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Effect of Dominance Balance in Many-objective Optimization

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Abstract. This paper examines the effect of dominance balance in many-objective optimization. The dominance balance can be defined by the ratio of the dominating space to the objective space. Here, CDAS, which is one of the most powerful evolutionary many-objective optimization algorithms, is known to be able to change the ratio of the dominating space by relaxing the definition of Pareto dominance with its user-specified parameter. However, the dominance balance is too difficult to control for the parameter in the higher-dimensional objective space. Therefore, we analyze the performance of CDAS by changing the ratio of the dominating space directly in even steps from the minimum to the maximum according to the number of objectives. The corresponding user-specified parameter in CDAS can be obtained from an equation which we assume in the paper. As benchmark test problems, we use DTLZ1, DTLZ2, DTLZ3, and DTLZ4 with two to ten objectives. From computational experiments, we can conclude that the optimal ratio of the dominating space differs depending on the problem at hand. It can be also said that the performance of CDAS is good especially when the ratio of the dominating space is small enough but not the minimum. Based on these observations, we propose a new version of CDAS called CDAS-D which controls the ratio of the dominating space dynamically during optimization.

Keywords: Pareto dominance, dominance relation, dominance area, many-objective optimization, multi-objective optimization.

1 Introduction

Recently many-objective optimization has attracted much attention in evolutionary multi-objective optimization (EMO) which is one of the most active research areas in evolutionary computation [9]. In the past two decades, a lot of EMO algorithms have been proposed in the hope of working for optimization problems with an arbitrary number of objectives [2, 3]. However, it can be found these days that they can hardly handle optimization problems with more than three objectives [9]. For example, NSGA-II, which is one of the most reputable EMO algorithms, compares solutions based on their rank values which generate search

pressure in the objective space [4]. In the higher-dimensional objective space, however, only the first rank may be assigned to every solution as almost all solutions in the population will become non-dominated. Without a variety of rank values, NSGA-II can not keep the search pressure anymore in many-objective optimization. Thus, we have to adopt totally different approaches.

There are mainly three approaches to tackle this issue in many-objective optimization. One is to use a scalarizing function. The scalarizing function yields a scalar value from multiple objective values with a weight vector. MOEA/D is one of representative algorithms that use the scalarizing function with multiple weight vectors in a single run of optimization [17]. Another approach is to have an indicator for comparing solutions sets. IBEA [20] and SMS-EMOA [1] are well-known indicator-based algorithms. As the indicator, the hypervolume is often used. The third approach is to modify the dominance balance. CDAS [13–15], for example, can keep the dominance balance by relaxing the definition of Pareto dominance. Although the performance of CDAS is good for both benchmark test problems and real-world applications [12], its theoretical understanding is not enough in the literature.

In the present paper, we examine the effect of the dominance balance on the performance of CDAS, theoretically and experimentally. The dominance balance is meant by the ratio of the dominating space to the objective space. CDAS can change the ratio of the dominating space by relaxing the definition of Pareto dominance with its parameter S. However, the ratio of the dominating space (i. e., dominance balance) is too hard to handle for the parameter S in the higher-dimensional objective space as the ratio is proportional to a^{d-1}, where $a = \pi/2 - S \cdot \pi$, d is the number of objectives. Therefore, we analyze the performance of CDAS by changing the ratio of the dominating space directly in even steps from the minimum ($(1/2)^d$) to the maximum ($1/2$). The corresponding parameter S in CDAS can be calculated by an equation which we assume in the paper. As benchmark test problems, we use DTLZ1, DTLZ2, DTLZ3, and DTLZ4 with two to ten objectives. From computational experiments, we see that the optimal ratio of the dominating space differs depending on the problem at hand. It can be also said that the performance of CDAS is improved especially when the ratio of the dominating space is small enough but not the minimum. Based on these observations, we propose a new version of CDAS called CDAS-D which controls the ratio of the dominating space dynamically during optimization. In computational experiments, we examine the performance of CDAS-D with two different settings for a range of the ratio of the dominating space.

