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Optimisation of a Stator Blade Used in a Transonic Compressor Cascade with Evolution Strategies

Markus Olhofer[1] Toshiyuki Arima[2]
Toyotaka Sonoda[2] Bernhard Sendhoff[1]

[1]Honda R&D Europe (Germany)
Future Technology Research
D-63073 Offenbach/Main
Germany

[2]Honda R&D Co. Ltd.
Wako Research Centre
Wako-shi, Saitama, 351-0193
Japan

Abstract. The evaluation of fluid dynamic properties of various different structures in aerodynamic design optimisation is a computationally demanding process. For the application of evolutionary algorithms it would therefore be beneficial to restrict the population size to a minimum even if parallel genetic algorithms are employed. In this paper, we will show that specific evolution strategies can be successfully used for design optimisation even in the transonic regime with small population sizes and a full Navier Stokes solver for the evaluation. Furthermore, we analyse the self-adaptation properties of the evolution strategy.

1 Introduction

Evolutionary algorithms have recently been applied to the area of design optimisation, in particular to the air foil and wing design for aerodynamics [1–3] for a number of reasons. Firstly, gradient information, which would have to be estimated numerically, is not necessary during the optimisation and secondly, the stochastic component of evolutionary algorithms (EA) makes it possible to escape local minima. Furthermore, the population based approach inherent in EAs is very suitable for multi-objective optimisation [3,4]. On the other hand, the population size can pose a serious problem with regard to the computation time needed for one generation. In most applications parallel genetic algorithms are used, however, population sizes of the order 100 or more still seem infeasible for applications where a numerical solution of the Navier Stokes equation is needed. Indeed many applications reported in the literature use the Euler equation for computational fluid dynamics (CFD) calculations. However, as we will see in Section 2.1, the simplifications in the Euler method cannot be carried out for turbulent transonic flow, where viscosity and heat conduction have a large influence on the energy loss. Therefore, it would be beneficial if large populations could be avoided even if parallel algorithms are employed.

In this paper we will apply a specific version of the evolution strategy (ES), which extends the self-adaptation property of the standard ES by using a derandomised, cumulative step-size adaptation method. One of the main advantages of this approach is the possible reduction of the population size. In addition, for reasonably smooth fitness landscapes it was shown [5] that the ES outperforms the GA. According to our own experiments, the design optimisation task described in this paper can be regarded as reasonably smooth. In the following section, we will outline the encoding of the stator blade and describe the CFD simulation. In Section 3, we will briefly introduce the ES with particular attention paid to the lesser known extensions which allow the reduction of the population size. The results will be described in Section 4 together with an analysis of the self-adaptation of the strategy parameters during the optimisation. In Section 5 we will discuss our findings.

2 Encoding of the airfoil and CFD simulation

The encoding of the air foil geometry is one of the most critical steps in the optimisation since, together with the evolutionary operators it determines the modifications of the structure during the optimisation process and therefore strongly influences the evolution path. There are numerous constraints which should be fulfilled by the encoding, e.g.:

Completeness The encoding has to guarantee a maximal degree of freedom for the generated geometries, otherwise the underlying model introduces unnecessary constraints on the phenotype space. In the worst case it might not be possible to encode the optimal structures.

Causality Causality implies that small steps on the genotype space lead to small steps in the phenotype space [6]. This attribute is especially necessary for the adaptation of the strategy parameters in evolution strategies.

Compactness It is necessary to minimise the dimensionality of the encoding in order to reduce the dimension of the search space and in turn the calculation time.

Our initial encoding used an intuitive model, where each parameter had a specific aerodynamic meaning. It was based on engineering knowledge for the description of the stator blade and was designed to use heuristics for the optimisation. As such its applicability was confined to conventional designs. For a more complete and less biased description of the search space it was too restrictive.

When we devised an alternative model, it was our aim to comply with the above described conditions to a greater extent. The model used fulfils the demand for strong causality and is a trade-off between a high degree of freedom for the structure generation and a low dimensionality of the genotype space: The upper and lower side of the air foil are each described by five points fixing two 4^{th} order polynomials. To reduce the dimensionality of the problem the x-coordinates of the points are fixed and only the y-coordinates are used as parameters for the optimisation. This method is feasible because of the special structure of the compressor cascade stator air foils. The leading and the trailing edges are described by a circle, which was given as a constraint for the design, see Figure 1. In this way, the encoding is reduced to 11 parameters, of which 10 represent the y-coordinates of the control points in a Cartesian coordinate system and one parameter determines the distance between two successive blades in a cascade.

