Target shape design optimization by evolving splines

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***Abstract*— Target shape design optimization problem (TSDOP) is a miniature model for real world design optimization problems. It is proposed as a test bed to design and analyze optimization approaches for design optimization with tremendously reducing the running period of optimization process, while, the merit can be only achieved by correctly approximating the real design situation and satisfying the causality of design and evaluation.**

The Representation of the designed object is mostly described by parameterization techniques. To realize the design optimization, is to vary the parameterized object by means of operating the relevant parameters. The solution of design optimization often involved the choice of suitable description for the designed object, which can be obtained by expanding the design freedom. When changing the description length, the original parameters of the designed object will then varied. This bring about the requirements for optimization algorithms to self-adapt their strategy parameters and related variables to perform consistently searching.

We first put forwards a revised fitness evaluation mechanism for the TSDOP in order to more reasonably check the designed shape and direct optimization procedures. Based on the revised TSDOP framework, we further discuss the parameter setting problem for algorithms, especially evolution strategies, to adapt and initial their search strategy parameters. A solution method is proposed with solving a linear equations by a recursive way with linear time complexity. All discussions are limited with the B-spline parameterization framework, but may generally suit other parameterization techniques. Experiments are used to verify the causality of the revised fitness evaluation mechanism and to study the significance of the proposed method for suitable parameter settings of optimization algorithms during the adaptation of the description length for design optimization.

I. Introduction

Optimization is the process of maximizing a desired quantity or minimizing an undesired one. By term of an optimal design, we mean the best of all feasible designs. Design optimization in structural engineering is generally classified into three major categories [1] [2]: (1) size optimization, (2) shape optimization, and (3) topology optimization.

Structural optimization procedures usually start from a given design topology, which determines a fundamental frame and provide a starting point, and then vary proportions (size) or boundary shapes of the design to achieve optimality of an objective under a range of constraints, which may effectively improve structural performance.

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In recent years, a lot of studies have been conducted to utilize evolutionary algorithms for structural design optimization [2], especially in the field of aerodynamics, e.g.[3]. For the real world applications, especially in the arena of aerodynamic design optimization, evaluation of a given design solution is always at the huge expense of high computational efforts such as the evaluation with Computational Fluid Dynamics (CFD) tools. On the other hand, the models of evolutionary computation such as Evolution Strategies (ESs), Genetic algorithms (GAs) and so on are working in the way of generate and test which inevitably leads to a great amount of evaluations for all testing points. These would be a disaster for the extensive test of searching an optimal design or even an acceptable one. Therefore, miniature models for design optimization are frequently constructed which manage to resemble to original optimization problem as closely as possible while extremely alleviating the computation cost. Here the miniature model is called the *target shape*. Given the target shape, the *target shape design optimization problem* (TSDOP) is defined as minimizing the distance measured between the target shape and the designed shape. For shapes are represented by a set of coordinate points, the target shape design optimization can be regarded as minimizing the two sets of points sampled from the target shape and designed shape with respect to an appropriate distant. The TSDOP is employed as a test bed to check algorithmic performance for the general shape design. The introduction of target shape design optimization problem aims at checking how well a given optimization method can conduct the shape design searching under a range of specific geometric representation. The performances are threefold: effectiveness, the ability to search complete design space, i.e. visit all feasible solutions; flexibility, the representation suitability to sufficiently fit the optimization design; efficiency, the ability to track the appropriate directions to conduct search as fast as it can be.

The TSDOP problem was first introduced in [4] and then obtained further studies in [5] and [6]. In [4], the adaptive encoding approach was proposed by means of an growing representation for spline coded structures. The design process was started with a representation with a minimal description length; and then the number of description length was adapted by structural mutations. In this way, the merit of "minimal description length for sufficient degree of freedom for the optimization" is expected to be achieved. Meanwhile, the fitness function for the evaluation of the two curves of the designed and the target, was presented and several ESs were employed to perform the optimization task. In [5], the TSDOP was further explicit and intensive experimental results were provided for this optimization problem. Additionally, the applicability of polygon shape morphing methods as recombination operators was studied in the standard ESs for the design optimization, where the target shape design was employed as a benchmark problem.

In this paper we will present an improved evaluation scheme for the TSDOP problem in order to fit this kind of design optimization benchmarks to more reasonable expression for real world design situations. Furthermore, we will propose an particular mechanism for evolutionary optimization algorithms, in particular the ESs, to perform the adaptive searching smoothly. For convenience and without loss of generality, all discussions are within the framework of the representation with B-spline designed boundary curves and optimization by means of ESs. In the following section, we will outline the original TSDOP problem including the dynamical representation, the fitness evaluation and solution techniques. In sections III, we will analyze the way to evaluation of the two shapes and present a improved evaluation function. In section IV the variation operations will be analyzed and drawbacks will be presented. Several invariance principles will be used to derive the approach to update algorithmic parameters for adaptation to varying the design length. The comparative experiments are conducted to study the significance of the modified TSDOP problem and the proposed algorithm in section V. Finally, section VI concludes with a brief summary and comments on the results.