This paper is organized as follows. We give a detailed description of the dominance balance with a brief introduction to multi-objective optimization and CDAS in section 2. The relation among the parameter S in CDAS, the ratio of the dominating space, and the number of objectives is also given in the form of an equation and a figure. In section 3, we examine the performance of CDAS on benchmark test problems. Based on our observations in section 3, we propose CDAS-D in section 4. The paper concludes in section 5 with discussions on results and future works.

2 Dominance Balance

2.1 Multi-objective Optimization

In this paper we consider minimization problems. Generally a d-objective minimization problem is formulated as follows:

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_d(\mathbf{x})), \text{ subject to } \mathbf{x} \in \mathbf{X}, \tag{1}$$

where $\mathbf{f}(\mathbf{x})$ is the d-dimensional objective vector, $f_i(\mathbf{x})$ is the i-th objective value to be minimized, $\mathbf{x}$ is the decision vector, $\mathbf{X}$ is the feasible region. When two feasible solutions $\mathbf{a}$ and $\mathbf{b}$ of (1) satisfy the following conditions, we can say that $\mathbf{a}$ dominates $\mathbf{b}$.

$$\forall i : f_i(\mathbf{a}) \leq f_i(\mathbf{b}) \text{ and } \exists j : f_j(\mathbf{a}) < f_j(\mathbf{b}). \tag{2}$$

If $\mathbf{a}$ is not dominated by any other feasible solutions, $\mathbf{a}$ is referred to as a Pareto-optimal solution. The entire set of Pareto-optimal solutions forms the Pareto front. The task in multi-objective optimization is to find a set of non-dominated solutions that approximates the whole Pareto front well [4, 18].

2.2 Controlling Dominance Area of Solutions (CDAS)

As explained in section 1, CDAS can change the ratio of the dominating space (i. e., dominance balance) by relaxing the definition of Pareto dominance indirectly with its parameter S [13–15]. In order to relax the definition implicitly, objective values are transformed in CDAS. The transformation equation for the i-th objective value of $\mathbf{f}(\mathbf{x})$ is given with the parameter S as follows:

$$\begin{aligned} f_i'(\mathbf{x}) &= r\cos(\omega_i) + r\sin(\omega_i) \cdot \frac{\cos(\varphi)}{\sin(\varphi)} \\ &= \frac{r\sin(\varphi + \omega_i)}{\sin(\varphi)}, \ \varphi = S \cdot \pi \ \left(\frac{1}{4} \leq S \leq \frac{1}{2}\right), \end{aligned} \tag{3}$$

where r is the norm of $\mathbf{f}(\mathbf{x})$, and ω_i is the declination angle between $f_i(\mathbf{x})$ and $\mathbf{f}(\mathbf{x})$. The transformation is also illustrated in Fig. 1 for the two-objective case. Based on transformed objective values by (3), CDAS assigns rank values and crowding distances to solutions in the same manner as NSGA-II. It should be noted that the algorithm of CDAS is exactly the same as that of NSGA-II except that transformed objective values are used to calculate rank values and crowding distances in CDAS. Although CDAS can handle multiple parameters S_i $(i = 1, 2, ..., d)$, only one parameter S is considered in this paper for the sake of simplicity.

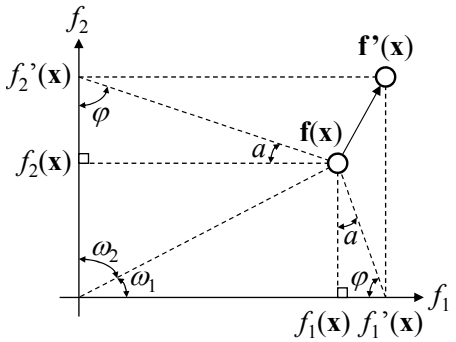


Fig. 1. Transformation of objective values in the two-objective space

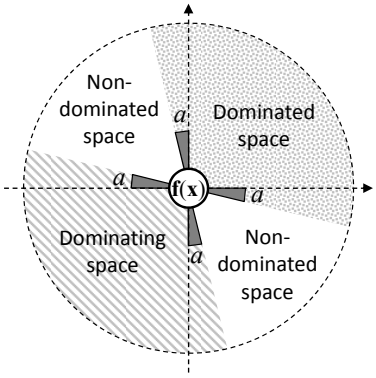


Fig. 2. Dominating, non-dominated, dominated spaces after transforming objective values are shown.

