

# **Sparse codes of V1 simple-cells and the emergence of globular receptive fields - a comparative study**

**Jörg Bornschein, Marc Henniges, Gervasio Puertas,  
Jörg Lücke**

**2011**

**Preprint:**

This is an accepted article published in Cosyne 2011. The final authenticated version is available online at: [https://doi.org/\[DOI not available\]](https://doi.org/[DOI not available])



## Summary

### Abstract

- multiple cause models with sparse priors
- linear or non-linear superposition of basis functions
- maximization of the data likelihood on image patches
- likelihood maximization using a novel form of variational EM (ET)
- same parameter set and training method for both models
- comparative analysis of the obtained basis functions

### Results

- Gabor-like basis functions are obtained in both cases
- more elongated basis functions when using the non-linear model
- higher fraction of globular basis functions for the non-linear model

## Linear vs. non-linear component extraction

$$p(\vec{s} | \theta) = \prod_h \pi^{s_h} (1 - \pi)^{1-s_h} \quad (\text{Bernoulli prior})$$

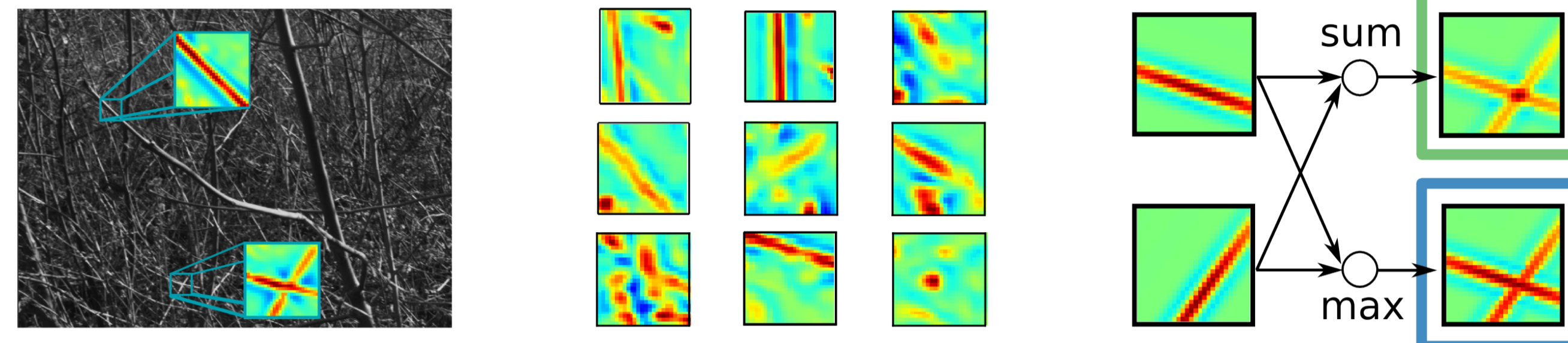
$$p(\vec{y} | \vec{s}, \theta) = \mathcal{N}(\vec{y}; \sum_h s_h \vec{W}_h, \sigma^2) \quad (\text{BSC; linear superposition})$$

$$p(\vec{y} | \vec{s}, \theta) = \mathcal{N}(\vec{y}; \max_h \{s_h \vec{W}_h\}, \sigma^2) \quad (\text{MCA; non-linear superposition})$$

$\vec{y} \in \mathbb{R}^D$  observed variables       $\pi$  prior parameter  
 $\vec{s} \in \{0, 1\}^H$  hidden variables       $\sigma$  observation noise level  
 $W \in \mathbb{R}^{D \times H}$  basis functions

We study two generative models: Binary Sparse Coding (BSC; [1]) and Maximal Causes Analysis (MCA; [2, 3]). As in standard approaches such as Sparse Coding [4] or Independent Component Analysis, both BSC and MCA assume a sparse prior with independent hidden variables. In the place where standard approaches and BSC use the sum to combine basis functions, MCA uses a (pixel-wise) maximum operation. To derive tractable approximations for parameter estimation we, for both models, apply Expectation Truncation (ET; [5]) - a variational EM approach. The resulting learning algorithms are applicable to large-scale problems with hundreds of observed and hidden variables. Furthermore, ET allows one to infer all model parameters including observation noise,  $\sigma$ , and the degree of sparseness,  $\pi$ .

