m(\vec{x}))^T, \tag{1}$$

where $\vec{x} = (x_1, \cdots, x_n) \in \Omega$ is the decision vector. $\Omega \subseteq \mathbb{R}^n$ is the decision space, and $n$ is the number of decision variables. $F : \Omega \to \mathbb{R}^m$ consists of $m$ objectives. When $m = 2$ or 3, the problem is referred to as a multi-objective problem (MOP); if $m \geq 4$, as a many-objective problem (MaOP).

Many strategies are proposed to deal with MOPs, such as the Pareto dominance-based approaches and weighted sum approaches. The Pareto dominance-based approaches include the non-dominated sorting genetic algorithm (NSGA-II) [3], the strength Pareto evolutionary algorithm (SPEA2) [4], and the niched Pareto genetic algorithm (NPGA) [5]. Examples of the weighted sum approaches are genetic algorithms with variables weights [6], dynamic weighted aggregation evolution stratgy [7], and the decomposition-based multiobjective evolutionary algorithm (MOEA/D) [8]. However, when these algorithms are extended to solve MaOPs, the Pareto-dominance cannot distinguish the relationship between the solutions in high-dimensional objective space [9], and the weighted sum approaches are ineffective in solving MOPs with a non-convex shape of the Pareto front. To address these limitations, reference-based and the preference-based strategies are proposed for MaOPs [10], [11].

Generally, algorithms introducing a set of references (including the weight vectors, reference points, or reference vectors) to assist the optimization are referred to as the reference-based strategies. One class of the methods is based on decomposition [8], [12]. They decompose MaOP into a set of subproblems via the scalar functions (weighted sum approach [13], Tchebycheff approach [13], or penalty-based boundary intersection (PBI) approach [8]). Then the subproblems are simultaneously optimized to obtain the optimal solutions constituting the representative subset of Pareto optimal front (PoF). Another class of methods adopts reference vectors or reference points to guide the optimization towards different subregions of the PoF, then simultaneously search the optimal solutions of the subregions, such as the reference vector guided evolutionary algorithm (RVEA) [14], nondominated sorting genetic algorithm III (NSGA-III) [15], MOEA/dominance and decomposition (MOEA/DD) [16], and $\theta$-dominance based evolutionary algorithm ($\theta$-DEA) [17], as well as decomposition of MOP into a set of MOPs (MOEA/D-M2M) [18].

Preference-based strategies, which can be divided into *a priori*, interactively, and *a posteriori* approaches, introduce the preference information into the optimization to guide the search towards the regions of interest (ROIs) [19], and finally present a small number of preferred solutions to the decision maker (DM) [11], [20]–[23]. In preference-based methods, there are different types of preference articulations, such as goal attainment [21], weight vectors [24]–[26], reference vectors [14], [27]–[32], preference relations [33]–[35], fuzzy preferences [36], utility functions [37]–[39], outranking [40], [41], implicit preferences (like knee points [42]–[46], extreme points or the nadir point [47], [48]).

Although the reference-based and preference-based strategies are widely used to deal with MaOPs, they both have some inherent limitations, which have been largely neglected in the literature.

The main limitations of the reference-based approach to MaOPs include the following.

1) It has been demonstrated that the performance of the aggregation-based approaches strongly depends on the shapes of the PoFs [49].
2) The performance of the reference-based methods also depends on the distribution of the predefined references [14] because the predefined subproblems may waste computational resources and fail to explore some subregions.
3) In dealing with MOPs, a manageable number of solutions is able to present a good approximation to the PoF. However, it is hardly practical when the same population size is adopted in dealing with MaOPs.
4) The performance indicators may introduce biases in specifying the reference points [50].

Main limitations of the preference-based approaches are listed below.