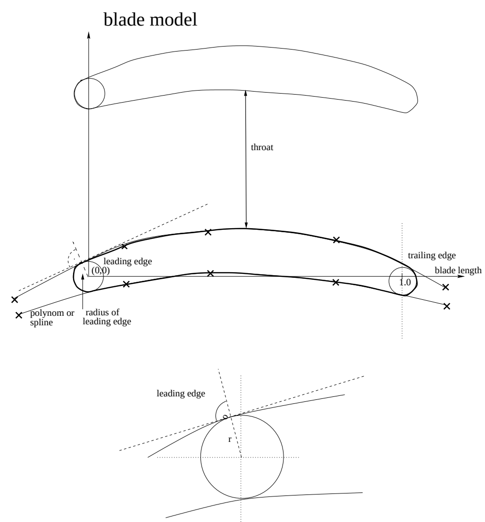

Fig. 1. The underlying air foil model based on two 4^{th} order polynomials is fixed by 10 control points, which are the parameters of the model. The circles which depict the leading and trailing edges are generated in such a way that the polynomials are tangential to the circle. If this construction is not possible, then the geometry is marked as invalid.

2.1 CFD Simulation

In order to evaluate the quality of the generated geometries, a 2D Navier Stokes flow solver with a low-Reynolds number k-ϵ turbulence model is used [7]. The grid size is fixed during the optimisation to 191×55. With grid sizes which are equal or larger to this value, the calculation is nearly independent from the number of grid points for the range of geometry modifications and the necessary computation time is acceptable. However, it should be mentioned that the design of an universal grid, for which the numerical accuracy for all different geometries encountered during optimisation is roughly the same, is still an unsolved and interesting problem.

Additionally, we continuously monitored whether the flow solver converged. If no convergence could be achieved after a given time, the results were marked as invalid and the quality function was replaced by a sufficiently large penalty term. At the same time, we used the convergence monitor to stop the calculation in case of earlier convergence to reduce the computation time to a minimum. Due mainly to this technique the usage of the full Navier Stokes flow solver with a turbulence model becomes feasible. The need for the Navier Stokes solver as opposed to the simpler and computationally less demanding Euler method is demonstrated in Figure 2. In the correlation diagram, the x-coordinate describes the resulting quality value using the Euler method and the y-coordinate the value resulting from Navier Stokes method. A correlation cannot be seen, which clearly indicate that the simplifications in the Euler method cannot be made under these streaming conditions.

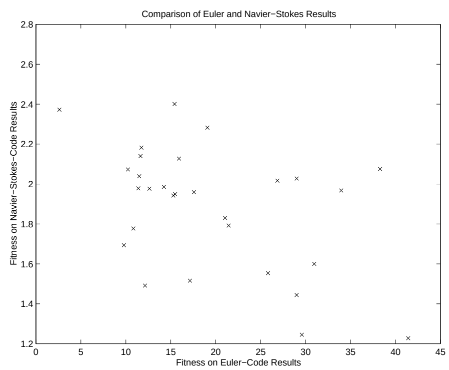

Fig. 2. Correlation diagram between the fitness values determined by the Euler and the Navier Stokes flow solver. The absolute difference between both solutions is not important for the optimisation, however, both results have to be strongly correlated in order not to mislead the search process during optimisation. The figure shows that this constraint is not satisfied, therefore, for the transonic regime the full Navier Stokes model has to be employed.

3 Evolution strategy with covariance matrix adaptation (CMA)

Standard evolution strategies have been described in several textbooks, e.g. [8], and therefore we will only briefly outline the lesser known extensions which have been proposed by Ostermeier [9] and Hansen [10,11]. Firstly, this is the derandomised strategy which reduces the stochastic influence on the self-adaptation of the strategy parameters. In the original mutative self-adaptation scheme, as proposed by Schwefel [8], both the strategy parameters, as well, as the objective parameters are subject to independent stochastic mutations. The idea behind the derandomised strategy is to use one stochastic source for both the adaptation of the objective and of the strategy parameters. In this case, the *actual* step length in the objective parameter space is used to adapt the strategy parameter, e.g. $\sigma(t)$ ($N(\mathbf{0},\mathbf{1})$ denotes a random vector whose components are Gaussian distributed random variables with zero mean and variance equal to one):

$$\sigma(t) = \sigma(t-1)\exp\left(\frac{1}{d}\left(|\boldsymbol{z}| - E[|N(\mathbf{0},\mathbf{1})|]\right)\right), \quad \boldsymbol{z} \sim N(\mathbf{0},\mathbf{1}) \tag{1}$$