II. Target Shape Designed Optimization Problem

The solution of the target shape design optimization problem is to operate the designed shape to fit the given target shape. Naturally, the TSDOP involves three basic components: how to represent the designed shape, how to gauge the distance between the two shapes, and how to operate the designed shape.

A. Representation

Representing the designed shape is critical for optimization algorithms and further the realization of design optimization. An suitable representation is always required possessing the characteristics such as [3]:

- **Completeness** the representation should guarantee a maximal degree of freedom for the generated geometries and at leat can represent the optimal shape.
- **Causality** This requires that small steps on the genotype space lead to small steps in the phenotype space [7] and vice versa.
- **Compactness** an adequate representation should be realized with the minimum encoding dimensionality which correspondingly reduces the size of the search space and in turn the calculation time.

One of the suitable ways is to utilize the parameterized curve or surface to encode the designed shapes. There are several shape parametrization techniques for the shape representation and manipulating. For convince and without loss of generality, we here only focus on the problem with the two 2-dimensional boundary curves as the target and corresponding designed curve which is defined by the B-splines. Let $\{\vec{P}_i \mid i = 1, \ldots, n, \vec{P}_i \in R^s\}$ be n s-dimensional control points and also let the vector $U = (u_0, \ldots, u_{n+p})$ be the knot vector where $a \leq u_0, \leq \cdots \leq b$, a B-spline curve can be formulated as [8]:

$$\vec{C}(u) = \sum_{i=0}^{n-1} B_{i,p}(u)\vec{P}_i \qquad a \leq u \leq b, \tag{1}$$

where, the p-degree B-spline basis functions $B_{i,p}(u)$ can be formulated as the *Cox-de Boor recursion formula*:

$$B_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \leq u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$\begin{aligned} B_{i,p}(u) &= \frac{u - u_i}{u_{i+p} - u_i} B_{i,p-1}(u) \\ &+ \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} B_{i+1,p-1}(u) \end{aligned} \tag{3}$$

By using the B-spline family of curves, we have the following merits for optimization algorithms to manipulate the designed curve to fit the target:

- The B-spline curves have a local behavior, i.e., a point on the curve is only affected by several neighbored control points.
- The low-degree B-spline form can represent complex curves efficiently and most accurately.
- The sensitivity derivatives of geometry $C(u)$ with respect to P_i is the B-spline basis function $B_{i,p}$, which stay fixed when providing a fixed knot vector.

Therefore, each designed curve can be encoded by the coordinates of the control points and its knot vector. Furthermore, for evolutionary algorithms to operate the designed curves, e.g. Evolution Strategies, an individual can be represented basically by the components of the relevant curve: control points, knots; and by the strategy parameters for those control points. For the adaptive representation, which has the growing number of control points and related additional knots, the structural element is also encoded into the individual representation [4].

B. Fitness function

The way to measure the distance between the designed curve and target curve is based on a modified hausdorff distance which calculates the value of two sets of points sampled from the two curves. Formally it calculates the distance based on a kind of averaged symmetric Hausdorff distance, e.g. [4]:

$$\begin{aligned} f_{dist} &= \frac{1}{2}\Big(\sum_{i=1}^{|z_1|} min|a_i - \mathbf{b}|^2; \mathbf{b} \in z_2 \\ &\quad + \sum_{j=1}^{|z_2|} min|b_j - \mathbf{a}|^2; \mathbf{a} \in z_1\Big) \\ &= \frac{1}{2}(A + B) \end{aligned} \tag{4}$$

where z_1 is the point set sampling from the target curve and z_2 is that from designed curve. $|z_1|$ and $|z_2|$ are the cardinalities of the sets. The first term A is to collect the distance values for the points in the target curve to their nearest points in the designed curve; the second term B is that from the points in the designed curve to the nearest points in the target curve.

In practice, to avoid the self-loops in the designed curve, a penalty value for each loop is often applied to the curve's fitness value.

III. Evaluation

The TSDOP mimics the real world design optimization problem such as aerodynamic design problems where the designed process is interactive with outside evaluations such as decisions by engineers or CFD simulations. These evaluation phrases are blind to the optimization algorithms. That is, the employed optimization algorithms is dealing with an almost black-box problem and the generate-and-test process can hardly use other information except for the feedbacks of the evaluation results. Therefore, this black-box characteristics need to be incorporated in to the evaluation function of the TSDOP.

On the other hand, in the real design, a good designed shape should be assigned the corresponding good evaluation value, otherwise a failure shape should be recognized and set low performance. This is another casuality requirement for the evaluation function of the TSDOP.