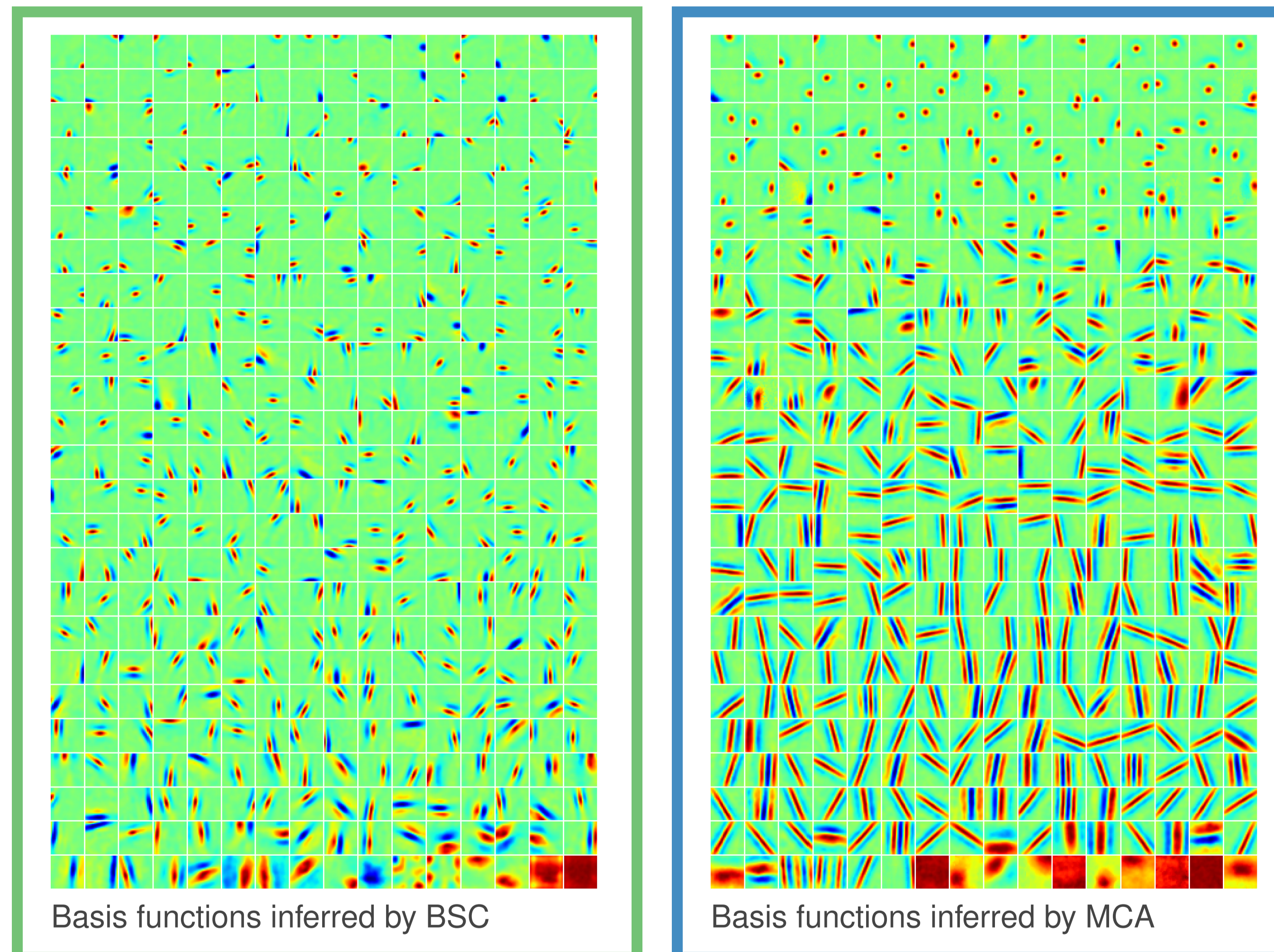
## Application to natural image patches



The strong non-linearity of the MCA generative model may represent a more plausible assumption for the superposition of components in preprocessed image patches.