1) How to properly specify the preference is an issue. On the one hand, there are so many preference articulation methods, and different preference methods may lead to different results. On the other hand, the lack of a priori knowledge makes it challenging for the DM to specify his or her preferences in advance in the preference-based evolutionary optimization.
2) Although different interactive methods [29], [51], [52] are able to allow the DMs to tune their preferences in terms of the acquired solutions in different stages, the articulation of the preferences and the tuning process are challenging and sometimes intractable.
3) In an *a posteriori* process, selecting preferred solutions among a representative solution set becomes increasingly difficult as the number of objectives increases because it is resource-intensive and time-consuming to get a good representative solution set to cover the whole PoF of an MaOP.
4) How to fairly evaluate the preference-based optimization is an open issue.

This paper aims to take a closer look into the similarities and differences in reference-based and preference-based approaches. Our analysis reveal that a proper combination of both approaches will be of great help in addressing the limitations of both methods. For example, a promising approach is to find some naturally interested solutions such as the knee points or regions and then use the acquired information about the knee points or other solutions of interest to guide the search using a reference-based strategy.

The rest of the paper is organized as follows. Section II presents the background of the reference-based and preference-based evolutionary optimization. The methodologies, similarities, and differences of the two strategies are analyzed in Section III. Section IV discusses the limitations of the strategies. Promising future research topics are suggested in Section V. Finally, Section VI concludes the paper.

## II. Background

In real-life optimizations, it is impractical to provide the DM with the entire set of Pareto optimal solutions of an MOP. Ideally, a representative subset of solutions that are evenly distributed in the whole PoF can be obtained and presented to the DM. This has been the basic assumption behind most *a posteriori* approaches in evolutionary multi-objective optimization. Following this assumption, reference-based strategies predefine a set of evenly distributed references, such as reference vectors, reference points or weight vectors in the objective space. By associating solutions to the references, an MOP or MaOP can be decomposed into a set of subproblems. Then the optimal solutions of the subproblems are achieved by simultaneously optimizing the subproblems. The acquired optimal solutions guided by the references can be regarded as the representative solution set of the PoF. Reference-based approaches are usually able to achieve a set of diverse and accurate solutions, if the PoF spans over the whole Pareto front.

By contrast, preference-based strategies focus on one or several small regions of the PoF by embedding the preference information specified by the DM into the optimization. Preference-based approaches have two main advantages. First, they can provide a concentrated search towards the ROIs so as to avoid the exploration of uninterested regions. Thus, the computation resources can be reduced. Second, the obtained solution set, contains a much smaller number of solutions than the one obtained by preference-based approaches, making it easier for the DM to select preferred solutions.

## III. Methodologies

### *A. Reference-based strategies*

Reference-based strategies decompose an MOP or MaOP (1) into a set of subproblems (single-objective problems or multi-objective problems) by using a number of reference vectors, a set of reference points, or a set of weight vectors. Then the subproblems are simultaneously optimized to find the optimal solutions or sets ($S_1,\cdots,S_N$), where $S_i$ is the optimal solution (or set) of $i$th subproblem and ($S_1,\cdots,$ $S_N$) is the obtained representative solution set of the PoF. There is a hypothesis in reference-based strategies that a relatively small number (typically defined by the number of references) of subproblems is able to represent an MOP or MaOP.

There are two widely used methods for decomposition in reference-based strategies.

1) The first construction method applies a scalarization function to decompose an MOP (MaOP) into a set of single-objective problems. Given a set of evenly

distributed weight vectors $\lambda_1, \lambda_2, \cdots, \lambda_N$, where $\lambda_i = (\lambda_i^1, \lambda_i^2, \cdots, \lambda_i^m)$, $\sum_{j=1}^{m} \lambda_i^j = 1$, $i = 1, \ldots, N$, and $m$ is the number of objectives,

• **Weighted sum approach** [13]

$$\begin{aligned} &\text{minimize } \mathrm{g}^{ws}(\vec{x} \mid \lambda_i) = \sum_{j=1}^{m} \lambda_i^j f_j(\vec{x}) \\ &\text{subject to } \vec{x} \in \Omega, \end{aligned} \tag{2}$$

• **Tchebycheff approach** [13]

$$\begin{aligned} &\text{minimize } \mathrm{g}^{tche}(\vec{x} \mid \lambda_i, \vec{z^*}) = \max_{1 \le j \le m} \lambda_i^j \mid f_j(\vec{x}) - z_j^* \mid \\ &\text{subject to } \vec{x} \in \Omega, \end{aligned} \tag{3}$$

where $j \in \{1, \cdots, m\}$, $\vec{z^*} = (z_1^*, \ldots, z_m^*)^T$ is the ideal point, i.e., $z_j^* = \min\{f_j(\vec{x}) | \vec{x} \in \Omega\}$.