2.3 Ratio of Dominating Space

In order to understand the ratio of the dominating space to the objective space, the dominating space that dominates $\mathbf{f}(\mathbf{x})$, the non-dominated space that is non-dominated in comparison with $\mathbf{f}(\mathbf{x})$, and the dominated space that is dominated by $\mathbf{f}(\mathbf{x})$ after the transformation by (3) are illustrated in Fig. 2 with $a = \pi/2 - S \cdot \pi$ ($0 \leq a \leq \pi/4$, see (3) and Fig. 1). Here, we assume a hypersphere to consider these spaces in the higher-dimensional objective space. From Fig. 2, we can see that the ratio of the dominating space to the objective space (i. e., circle) is proportional to the value of a for the case of two objectives. In the three-dimensional objective space, the ratio of the dominating space to the objective space (i. e., sphere) is proportional to the value of a to the second power. Therefore, we can assume that the ratio of the dominating space is proportional to the value of a to the $(d-1)$-th power in the d-dimensional objective space. It can be also assumed that the minimum and the maximum of the ratio of the dominating space is $(1/2)^d$ and $1/2$, respectively. From these assumptions, we obtain the following relational expression of the ratio of the dominating space,

the value of a, and the number of objectives d.

$$\text{Ratio}_{\text{ Dominating Space}} = \left\{\frac{1}{2} - \left(\frac{1}{2}\right)^d\right\} \cdot \left(\frac{4}{\pi}\right)^{d-1} \cdot a^{d-1} + \left(\frac{1}{2}\right)^d \tag{4}$$

We can also visualize the equation (4) in Fig. 3. From Fig. 3, we can see that

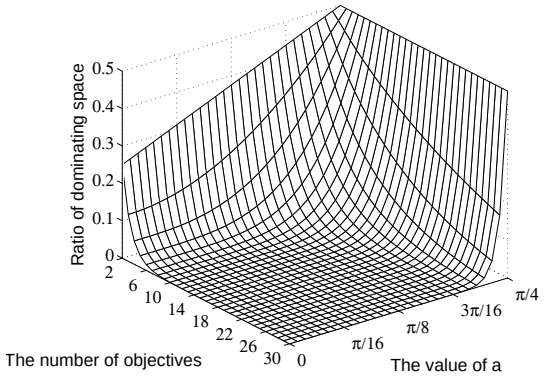


Fig. 3. Ratio of dominating area

the ratio of the dominating space is nearly zero in the objective space with more than ten objectives when the value of a is specified as $a = 0$. In other words, search pressure will be lost in the higher-dimensional objective space as the ratio of the non-dominated space is nearly one, thereby almost all solutions become non-dominated. However, when the value of a is specified as close to the maximum (i. e., $\pi/4$), the ratio of the dominating space is still more than zero even in the higher-dimensional objective space. It should be noted that the ratio of the dominating space (i. e., dominance balance) is difficult to specify for the value of a in the higher-dimensional objective space. In the 30-dimensional objective space, for example, the ratio of the dominating space is nearly zero when the value of a is not specified as close to the maximum (i. e., $\pi/4$). On the other hand, if it is close to the maximum, the ratio of the dominating space changes dramatically according to the value of a in its narrow range. In the next section, we examine the performance of CDAS by changing the value of a and the ratio of the dominating space in even steps, respectively.