To study the implications of the linear vs. non-linear superposition for visual data, both algorithms were applied to  $N = 200\,000$  image patches extracted from the van Hateren image database ( $26 \times 26$  pixels; preprocessed using a DoG filter and channel splitting to ensure non-negativity). Parameters of both models were inferred for the same set of patches using the same training scheme with the same parameter initialization.

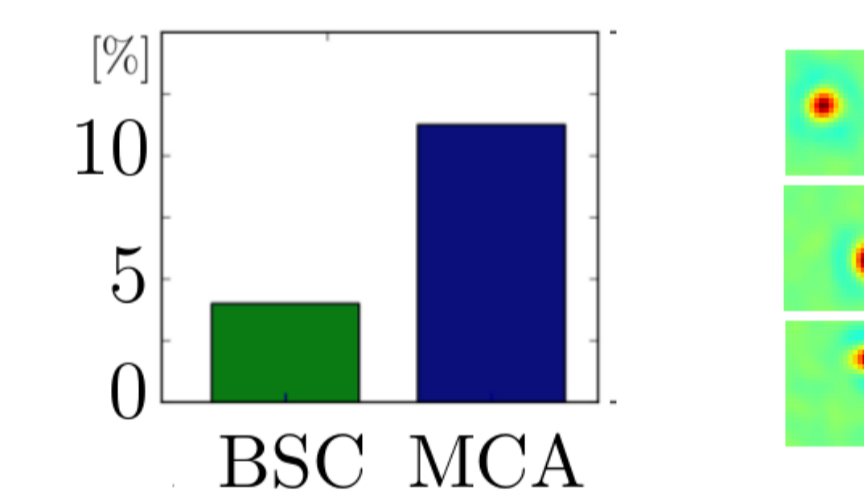
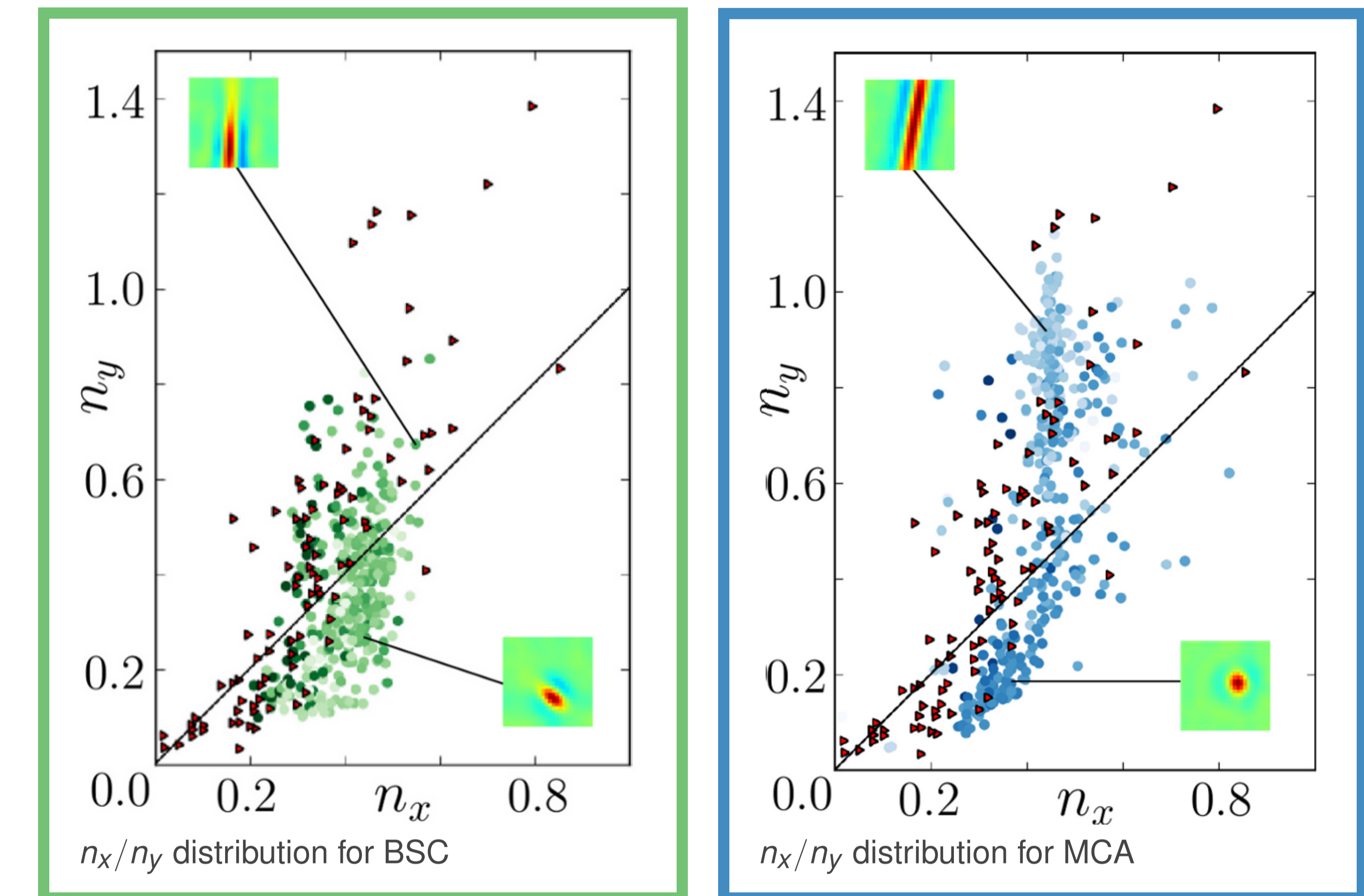
Inferred basis functions (H=400):