• **Penalty-based boundary intersection (PBI) approach** [8]

$$\begin{aligned} &\text{minimize } \mathrm{g}^{pbi}(\vec{x} \mid \lambda_i, \vec{z^*}) = d_1 + \theta d_2 \\ &\text{subject to } \vec{x} \in \Omega, \end{aligned} \tag{4}$$

where

$$\begin{aligned} d_1 &= \frac{\|(z^* - F(\vec{x}))^T \lambda_i\|}{\|\vec{\lambda}_i\|} \\ d_2 &= \|F(\vec{x}) - (z^* - d_1 \vec{\lambda}_i)\|, \end{aligned} \tag{5}$$

and $\theta > 0$ is a predefined penalty parameter. $\vec{z^*}$ is the same as in the Tchebycheff approach.

2) The second construction method decomposes an MOP or MaOP into a set of sub-MOPs by associating solutions to different reference vecors or points. To partition the problem into $K$ subproblems, given a set of uniformly distributed reference vectors (or points) $(v^1, \cdots, v^K)$, in RVEA [14], MOEA/D-M2M [18], and MOEA/DD [16], the association method is defined as follows:

$$\Phi^i = \{F(\vec{x}) \in \mathbb{R}^m | \langle F(\vec{x}), v^i \rangle \le \langle F(\vec{x}), v^j \rangle\} \tag{6}$$

where $i \neq j$, and $i \in \{1, \cdots, K\}$. $\Phi^i$ is the $i$th subregion or subproblem.

In NSGA-III [15], and $\theta$-DEA [17], the association method is defined as follows:

$$d_2 = \|F(\vec{x}) - (\vec{z^*} - d_1 \vec{v}_i)\|, \tag{7}$$

where $d_1$ is same as the $d_1$ in Eq. 5. $i \in \{1, \cdots, K\}$. $\vec{z^*} = (z_1^*, \ldots, z_m^*)^T$ is the ideal point, i.e., $z_j^* = \min\{f_j(\vec{x}) | \vec{x} \in \Omega\}$, $j \in \{1, \cdots, m\}$.

### *B. Preference-based strategies*

The construction method in different preference-based strategies varies, depending on the way in which the preferences are articulated by the DM [11], [22]. The preference information can be roughly categorized into two classes. The first class is explicit preferences such as the goal attainment, weight vectors, importance, reference vectors, preference relations, utility functions. The second class is the implicit preferences such as knee points, extreme points or the nadir point. According to different timings when the DM interacts with the optimization process, the construction methods can be classified into *a priori*, interactive, and *a posteriori* approaches [20], [21].

Here, we investigate three kinds of preference-based models. The first model adopts weight vectors. In Fig. 1 (a), the best candidate can be found along the preference direction. The second model is based on goal attainment (or reservation point). In Fig. 1 (b), the square is the goal specified by the DM. The solutions close to the goal and located in the region of interest (ROI) [19] are the preferred solutions, and the solution closest to the goal is regarded as the best individual.

Another one is the light beam search model [37], [53], [54]. In Fig. 1 (c), the preference direction is determined by the aspiration point and reservation point, which are specified by the DM. The best individual is the middle point of the obtained solutions located in the ROI, where $v_1$ and $v_2$ are the parameters to control the ROI. The light beam search model carries more information than that of other two models. The light beam search model can evaluate how close the preferred candidates to the goal point (reservation point). Meanwhile, it favors the solutions close to the aspiration point (or ideal point). Other articulations of the model are shown in [26], [55], [56].