While, for the previous evaluation function, i.e. the averaged symmetric Hausdorff distance in (4), the latter requirement for providing the evaluation function is less satisfied. In this averaged symmetric Hausdorff distance, there exist two cases where the evaluation approach in (4) tends to fail.

For the first case (see Fig.1), let us count the distance for the four line segments within the interval of $\overline{AB}$, since other parts are obviously independent of the four lines. Given that $h_1 < h_2 < \frac{1}{2}H < h_3$, $h_2 + h_3 = H$ and the numbers of sampling points for the four segment $D1$, $D2$ in the designed curve and $T1$, $T2$ in the target curve are $|D1|$, $|D2|$, $|T1|$, and $|T2|$ respectively. From the (4), we have that,

$$\begin{aligned} A &= \sum_{i=1}^{|T_1|} h_1 + \sum_{j=1}^{|T_2|} h_3 \\ &= |T_1| \cdot h_1 + |T_2| \cdot h_3 \end{aligned} \tag{5}$$

$$\begin{aligned} B &= \sum_{i=1}^{|D_1|} h_1 + \sum_{j=1}^{|D_2|} h_3 \\ &= |D_1| \cdot h_1 + |D_2| \cdot h_2. \end{aligned} \tag{6}$$

For the value h_1 could be too tiny to be ignored, the value f_{dist} in (4) has

$$f_{dist}(\overline{AB}) = \frac{1}{2}(A+B) = \frac{1}{2}(|T_2| \cdot h_3 + |D_2| \cdot h_2)$$

When the difference between $|T_2|$ and $|D_2|$ is not significant, which is common to have sampling points at regular intervals, the value of f_{dist} is determined by h_2 and h_3. While $h_2 + h_3 = H$ is satisfied, when moving the line of $D2$ in the direction close to T_2 and without overpass of the central line, i.e. $h_2 \le \frac{1}{2}H$, the f_{dist} is almost unchanged. In all, for the obvious bad designed curve, when an optimization algorithm try to improve it such as by the mutation operation, the feedback information is nearly unchanged, since the amount first term reduces is equal to that the second term adds. So although the designed shape improved, but the evaluation value fails to reflect it. This cause the intractability for the mutation-based algorithms to solve this TSDOP. While it is important to note that this is not the fact in the real world situation, the only mistake occurs in the fitness evaluation function.

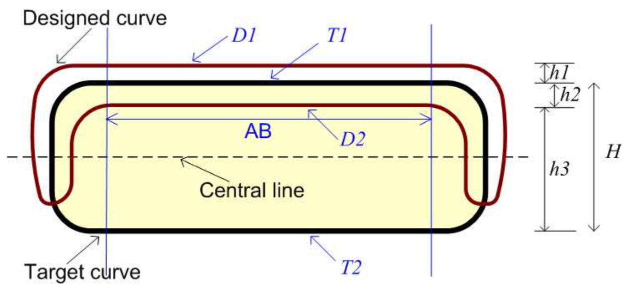


Fig. 1. Case I for evaluation

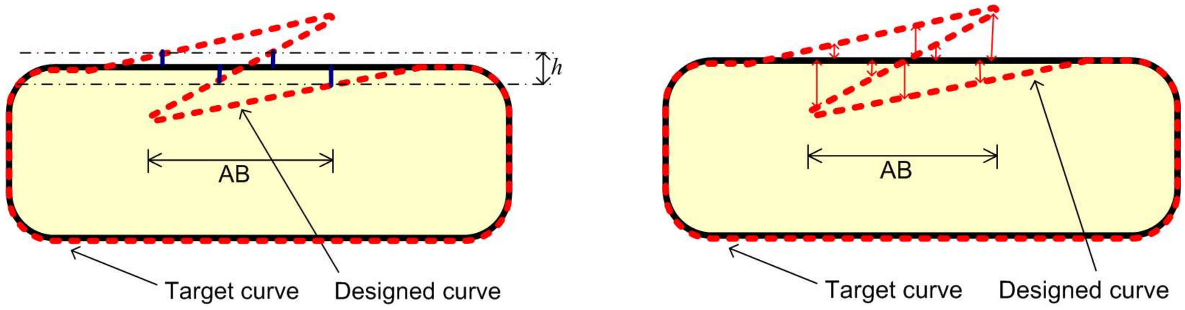
For the second case, we provide a bizarre shape but this is not uncommon for fitting the target curve by means of adaptation representation, in which control points and knots are gradually growing with the design requirement. Given the two curves in the Fig. 2, we focus on the two segments in the interval of $\overline{AB}$, where the segment of the target curve has 4 sample points and the segment of the designed curve has 8 points. The 2(a) in Fig. 2 provides the distances $\{\bar{h}_i \le h \mid i = 1,2,3,4\}$ for each sample point in the target curve and the 2(b) presents those in the designed curve, i.e. $\{h_1, h_2, \ldots, h_8\}$. Note that, although we sampled the two curves with the same number of points and in equal distances, the zigzag segment extended the designed curve and as a result there exists more sample points in the $\overline{AB}$ interval.