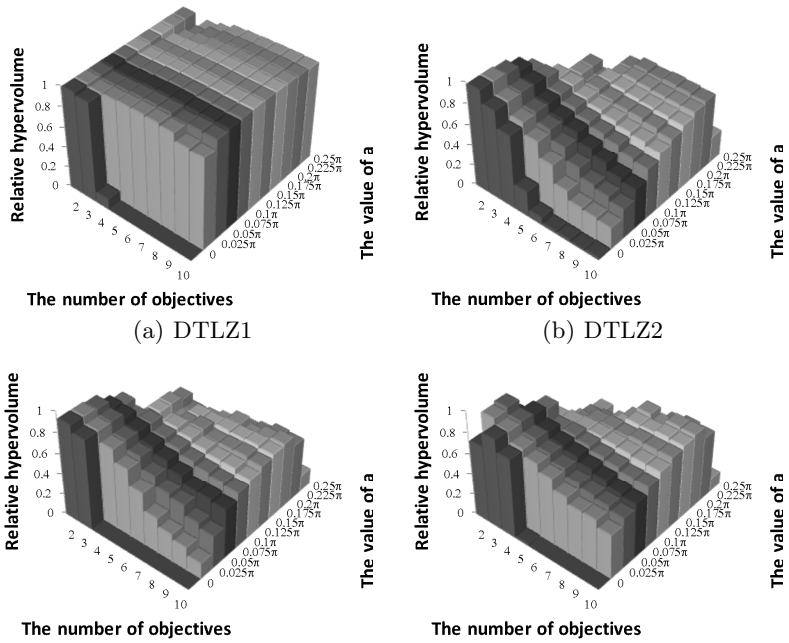
3 Effect of Dominance Balance

In the paper, we use DTLZ1, DTLZ2 ,DTLZ3, and DTLZ4 with two to ten objectives as benchmark test problems [5, 6]. They are well-known benchmark test problems where the number of objectives can be specified by the user. We

use the relative hypervolume (also called as covered fraction in [7]) to assess the performance of CDAS on benchmark test problems. The relative hypervolume is calculated as follows. First the hypervolume of non-dominated solutions in a population is calculated with a reference point. The reference point for calculating the hypervolume should be specified in advance. In this paper, we use the reference point $\mathbf{p} = 0.7^d$ for DTLZ1, $\mathbf{p} = 1.1^d$ for DTLZ2, DTLZ3, and DTLZ4, where d is the number of objectives. We can also calculate the hypervolume of the Pareto front analytically as the Pareto front of each problem is known. Then the hypervolume of non-dominated solutions divided by the hypervolume of the Pareto front yields the relative hypervolume. When the relative hypervolume is one, it means that the Pareto front is obtained completely. On the other hand, when the relative hypervolume is zero, no solutions which dominate the reference point are found. It should be noted that the hypervolume and its relative value give the same conclusion because the hypervolume is just divided by a constant. Regarding the other settings for our computational experiments like a crossover probability and a mutation probability and so on, we use the same settings as [16] except that the number of generations is set to 1000.

First, we show the relative hypervolume of non-dominated solutions obtained by CDAS for combinations of $a = (0, 0.025\pi, \ldots, 0.25\pi)$ and $d = (2, 3, \ldots, 10)$ in Fig. 4. The average of the relative hypervolume over 20 trials is given throughout the paper. We can see from Fig. 4 that the relative hypervolume is larger for each problem when the number of objectives is smaller and the value of a is $a = 0$. It should be noted that results with $a = 0$ correspond to results of NSGA-II as objective values do not change when the value of a is specified as $a = 0$. When the number of objectives is larger, the performance of CDAS with $a = 0$ (i. e., NSGA-II) easily deteriorates. This is justified by the fact that the performance of NSGA-II deteriorates with an increase in the number of objectives [9]. From Fig. 4(a), the relative hypervolume is nearly one in almost all cases for DTLZ1 when the value of a is set to $a \neq 0$. On the other hand, we obtain different results for DTLZ2, DTLZ3, and DTLZ4 from Fig. 4(b), Fig. 4(c), and Fig. 4(d), respectively. From these results, when the number of objectives is smaller, the best performance is achieved with small values of a. However, the best relative hypervolume is obtained by large values of a excluding the maximum when the number of objectives is larger. As discussed in Section 2.3 for Fig. 3, the ratio of the dominating space is still more than zero even in the higher-dimensional objective space when the value of a is specified as large values in its range. Therefore, we can say that the ratio of the dominating space is related to the performance of CDAS.