### *C. Similarities and differences*

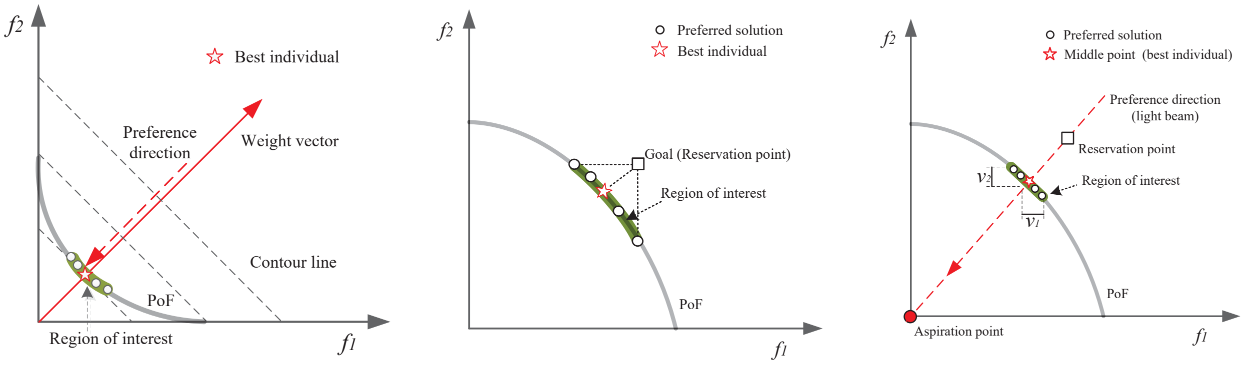
According to Section II and III, a brief summary can be made on the similarities and differences between the references-based and preferences-based strategies.

The two approaches have the following main similarities.

- They both work on solving MOPs and MaOPs. The references-based methods simplify the problem by optimizing a set of subproblems [10], [57]. The preference-based methods introduce the preference information to guide the search process to the preferred regions of the PoF [11].
- Some preference articulations are similar to references, such as the weight vectors and reference vectors (or points).
- The preference-based search can be regarded as a specific subproblem of the reference-based search. On the contrary, the reference-based search can be seen as a preference-based strategy with multiple preferences.

These two approaches also show clear differences as described below.

- Reference-based strategies aim to find a representative solution set of the entire PoF, while preference-based strategies focus on specific ROIs of the PoF.
- Reference-based methods are limited to a subset of preference articulation methods, such as weight vectors, reference points, and reference vectors. By contrast, many other preference articulation methods can be adopted in the preference-based models.
- The preference-based methods are suited for dealing with problems with different shapes of the PoFs, but




(a) (b) (c)

Fig. 1. (a) The preference model with weight vector. (b) The preference model with goal information. (c) The light beam search model.

the reference-based methods with a predefined set of references are not well suited for solving problems with complex PoFs because the performance of the reference-based methods strongly depends on the shapes of the PoF [49].

- Reference-based strategies belong to *a posteriori* approaches in multi-objective optimization. From this point of view, reference-based strategies are one specific type of preference-based strategies.

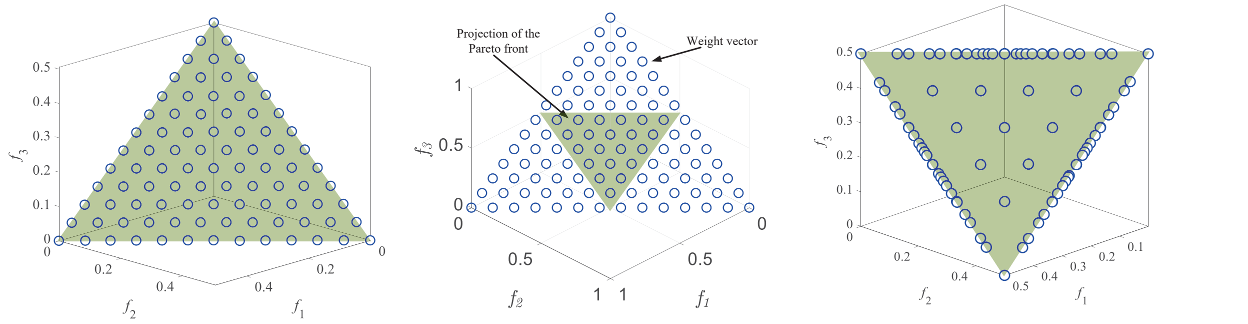
Both strategies suffer from a number of limitations, which will be discussed in the following section.