From the averaged symmetric Hausdorff distance (4), we have that,

$$A = \sum_{i=1}^{4} \bar{h}_i \tag{7}$$

$$B = \sum_{j=1}^{8} h_j \tag{8}$$

where since the A value is much smaller than the B value, the value of f_{dist} is dominated by the B value. This is reasonable to denote that the shape has low performance, but due the existence of the term B, the significance for designating the unexpected shape will be cutting down by means of the averaged value. This result will directly cause the algorithm to assent this kind of shape just because it is "not too bad". The fact is that this kind of zigzag shapes has relatively less effects on the shapes' performance evaluations for the way to



(a) Term A in $f_{dist}(\overline{AB})$

(b) Term B in $f_{dist}(\overline{AB})$

Fig. 2. Case II for evaluation

evaluation is by comparison of distance between the sample points and the average values will then further lessen the feedbacks of poor shapes. Therefore, for the second case, the fitness function also shows weak to reflect the causality in the real design,

Instead, we here provide a modification for the TSDOP that choose the worse evaluation value between the term A and B as the fitness evaluation rather than the averaged one. It is formally presented as below:

$$
\begin{aligned}
f_{dist} &= \max(\sum_{i=1}^{|z_1|} min|a_i - \mathbf{b}|^2; \mathbf{b} \in z_2, \\
&\quad \sum_{j=1}^{|z_2|} min|b_j - \mathbf{a}|^2; \mathbf{a} \in z_1) \\
&= \max(A, B)
\end{aligned} \tag{9}
$$

This modified fitness evaluation not merely holds the basic causality that well fitted designed curve receives good evaluation value, but also can improve the evaluation mechanism for both of the two preceding cases. For the first one, the fitness for the four segment is determined by the term A without any disturbance of the term B, which will then direct the designed moving to the right direction with respect to a reasonable optimization method. On the other hand, for the second case, the fitness for the zigzag part will then decided by the term derived form the designed curve to the referenced target and the other disturbing term will be ignored.

IV. Variation and Strategy Parameters

Evolution strategies have been widely applied into the field of the design optimization. Also this class of algorithms have been studied (e.g. [4] [5] [6]) in the solution of the TSDOP by controlling the parameterized curves, such as B-splines and NURBS. The default way, for ESs to solve the TSDOP, adopts the following form:

- An individual consists of two parts: strategy parameters and control points, in which each dimensional component of a control point takes a position in the chromosome, e.g. for 2-dimensional curve with n control points we have the individual representation as $(c_{x_1}, c_{y_1}, c_{x_2}, c_{y_2}, \ldots, c_{x_n}, c_{y_n})$.
- The strategy parameters determine the strength of the variations for the relevant components. It can be all components share a common strategy parameter for the isomorphic mutation; each component has its own strategy parameter for the independent mutation or even more strategy parameters for the correlated mutation.
- The variation of the designed curve is realized by means of moving the control points. Together with the bases of the relevant splines, the exact variation step for each point in the designed curve can be calculated.

Obviously, the design optimization process is conducted with operating the control points. Thus the strategy parameter is the critical for its corresponding control point to perform the variation operation. While much attention has been payed to the adaptation of the strategy parameters, a side effect on the adaptation of strategy parameters, brought out by the consecutive variation mechanism, has been neglected.

Given a designed B-spline curve $C(u)$ defined as (1) with n control points $\vec{P_i}$ and the basis functions $B_{i,p}(u)$. For convenience and without loss of generality, let each control point has a strategy parameter vector $\vec{\sigma}_i$. Therefore, the representation of an individual is the set of pairs $(\vec{P}_i, \vec{\sigma}_i)$ where usually $\vec{\sigma}_i$ are standard deviations of the normal distributions. The adaptation process of $\vec{\sigma}_i$ is shown in Fig. 3

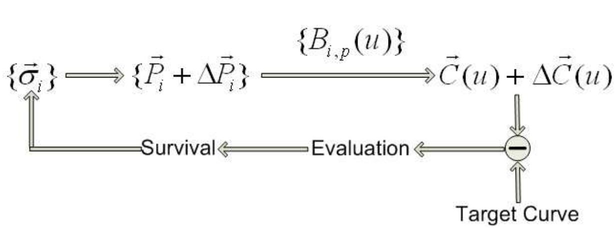


Fig. 3. Adaptation for strategy parameters

On one hand, the adaptation of $\vec{\sigma}_i$ is to adjust its value to vary the designed curve $\Delta\vec{C}(u)$ to realize gradual improvement for the design task; on the other hand, the operation of $\vec{\sigma}_i$ on the designed curve is indirect, but only to trail by means of affecting its related control point $\vec{P}_i$ and then combined with the basis functions $B_{i,p}(u)$ to determine the amount to change the designed curve. As a result, the goal of adaptation of strategy parameters lies in the current state of the designed curve, rather than the coordinates of control points.