Equation (1) results in the following, simple, however successful, effect: If the mutation was larger than expected ($|\boldsymbol{z}| > E[|N(\mathbf{0},\mathbf{1})|]$), then the strategy parameter is increased. This ensures that if this larger mutation was successful (i.e. the individual was selected), then such a larger mutation will again occur in the next generation, since $\sigma(t)$ was increased. The same argumentation holds if ($|\boldsymbol{z}| < E[|N(\mathbf{0},\mathbf{1})|]$). Therefore, the self-adaptation of the strategy parameters depends more directly on the local topology of the search space (a topic which has received some criticism as being less "evolutionary" [8]).

The second method is the introduction of the cumulative step size adaptation. Whereas the standard evolution strategy extracts the necessary information for the adaptation of the strategy parameters from the population (ensemble approach), the cumulative step size adaptation relies on information collected during successive generations (time averaged approach). This leads to a reduction of the necessary population size. The main idea is to avoid strong correlations (positive or negative) in successive step sizes, because such cumulative steps can be more efficiently realized by single steps.

In the CMA algorithm, the full covariance matrix of the probability density function

$$f(\boldsymbol{z}) = \frac{\sqrt{\det(\boldsymbol{C}^{-1})}}{(2\pi)^{n/2}} \exp\left(-\frac{1}{2}(\boldsymbol{z}^T\,\boldsymbol{C}^{-1}\,\boldsymbol{z})\right). \tag{2}$$

is adapted for the mutation of the objective parameter vector. This has the advantage that the mutation direction is independent from the choice of the coordinate system and correlations between parameters can be represented. If the matrix $\boldsymbol{B}$ satisfies $\boldsymbol{C} = \boldsymbol{B}\boldsymbol{B}^T$ and if $z_i \sim N(0,1)$ is a Gaussian distributed random variable with zero mean, then $\boldsymbol{B}\,\boldsymbol{z} \sim N(0,\boldsymbol{C})$. The adaptation of the objective vector can then be written as:

$$\boldsymbol{x}(t) = \boldsymbol{x}(t-1) + \delta(t-1)\,\boldsymbol{B}(t-1)\,\boldsymbol{z}, \quad z_i \sim N(0,1), \tag{3}$$

where $\delta(t-1)$ denotes the global step-size of the strategy. Thus, the overall mutation length can be adapted on a faster time scale (one parameter) than the direction which needs the adaptation of the covariance matrix. Since $\boldsymbol{C}^{-1}$ has to be positive definite with $\det(\boldsymbol{C}^{-1}) > 0$, the different matrix entries cannot be determined independently and the detailed adaptation algorithm combined with the cumulative step-size approach is more involved, see [11,10,12] for a detailed description.

3.1 The quality function

The quality function of the objective parameter vector $\boldsymbol{x}$ consists of four terms. The first one depends on the pressure loss (ω), which is a direct measure of the energy loss and should be minimised. The second term measures the deviation of the averaged angle of the gas stream at the end of the blade

α_2 from the specified value $\tilde{\alpha}$, which is the target for the optimisation. A tolerance range ϵ is given for this difference. Finally, a third and a fourth term take geometrical constraints into account. The minimal diameter of the blade d_1 has to be larger than a given minimal thickness d_{min} and the maximal diameter d_2 of the blade has to be larger than a given value d_{max} to fix the minimal diameter at the thickest part, which is necessary for stability and manufacturing.

$$f(\boldsymbol{x}) = \eta_1 \cdot \omega + \eta_2 \cdot f_2(\alpha_2) + \eta_3 \cdot f_3(d_1) + \eta_4 \cdot f_4(d_2) \tag{4}$$

$$f_2(\alpha_2) = \begin{cases} 0 & , \quad \tilde{\alpha} - \epsilon \leq \alpha_2 \leq \tilde{\alpha} + \epsilon \\ (\tilde{\alpha} - \alpha_2)^2 - \epsilon^2 & , \alpha_2 < \tilde{\alpha} - \epsilon \text{ or } \alpha_2 > \tilde{\alpha} + \epsilon \end{cases} \tag{5}$$

$$f_3(d_1) = \begin{cases} 0 & , d_1 \geq d_{min} \\ (d_{min} - d_1)^2 & , d_1 < d_{min} \end{cases} \tag{6}$$

$$f_4(d_2) = \begin{cases} 0 & , d_2 \geq d_{max} \\ (d_{max} - d_2)^2 & , d_2 < d_{max} \end{cases} \tag{7}$$