## IV. Limitations

Both reference-based and preference-based strategies are popular in multi-objective optimization, however, their limitations have largely been neglected in the community, especially when they are applied to solve MaOPs.

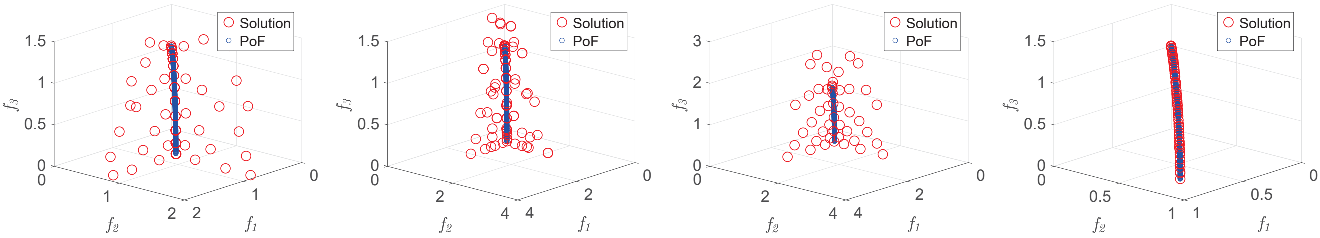
### A. References-based strategies

1) The performance of the aggregation-based approaches heavily depends on the shapes of the Pareto fronts [49]. Authors [49] investigated the performance of four reference-based algorithms, including MOEA/D [8], nondominated sorting genetic algorithm III (NSGA-III) [15], MOEA/dominance and decomposition (MOEA/DD) [16], and $\theta$-dominance based evolutionary algorithm ($\theta$-DEA) [17] on DTLZ [58] and WFG [60] test suites and their variants with rotated PoFs. The investigation demonstrates that a slight change of the problems will seriously deteriorate the performance of the algorithms. As shown in Fig. 2 (a), MOEA/D-PBI performs very well on DTLZ1 with a regular PoF (triangle) because the weight vectors perfectly matches the structure of the whole PoF. Based on the same weight vectors, MOEA/D-PBI exhibits much poorer performance on IDTLZ1 with an inverted PoF (inverted triangle), as shown in Fig. 2 (c). The deterioration of the performance can be attributed to the fact that only a small part of the weight vectors match the structure of the PoF, as shown in Fig. 2 (b) and the optimal solutions of the subproblems embedded with inconsistent weight vectors will be crowded around the boundaries of the PoF.
2) The performance of the reference-based methods also relies on the distribution of the predefined references [14]. For example, Fig. 3 presents the performance of three algorithms including MOEA/D-PBI, MOEA/DD, and RVEA on three-objective DTLZ5 [58]. The reference vectors (points) are shown in Fig. 2 (b). From Fig. 3, we can see that on the one hand, the degenerated PoF deteriorates the performance of these reference-based methods. On the other hand, a great deal of computing resources is wasted because the obtained solutions are far away from the true PoF. Meanwhile, it also reflects that some regions of the PoF are not well explored. It is easy to see that in dealing with MaOPs with irregular PoFs, the performance will deteriorate more seriously. Thus, a set of predefined evenly distributed references works well only if the PoF has a smooth, continuous and well spread geometrical structure [14].
However, in Fig. 3 (d), one variant of RVEA (RVEAa) [14] shows better performance. It utilizes the information from the subproblems (preferences information) associated with solutions to randomly generate references vectors and replaces the inactive reference vectors with which no solutions are associated. During the optimization, similar to the interactive process, an adaptation mechanism is introduced to tune the reference vectors towards the PoF.
3) In dealing with MOPs, a manageable number of solutions is able to have a good approximation to the PoF. Unfortunately, it is impractical to use a small number of references to achieve a set of representative solutions for MaOPs. Fig. 4 presents the performance of three reference-based algorithms (NSGA-III, RVEA and $\theta$-DEA) on 10-objective DTLZ2. Comparing with the PoF