Generally, ESs are working in the following manner for design optimization:

$$\vec{\sigma}_i \text{ adaptation} \tag{10}$$

$$\vec{P}'_i = \vec{P}_i + \vec{\sigma}_i \cdot \mathcal{N}(0,1), \tag{11}$$

$$\begin{aligned} \vec{C}'(u) &= \sum_{i=0}^{n-1} B_{i,p}(u)(\vec{P}'_i) \\ &= \sum_{i=0}^{n-1} B_{i,p}(u)(\vec{P}_i) \\ &+ \sum_{i=0}^{n-1} B_{i,p}(u)(\vec{\sigma}_i \cdot \mathcal{N}(0,1)) \end{aligned} \tag{12}$$

where $\mathcal{N}(0,1)$ is a normally distributed random number.

We always expect that with a sophisticated adaptation mechanism for strategy parameters $\vec{\sigma}_i$, the designed curve $\vec{C}(u)$ can gradually move close to the target curve by means of the variation of the control points $\vec{P}_i$. This may be achieved when the basis functions $\{B_{i,p}(u) \mid i = 0, 1, \ldots, (n-1)\}$ are static, i.e. the knot vector $U = (u_0, u_1, \ldots, u_{n+p})$ is fixed and the relationship between these control points and the designed curve is linear, since for given u, $B_{i,p}(u)$ are constants with respect to i. Therefore, the goal of following the variation of the designed curve is almost equivalent to moving the control points expect for the combination of variations of the control points. Unfortunately, when the bases are constantly changing, which occurs in the context of the adaptive encoding (see [4]), this relationship shift to an undecidable mapping. Specifically when inserting a control point, the knot vector will then change with adding a knot or be completely rebuilt and the basis functions will accordingly change. Such a variation of bases serves as nonlinear disturbance for the whole adaptation of strategy parameters.

In practice, for expanding the description length of the designed curve we usually utilize the *knot insertion* method rather than completely rebuild knot vectors (for details, see [8]). By means of the knot insertion method, we can adding a new knot into the knot vector without changing the curve's shape. While a knot is inserted, the basis functions are then locally distorted, e.g., that shown in Fig. 4; and the neighbored control points have to been rearranged in order to keep the original shape of the curve. This brings about two matters for ESs to conduct consecutive searching:

- related control points are rearranged but the corresponding strategy parameters remain previous values;
- the settings of strategy parameters for the newly adding control point need to be determined reasonably.

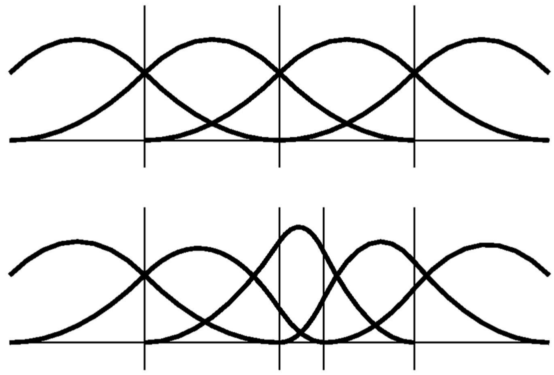


Fig. 4. Illustration of basis functions with inserting a knot

Both of the problems are related to the settings of the strategy parameters when extending the description length for design optimization. The principle for suitable settings is that: while holding the curve unchanged with the knot insertion, we need a further invariance that the strength of random variation keeps for each point in the curve.

Given the original curve $\vec{C}(u)$ and the modified curve $\vec{C}^I(u)$ obtained after the knot insertion operation. By the standard knot insertion method, we have the *basic invariance*, i.e., $\vec{C}(u) = \vec{C}^I(u)$. Further, let $\vec{C}'(u)$ and $\vec{C}^{I\prime}(u)$ be the one step modification obtained from mutations of $\vec{C}(u)$ and $\vec{C}^I(u)$, both of which are performed by their respective strategy parameters. The further principle for the invariance of strength of random variation requires that $E[\vec{C}'(u)] = E[\vec{C}^{I\prime}(u)]$ and $Var[\vec{C}'(u)] = Var[\vec{C}^{I\prime}(u)]$ are satisfied, where $E[\cdot]$ is the expectation of $[\cdot]$ and $Var[\cdot]$ is the variance of $[\cdot]$. Moreover, the first term is referred to as the *first-order invariance* and the second term is denoted as the *second-order invariance*.