In equation (4), η_i denote the factors which allow a weighting between different objectives; they have to be chosen manually. In order to include given constraints concerning outlet angle and geometry into the fitness function the nonlinear functions f_i are introduced. As long as the corresponding values are in the given tolerance range or larger than the minimal value the function is equal to zero. If the deviation angle is too large or the geometrical constraints are not fulfilled the corresponding terms act as a penalty term.

The weighting factors η_i are necessary to normalise the range of possible values for the different terms. Furthermore, they allow certain criteria to be prioritised. The pressure loss ω varies during the optimisation between 0.095 at the beginning and 0.055 at the end, see Figure 5. The factor for the outlet angle was set to $\eta_2 = 10$ and the factor for the geometric constraints was set to $\eta_3 = \eta_4 = 10^7$. If we take the range of the values of the angle and the thicknesses during the optimisation into account, the relation between the different terms can be estimated. The maximal difference between the outlet angle and the design value was 2.5°. The tolerance in equation (5) was set to $\epsilon = 0.3°$. The maximal differences between the stipulated and the measured minimal thickness at the thickest and the thinnest part were maximal 1% during the optimisation. Therefore, the relation between the terms in the order given in equation (4) can be estimated as (1 : 100 : 30 : 30) at the beginning of the optimisation. As a consequence of the small influence of the pressure loss and the high influence of the constraints, the shape is modified to an "allowed" shape in the first step. After the constraints for the outlet angle and the blade geometry have been fulfilled, minimisation of the pressure loss becomes the main target of optimisation. This has the advantage that during the whole optimisation valid shapes are created.

Alternatively, instead of combining the different terms into one quality function, which makes it necessary to fix the unknown parameters η_i, it would have been possible to strive for the determination of the Pareto set of possible solutions in the framework of a multi-objective approach to design optimisation. The main drawback is that in order to determine the Pareto set the population size has to be increased significantly.

4 Experimental Results

The presented results are calculated with a (1,12) evolution strategy with covariance matrix adaptation as described in Section 3. The population was initialised with an existing air foil geometry in order to start with an air foil whose quality can be calculated with the Navier Stokes flow solver. In Figure 3 the history of the overall quality during the optimisation process is shown. The number of quality evaluations (calculations) rather than generations is shown on the x-axis of all plots. Figure 5 and 6 show the development of the pressure loss (ω) and the deviation angle (α_2) during the optimisation. The outliers in Figure 5 correspond to solutions which did not converge and which were removed from the population. Since the deviation angle has a strong influence on the quality function (η_2 is comparably large), mainly (α_2) is optimised during the very first generations ($\sim$ 10). Thereafter, the deviation angles of the individuals fluctuate near the given tolerance range and the optimisation of the pressure loss becomes more important.

In order to observe the self-adaptation of the strategy parameters, the global step size δ and the condition of the covariance matrix during the optimisation are shown in Figure 7 and in Figure 8. The condition of a matrix is the relation between its largest and smallest eigenvalue. The eigenvalues are shown in Figure 4. If all eigenvalues are equal (thus the condition of the matrix is one), the iso-density lines of the probability density function for the mutation of the parameters are circles and no adaptation of the strategy parameters has taken place. The larger the difference between the eigenvalues the more pronounced is the "cigar-shape" of the iso-density lines and the more distinct is the preference to one direction. The magnitude of the mutation is controlled by the global step size.