Although CDAS can change the ratio of the dominating space, it is not changed in even steps in Fig. 4. Therefore, we measure the performance of CDAS for combinations of $r = (r1, r2, \ldots, r11)$ and $d = (2, 3, \ldots, 10)$, where r is the ratio of the dominating space, d is the number of objectives, respectively. Here, $r1$ corresponds to the minimal ratio of the dominating space (i. e., $(1/2)^d$), r11 corresponds to the maximal ratio of the dominating space (i. e., $1/2$). The values of $r = r1, r2, \ldots, r11$ are evenly distributed. It should be noted that the



(a) DTLZ1 (b) DTLZ2

(c) DTLZ3 (d) DTLZ4

Fig. 4. The relative hypervolume of solutions obtained by CDAS for each benchmark test problem is shown for the value of a and the number of objectives.

corresponding parameter S is calculated from (4), which is needed by CDAS to transform objective values (see (3)). We summarize the performance of CDAS on benchmark test problems in Fig. 5. From Fig. 5(a) for DTLZ1, we can say that the relative hypervolume is nearly one for $r = r1, r2, \ldots, r10$ when the number of objectives is two. In the three-dimensional objective space, the relative hypervolume decreases with an increase in the ratio of the dominating space. When we consider the objective space with more than four objectives, CDAS with $r = r1$ (i. e., NSGA-II) does not find any non-dominated solutions which dominate the reference point. We can also see that the relative hypervolume is almost one for $r = r2, r3, \ldots, r11$ when the number of objectives is more than three. On DTLZ2 in Fig. 5(b), when the number of objectives is specified as two to four, $r = r1, r2, r3$ gives good results. In the more than four-dimensional objective space, $r = r1$ yields the worst performance for each objective. On the other hand, $r = r2, r3, \ldots, r10$ gives similar nice results. Here, we should notice that worse results are obtained when the ratio of the dominating space is specified as the maximum. Regarding the performance of CDAS on DTLZ3, the best relative hypervolume is obtained by $r = r1$ in the objective space with two and three objectives in Fig. 5(c). In the more than three-objective space, $r = r1$ gives the worst performance and the optimal ratio of the dominating space is

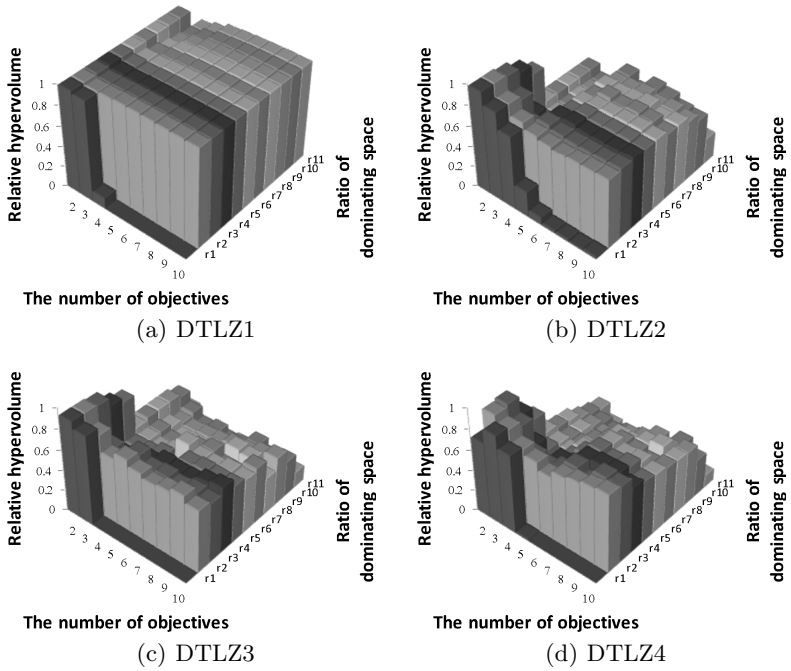


Fig. 5. The relative hypervolume of solutions obtained by CDAS for each benchmark test problem is shown for the ratio of the dominating space and the number of objectives.

different depending on the number of objectives. With respect to DTLZ4 from Fig. 5(d), surprisingly, $r = r3$ gives the best performance when the number of objectives is two. In the three- and four-objective space, $r = r1$ gives the best relative hypervolume. The optimal ratio of the dominating space is different depending on the number of objectives as well as DTLZ3 with more than four objectives. It should be noted that CDAS with $r = r1$ (i. e., NSGA-II) does not work anymore for more than five objectives here too.