According to the scheme of (10), let the variation of an arbitrary curve be,

$$\vec{C}^*(u) = \sum_{i=0}^{n-1} B^*_{i,p}(u)(\vec{P}^*_i) + \sum_{i=0}^{n-1} B^*_{i,p}(u)\vec{\sigma}^*_i \cdot \mathcal{N}(0,1)) \tag{13}$$

and the expectation and variance be

$$\sum_{i=0}^{n-1} B^*_{i,p}(u)(\vec{P}^*_i) \tag{14}$$

and

$$\sum_{i=0}^{n-1} (B^*_{i,p}(u))^2 (\vec{\sigma}^*_i)^2. \tag{15}$$

Then given the original curve $\vec{C}(u)$ and the modified curve $\vec{C}^{I\prime}(u)$ with a knot insertion, we have the first-order and second-order invariance for this modification, that is, $E[\vec{C}'(u)] = E[\vec{C}^{I\prime}(u)]$ and $Var[\vec{C}'(u)] = Var[\vec{C}^{I\prime}(u)]$.

Therefore, The following conditions,

$$\sum_{i=0}^{n-1} B_{i,p}(u)(\vec{P}_i) = \sum_{i=0}^{n-1} B'_{i,p}(u)(\vec{P}'_i) \tag{16}$$

$$\sum_{i=0}^{n-1}(B_{i,p}(u))^2(\vec{\sigma}_i)^2 = \sum_{i=0}^{n-1}(B'_{i,p}(u))^2(\vec{\sigma}'_i)^2 \tag{17}$$

are obtained. Note that the first-order condition (16) is equivalent to the basic invariance for knot insertion and the second-order condition provide a channel to reasonable settings of strategy parameters for ESs to conduct further optimization.

Given an original curve of a degree p consisting of a control point vector $\{\vec{P}_0, \vec{P}_1, \ldots, \vec{P}_{n-1}\}$ and a knot vector $\{u_0, u_1, \ldots, u_{n+p}\}$. Suppose the knot t to be inserted lies in the span $[u_k, u_{k+1})$, according to the convex hull property, the knot insertion computation is restricted on control points $\vec{P}_k, \vec{P}_{k-1}, \ldots, \vec{P}_{k-p}$. The way to modify the control points is illustrated in Fig. 5 and formally stated as below:

$$\vec{q}_i = (1-\alpha_i)\vec{P}_{i-1} + \alpha_i\vec{P}_i \tag{18}$$

where

$$\alpha_i = \frac{t-u_i}{u_{i+p}-u_i} \qquad \text{for} \quad k-p+1 \le i \le k.$$

With (18), the whole control point vector can be reconstructed.

Furthermore, to rebuild the individual representation for ESs, we also need to finding the values for the strategy parameters of the related control points. Due to the local behavior in a B-spline curve, calculation can be restricted on part basis functions rather than that for the whole curve. The strategy parameters are determined by the second-order invariance and calculated as follows. Let choose $2p+1$ knots as $\{u_{k-2p+1}, u_{k-2p+2}, \ldots, u_k, t\}$, where t is the new knot and p is the degree of relevant. According to these parameters, $2p+1$ points can be calculated for each of both original and modified curves. From the second-order invariance presented in (17), we have the linear equations in (19) with respect to the given $2p+1$ knots. Note that the matrix $\mathbf{B}$ is a lower triangular matrix, therefore, the settings σ for strategy parameters can be recursively computed.

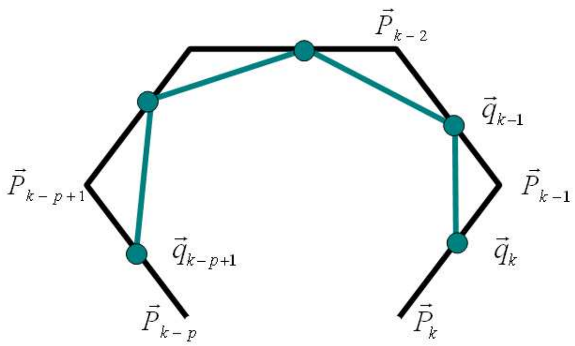


Fig. 5. Illustration of knot insertion

V. Experimental Results

A. Setup of the experiments

Experiments have been conduct to verify the causality of the revised fitness evaluation function and to demonstrate the effectiveness for individuals to update their parameters including the reconstructed control points and their strategy parameters during the process of variation of the description, i.e. the sequence of knot insertion operations.