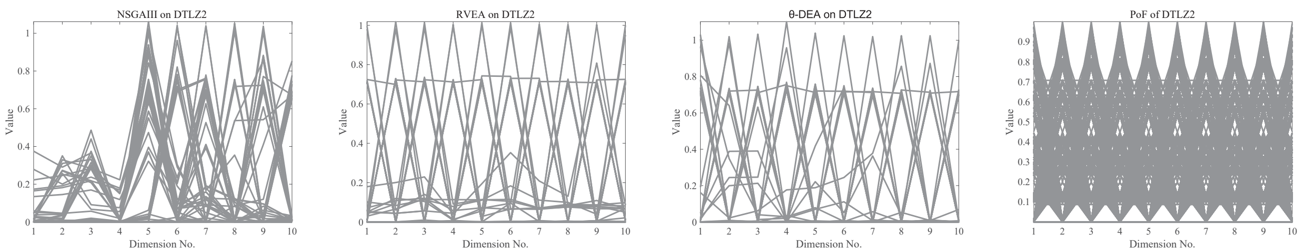

(a) (b) (C)

Fig. 2. (a) The performance of MOEA/D-PBI on DTLZ1 [58]. (b) An example of uniformly distributed weight vectors on the PoF of IDTLZ1 problem [59]. (c) The performance of MOEA/D-PBI on IDTLZ1.




(a) (b) (C) (d)

Fig. 3. (a) The performance of MOEA/D-PBI on DTLZ5. (b) The performance of MOEA/DD on DTLZ5. (c) The performance of RVEA on DTLZ5. (d) The performance of RVEAa on DTLZ5.




(a) (b) (C) (d)

Fig. 4. (a) The performance of NSGA-III on DTLZ2. (b) The performance of RVEA on DTLZ2. (c) The performance of $\theta$-DEA on DTLZ2. (d) The PoF of DTLZ2 with 10 objectives.

in Fig. 4 (d), we can see that the achieved population (91 solutions) in Fig. 4 (a), (b) and (c) is far from being sufficient for approximating the whole PoF (10 000 reference points) of the 10-objective DTLZ2. Although the acquired solutions have good convergence, there are many regions remain unexplored (the objective values around 0.1, 0.3, 0.5, 0.9 are missing). Furthermore, if the PoF is irregular and complex, it becomes more challenging to get a representative solution set of the PoF of MaOPs.

4) The performance indicators may introduce biases because the specification of the reference points can be biased. Furthermore, the biased reference points will result in an unfair performance comparison [50], like the inverted generation distance (IGD) [61]. A similar issue can occur with the generation distance (GD) [62]. The reader is referred to [62] for more details. Moreover, many existing benchmarks are designed with regular PoFs [58]. The generation of references points for the PoF is the same as the generation of the reference-based methods, which may introduce bias as well.

### B. *Preferences-based strategies*

1) How to properly specify preferences is an issue because different preferences may lead to different results. In Fig. 5, solution $A$ is the most preferred solution for the DM. The weight vector is one of most easily attainable preference from the DM but two different results (solutions $A$ and $B$) are obtained, which can be

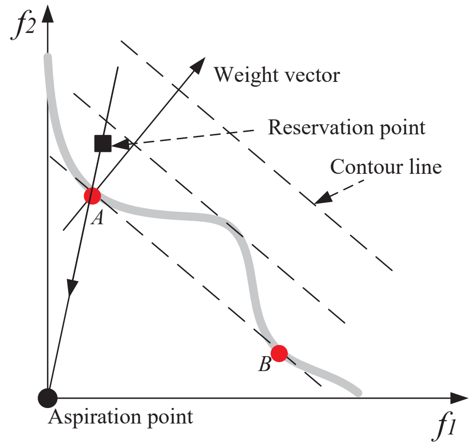



Fig. 5. An example of preference-based optimization.

referred to [63]. The light beam search model [37], [53] needs more preference information (the aspiration point and reservation point at least), but only solution $A$ can be acquired according to the model. Thus, it is challenging for the DM to select the proper method for articulation of preferences for different problems. Moreover, it is challenging for the DM to specify preferences in advance without sufficient *a priori* knowledge in preferences-based evolutionary optimization.