## Analysis of obtained basis functions

To analyze the receptive fields associated with the inferred basis functions, we convoluted (reverse-correlated) the basis functions and matched them with Gabor wavelets and with difference of gaussian kernels.

Shape of the gaussian envelope; shown simultaneously with data measured *in vivo* [6] (red triangles).



Fraction of globular fields; fields that are better matched by DoG kernels than by Gabor wavelet functions. The receptive fields extracted by MCA have a significantly higher fraction of globular shaped fields.

## Conclusions

- in both models Gabor-like basis functions are inferred
- linear and non-linear models result in very different RF distributions
- MCA infers a much higher fraction of globular RFs
- continuous linear models can represent globular structures by superimposing gabors

## References

- [1] M. Henniges, G. Puertas, J. Bornschein, J. Eggert, J. Lücke. Binary Sparse Coding. *LVA/ICA* 6365:450-457, 2010.
- [2] J. Lücke, M. Sahani. Maximal causes for non-linear component extraction. *JMLR* 9:1227-1267, 2008.
- [3] G. Puertas, J. Bornschein, J. Lücke. The Maximal Causes of Natural Scenes are Edge Filters. *Proc. NIPS* 23 1939-1947, 2010.
- [4] B. A. Olshausen, D. J. Field. Emergence of simple-cell receptive field properties by learning a sparse code for natural images. *Nature* 381:607 - 609, 1996.
- [5] J. Lücke, J. Eggert. Expectation Truncation and the Benefits of Preselection in Training Generative Models. *JMLR* 11:2855-2900, 2010.
- [6] D. L. Ringach. Spatial structure and symmetry of simple-cell receptive fields in macaque primary visual cortex. *Journal of Neurophysiology* 88:455 - 463, 2002. Data retrieved 2006.

This project was supported by the German Federal Ministry of Education and Research (BMBF) within the "Bernstein Focus: Neurotechnology Frankfurt" through research grant 01GQ0840, by the German Research Foundation (DFG) in the project LU 1196/4-1 and by the Honda Research Institute Europe.