All experiments used a common 2-dimensional blade-like target curve, shown in Fig. 7 and comparisons were carried out with 200 samples from both of the target curve and designed curve. The designed curve is a B-spline curve

$$\begin{bmatrix} \sum_{i=0}^{n-1}(B_{i,p}(u_{k-2p+1}))^2(\vec{\sigma}_i)^2 \\ \sum_{i=0}^{n-1}(B_{i,p}(u_{k-2p+2}))^2(\vec{\sigma}_i)^2 \\ \vdots \\ \sum_{i=0}^{n-1}(B_{i,p}(u_k))^2(\vec{\sigma}_i)^2 \\ \sum_{i=0}^{n-1}(B_{i,p}(u_t))^2(\vec{\sigma}_i)^2 \end{bmatrix} = \sigma \cdot \mathbf{B} = \begin{bmatrix} (\vec{\sigma}'_{k-2p+1})^2 & (\vec{\sigma}'_{k-2p+2})^2 & \ldots & (\vec{\sigma}'_k)^2 & \vec{\sigma}'_{k+1})^2 \end{bmatrix} \cdot$$

$$\begin{bmatrix} B'_{k-2p+1,p}(u_{k-2p+1}) & & & & & \\ \vdots & B'_{k-2p+2,p}(u_{k-2p+2}) & & & & \\ B'_{k-p+1,p}(u_{k-2p+1}) & \vdots & & & & \\ & B'_{k-p+2,p}(u_{k-2p+2}) & & & & \\ & & \ddots & & & \\ & & \vdots & B'_{k-p,p}(u_{k-p}) & & \\ & & \ddots & \vdots & \ddots & \\ & & & B'_{k,p}(u_{k-p}) & \ldots & B'_{k,p}(u_k) & \\ & & & & & B'_{k+1,p}(u_k) & B'_{k+1,p}(t) \end{bmatrix} \tag{19}$$

with degree 2 and initialized by sampling the three sampled control points along a given circle with equal knot span. Based on the sampled control points, an initial designed curve can be obtained (see 7). The whole testing curves have been randomly generated by this approach and this common testing curves were used in all experiments.

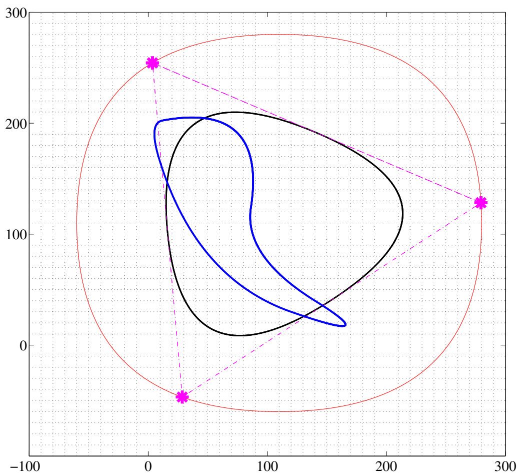


Fig. 6. Illustration of testing set

Two optimization algorithms were involved of optimizing the target curve: the individual mutative evolution strategy (iES) [9] and the covariance matrix adaptation evolution strategy (CMA) [10]. The framework proposed in [4], for adaptive encoding the designed curve which gradually expanding the control points, was utilized to conduct structural mutation.

For each structural mutation, the strategy parameters for each sequence of modified controls are obtained according to our proposed methods. More specific, for the individual mutative ES, the strategy parameters (step sizes) related to the modified control points are calculated by the equations in (19) and for the CMA, the corresponding diagnose elements in the covariance matrix of strategy parameter are setting with the values derived from the equations in (19), while, the elements of covariance are setting default 0 and the historical step size information are ignored.

B. Results

In this part, all Results were obtained by 30 independent runs. the individual mutative evolution strategy was incorporated into the adaptive encoding framework, where the interval for try structural mutation was set one and strategy parameter update for structural mutation was obtained from the equations (19). Fig. 7 depicts two kinds of relations: the first relation depicts the running results derived from proposed modified evaluation mechanism and the other compares two running results derived from the original evaluation way and the modified mechanism.

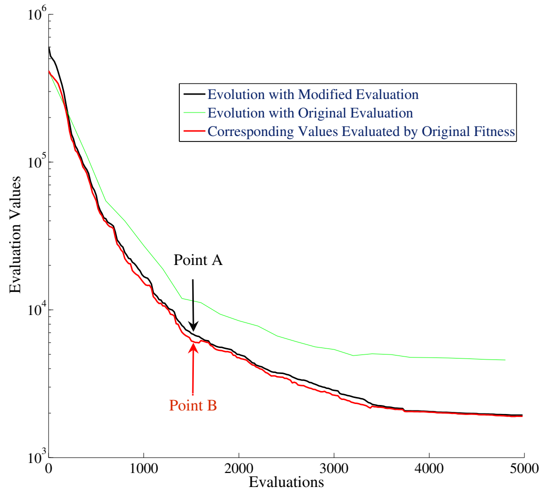


Fig. 7. Illustration of testing set

In the first relation, the mean best individual during the evolution process was presented by giving its modified evaluation and its corresponding original fitness. In Fig. 7 the point A in the modified evaluation curve and the point B in the original fitness curve reflect the common individual, while, the point A shows inferior performance. Although this individual seems to represent a good designed curve in view of the original fitness evaluation, but from the modified fitness evaluation, it is not; and even the descendent individuals had a bit worse performance evaluated by the original way, while it did still make progress in the optimization process evaluated by the modified evaluation mechanism. This conflict relation shows a clue for the foregoing discussion on the two exemplified cases for the original evaluation drawbacks that there exists some circumstances for the original evaluation to fail to reflect the real states of the designed curves.