For the structure optimisation task of the compressor cascade stator blade, we observe the following characteristics in the figures. After 50 to 100 calculations, which in the case of the used (1,12) ES corresponds to the 4^{th} to 9^{th} generation, the outflow angle lies within the tolerance range and the optimisation of the pressure loss starts. Overall, the self-adaptation of the covariance matrix during the whole evolution process is successful as is indicated by the trend of the condition of the matrix and the development of single eigenvalues in Figure 4. It is interesting to note that after generation 25 (300 calculations) the largest eigenvalue increases. This indicates that at this time adaptation of the covariance matrix has taken place and a direction for the search path has been found. At the same time, the global step size strongly increases by a factor of four. The behaviour of the covariance matrix monitored by the eigenvalues and the development of the global step size clearly indicates an adaptation of the strategy parameters to the local topology of the fitness function after about 25 generations. These observations imply that a significant adaptation of the covariance matrix occurs much earlier for this problem than the approximation of the adaptation time $O(n^2)$ given in [10,11]. After about 450 calculations the covariance matrix becomes unfavourable. Since the mutation rate is high according to the large global step size and since the largest eigenvalue of the covariance matrix is reduced, the diversity within the population is increased and no further progress is achieved at this point. As a result, the global step size becomes smaller and the variation in the eigenvalues indicates a new adaptation of the covariance matrix to the slightly different local topology of the fitness function[1]. Of course, we should be careful when drawing precise conclusions from the observations which only rely on one particular optimisation run (although similar results have been observed during other design optimisations of the stator blade). However, we can conclude that a self-adaptation of the strategy parameters indeed occurs and fortunately at an earlier stage than expected.

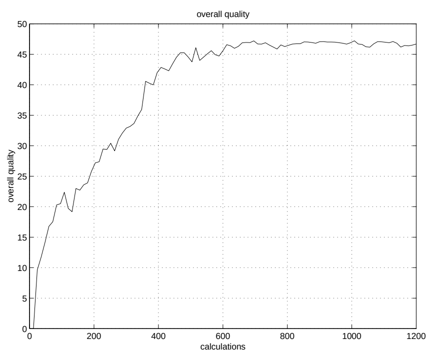

Fig. 3. The quality of the best individual in different generations.

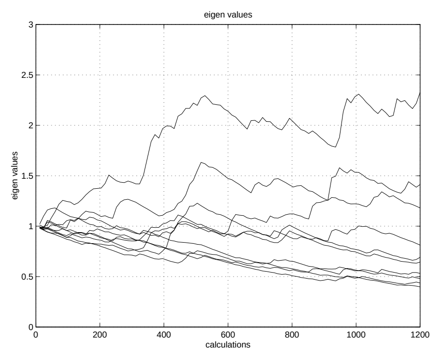

Fig. 4. The eigenvalues of the covariance matrix during optimisation.

5 Discussion and Conclusion

In this paper we successfully applied the derandomised evolution strategy with covariance matrix adaptation to the design of a transonic stator blade for a gas-turbine engine. With the CMA algorithm it was possible to reduce

[1] This behaviour is different from escaping local optima, where it is generally assumed that the step-size increases. In our case, we believe that the *new* adaptation occurs as a result of solely local properties, like for example if we overshot the minimum.

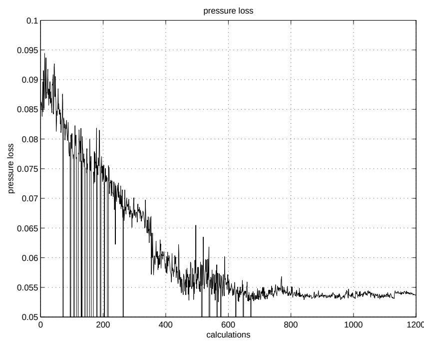

Fig. 5. The development of the pressure loss during optimisation

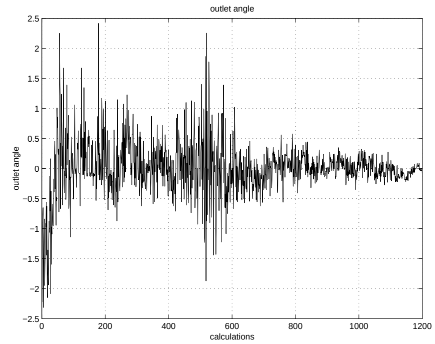

Fig. 6. The development of the deviation angle during optimisation

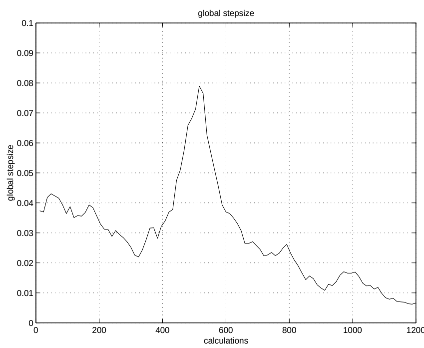

Fig. 7. Adaptation of the global step size parameter during optimisation.