2) Although different interactive methods [29], [51], [52] are able to allow the DMs to tune their preferences in terms of the acquired solutions in different stages, the articulation of the preferences is not straightforward and the tuning process is challenging and sometimes intractable. First, it is unclear how frequently the DM should interact with the optimization. Second, tuning the preferences is based on an assumption that the DM is always able to give well-informed preferences in terms of the gained information, but this assumption does not always hold true. Similarly, it is possible that different articulation methods of preferences need to be determined in different search stages.
3) In the *a posteriori* process, selecting preferred solutions among a representative solution set becomes increasingly difficult as the number of objectives increases, because it is resource-intensive and time-consuming to get a good representative solution set to cover the whole PoF of MaOP. What is worse, it is difficult to define a manageable size of a representative solution set of the PoF. Taking Fig. 4 as an example, the achieved population (91 solutions) is not able to represent the PoF in a high-dimensional objective space.
4) A proper evaluation of preference-based optimization algorithms remains an open issue. Although several preference indicators [64]–[67] are proposed for preference-based optimization, there are limited to specific preference articulation methods. Note, however, that reference point based performance indicators may introduce biases.

## V. Promising Research Topics

According to above analysis, we propose the following promising future research topics.

1) More emphases should be put on solving irregular problems, and some learning or interactive strategies, such as estimating the shapes of the PoFs, modeling the regions of interest, are essential. It has been demonstrated that the fixed references are not effective to deal with irregular problems, especially on problems with complex geometry shapes of the PoFs [14], [49]. Besides, in real applications, the PoFs of the problems are always unknown. Thus, the algorithms ought to be robust to different shapes of the PoFs of irregular problems. Some self-learning or interactive strategies can be used to detect the geometry of the PoF, which many help to acquire a good representative solution set of the PoF [14], [68]–[71].
2) In many-objective optimization, a limited population size is not able to obtain a good representative solution set for MaOPs. Therefore, it is much more practical to search specific regions of interest of the PoF, which can be well represented with a small number of solutions.
3) Due to the difficulties in preference articulation to obtain solutions in some specific regions of the PoF, a more general preference is the 'natural solutions of interest', such as the knee regions (points) [44]–[46], [72]. Besides, the knees are good representative solutions of the PoF, and the found knees can be used as the reference points for both reference-based or preference-based strategies to better explore other regions where the DM may be interested in.
4) Last but not the least, it is very important to develop new performance indicators that do not introduce biases for evaluating the quality of the solutions acquired by both reference-based and preference-based strategies. Hypervolume metric can be a good option but it is computational expensive [61], [73].

## VI. Conclusion

A large body of research has been done on developing reference-based and preference-based multi-objective evolutionary algorithms. Nevertheless, little research has been dedicated to the analysis of the similarities and differences between these two approaches and limitations of both strategies have largely been neglected.

This paper analyzes the similarities and differences between the reference-based and preference-based methods for many-objective optimization, and discusses the main limitations of both methods. With these discussions, we aim to clarify some confusions and misunderstandings about these two approaches.

Based on these analyses, we suggest that preference information becomes indispensable for developing reference-based evolutionary algorithms for solving many-objective optimization problems, because such preference information can help reference-based strategies work more robustly on irregular problems. However, it is difficult for a decision maker to provide informed preferences without sufficient knowledge about the problem to be optimized. Naturally preferred solutions such as knee points can be used as preference information, and learning strategies should also be developed to learning the structure of the Pareto fronts as well as the user preferences.

Our future research will focus on embedding preference information into the reference-based strategies to more precisely find the regions of interests. Developing proper indicators for preference-based methods are also one of our future research.

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