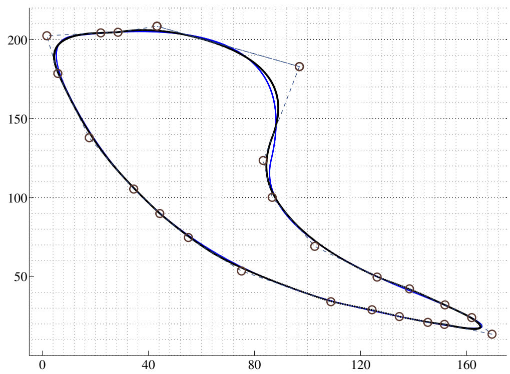


Fig. 8. Illustration of a designed curve

The second curve relation provides a comparison for the effectiveness of the two evaluation mechanism. The common algorithm, which individual mutative ES with adaptive

TABLE I

PERFORMANCE COMPARISON OF IES WITH g GENERATIONS FOR EACH STRUCTURAL MUTATION. M MEANS TO UPDATE STRATEGY PARAMETERS BY THE PROPOSED METHOD

Method	$g = 1$	$g =$ 1-M	$g = 10$	$g =$ 10-M
Min	415.8	202.8	409.2	247.6
Mean	1864.3	812.9	1323.1	659.0
STD	2342.4	590.1	1524.3	552.9

TABLE II

PERFORMANCE COMPARISON OF CMA-ES

Method	$g = 1$	$g =$ 1-M	$g = 10$	$g =$ 10-M
Min	1461.9	390.1	486.3	214.5
Mean	7014.1	969.0	6043.8	739.2
STD	7932.5	1021.7	7161.9	721.0

representation and the proposed strategy parameter settings, was employed and two evaluation mechanisms were used. Both of the curves depict in Fig. 7: the curve with modified evaluation and that with original evaluation. The causality, that good curve shows good evaluation, and effectiveness for the modified evaluation can be obtained by means of parallel comparison with the curve with original evaluation, in Fig. 7 and the result od a designed curve in Fig. 8.

Further experiments have been done to check the correctness and effectiveness for the proposed strategy parameter setting approach. The optimization framework of adaptive encoding, in which every g generations a structural mutation was tried (see [4]), were selected. The individual mutative ES (iES) and the CMA-ES were employed as optimization algorithms. The statistical results about minimum value, mean value and standard deviation are presented in TABLE I for individual mutative ES and the in TABLE I for CMA-ES, in which the interval for structural mutation was 1 and 10. The notion of $g =$ 1-M and $g =$ 10-M denote those using the proposed strategy parameter setting method and others without M are those using the default initial strategy parameter values for new control points.

The results showed that the structural mutation badly disturbed the optimization process. When ignoring the suitable settings to update the strategy parameters, the standard deviations were complete large and some of the designed results were completely unacceptable, which can be seen from the STDs of $g = 1$ and $g = 10$ in TABLE I and TABLE II. Although when extending the length of the generation interval g, the standard deviations reduced, it contrarily gave an example that reducing the frequency of structural mutation, the disturbance to the optimization process will then get weak. In comparison, the proposed method for setting strategy parameters have made significant improvement for evolution strategies to steadily and effectively obtain the good solutions. The standard deviations were reduced greatly and the solution quality have been improved. These results demonstrate that solution performance for designed curve can been steadily improved by our proposed method and the side effect on algorithmic parameters from structural mutation have been effectively controlled and further the good design can be relatively steadily achieved.

VI. CONCLUSIONS

The target shape design optimization problem have been further discussed and an improved fitness evaluation function has been proposed to form a reasonable test bed to facilitate optimization algorithms conducting design optimization. The causality of the modified evaluation mechanism for the designed object has also been illustrated with the experiment.

Based on the revised framework, we further discussed the parameter setting problem which is derived in the context of the variable length of description for representing a parameterized object. With the variation of description length, the parameters have to be reconstructed and so is the parameters of optimization algorithms. We proposed a reconstruction method for the parameters of algorithms, based on the proposed first-order invariance and second-order invariance. These requirements of invariance provides a basis for establish the relationship between the two consecutive solving steps. Taking the evolution strategies as an example, the general equations were proposed to directly obtain the needed values of strategy parameters. In the experiments, the corresponding statistics were provided. The proposed parameter setting method provided a reliable approach to tune the adaptation setting and realize steady solution.

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