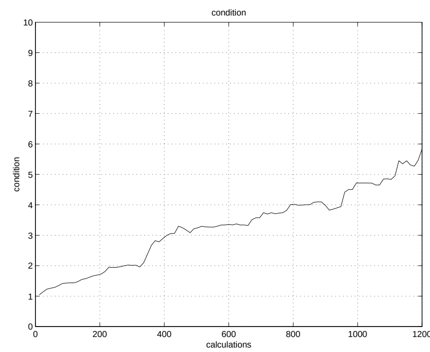

Fig. 8. The condition of the covariance matrix during optimisation.

the population size considerably, so that together with a coarse parallelisation (twelve Sun workstations) and a dynamic termination of the simulation process, we were able to run the optimisation with a full Navier-Stokes solver and the k-ϵ turbulence model for 100 generations (this corresponds to 1200 calculations, compared to a typical population size of 100, only 12 generations would have been possible).

The resulting blade design is considerably different from the “standard” engineering design, which indicates that the evolutionary optimisation resulted in more than a fine tuning (two patents have been submitted in Japan for the optimised blades [13]). The stator blade is currently being built and tested in a wind tunnel at the Virginia Polytechnic Institute and State University; first results are very promising.

We also analysed the self-adaptation of the strategy parameters during evolution and ascertained that the optimisation strategy is very well adjusted to the local topology of the fitness landscape. In addition it also seems to be able to cope with changes in this topology during search.

We did not employ a multi-objective approach which might be one of the directions for future work. However, in particular with reference to 3D optimisation, it seems difficult to further increase the number of structure evaluations. Instead it seems likely that "real-world" applications of multi-objective optimisation will only be possible together with some kind of meta-model for evaluation.

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References

1. A. Vicini and D. Quagliarella. Airfoil and wing design through hybrid optimization strategies. In *16^{th} Applied Aerodynamics Conference*. American Institute of Aeronautics and Astronautics, 1998. AIAA Paper 98-2729.
2. D.J. Doorly. Parallel genetic algorithms for optimization in cfd. In J. Périaux and G. Winter, editors, *Genetic Algorithms in Engineering and Computer Science*. Wiley, 1995.
3. S. Obayashi, Y. Yamaguchi, and T. Nakamura. Multiobjective genetic algorithm for multidisciplinary design of transonic wing planform. *Journal of Aircraft*, 34(5):690–693, 1997.
4. C.M. Fonseca and P.J. Fleming. Multiobjective optimization and multiple constraint handling with evolutionary algorithms – part II: Application example. *IEEE Transactions on Systems, Man and Cybernetics*, 28(1):38–47, 1998.
5. Th. Bäck and H.-P. Schwefel. An overview of evolutionary algorithms for parameter optimization. *Evolutionary Computation*, 1(1):1–23, 1993.
6. B. Sendhoff, M. Kreutz, and W. von Seelen. A condition for the genotype–phenotype mapping: Causality. In Thomas Bäck, editor, *Genetic Algorithms: Proceedings of the 7th Int. Conf. (ICGA)*, pages 73–80. Morgan Kaufmann, 1997.
7. T. Arima, T. Sonoda, M. Shirotori, A. Tamura, and K. Kikuchi. A numerical investigation of transonic axial compressor rotor flow using a low-Reynolds number k-ϵ turbulence model. *Journal of Turbomachinery*, 121:44–58, 1997. Transactions of the ASME.
8. H.-P. Schwefel. *Evolution and Optimum Seeking*. John Wiley & sons, New York, 1995.
9. A. Ostermeier. A derandomized approach to self adaptation of evolution strategies. *Evolutionary Computation*, 2(4):369–380, 1994.
10. N. Hansen and A. Ostermeier. Adapting arbitrary normal mutation distributions in evolution strategies: The covariance matrix adaption. In *Proc. 1996 IEEE Int. Conf. on Evolutionary Computation*, pages 312–317. IEEE Press, 1996.
11. N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation*, 2000. To appear.
12. M. Kreutz, B. Sendhoff, and Ch. Igel. *EALib: A C++ class library for evolutinary algorithms.* Institut für Neuroinformatik, Ruhr-Universität Bochum, 1.4 edition, March 1999.
13. M. Olhofer, Y. Yamaguchi, B. Sendhoff, and E. Körner. Stator blade of axial flow compressor. Japanese Patent No. 348577, December 1999. (in Japanese).