From Fig. 4 and Fig. 5, our consideration on the dominance balance is summarized as follows:

- CDAS with $a = 0$ or $r = r1$ (i. e., NSGA-II) performs well for a few objectives whereas it does not work anymore in the higher-dimensional objective space.
- CDAS with $a = \pi/4$ or $r = r11$ (i. e., when the ratio of the dominating space is $1/2$) gives poor results except for DTLZ1.
- In the higher-dimensional objective space, CDAS shows poor results for smaller values of a and nice results for larger values of a excluding the maximum except for DTLZ1.
- In the higher-dimensional objective space, CDAS shows stable and good results for the ratio of the dominating space $r = r2, r3, \ldots, r10$.
- CDAS with $r = r2$ often shows good performance.

4 CDAS-D

4.1 Proposal of CDAS-D

From the previous section, we can see that CDAS with the ratio of the dominating space $r = r2, r3, \ldots, r10$ frequently shows more stable and better performance than that with the value of a. However, we do not know the optimal ratio of the dominating space in advance as it changes depending on the problem and the number of objectives. Therefore we propose a new version of CDAS with the dynamic ratio of the dominating space, which we call CDAS-D in this paper. In CDAS-D, the hypervolume of solutions is calculated at each generation. As long as the hypervolume is improved (i. e., increased), we do not change the ratio of the dominating space. Once the hypervolume deteriorates (i. e., decreases), the ratio of the dominating space is replaced with a random value from a uniform distribution. The idea behind this is that we do not want the ratio to be fixed during optimization. As we do not know the optimal ratio, the random value is tried for the ratio when the hypervolume is decreased. There are two settings for the range of the random value. One is the wide range from $r1$ to $r11$. The other is the narrow range from $r1$ to $r2$. The values of $r1, r2, r11$ are explained in the previous section. The flow chart of CDAS-D is shown in Fig. 6.

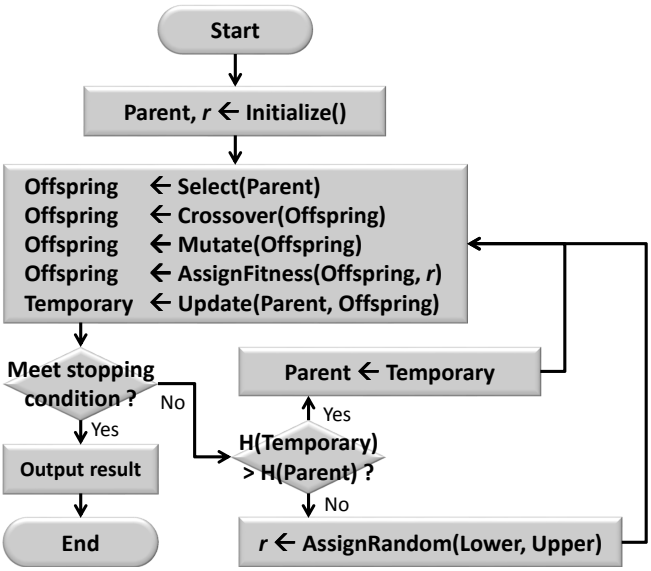


Fig. 6. The flow chart of CDAS-D is illustrated. In the chart, H(S) gives the hypervolume of S, and AssignRandom(Lower, Upper) returns a random value uniformly distributed between Lower and Upper.

4.2 Computational Experiments

We apply our CDAS-D to benchmark test problems in the same manner as the previous section. Computational results are summarized in Fig. 7. It should be noted that Fig. 7 is obtained by adding the results of CDAS-D to Fig. 5 for comparison. From Fig. 7 we can see that our CDAS-D is comparable to

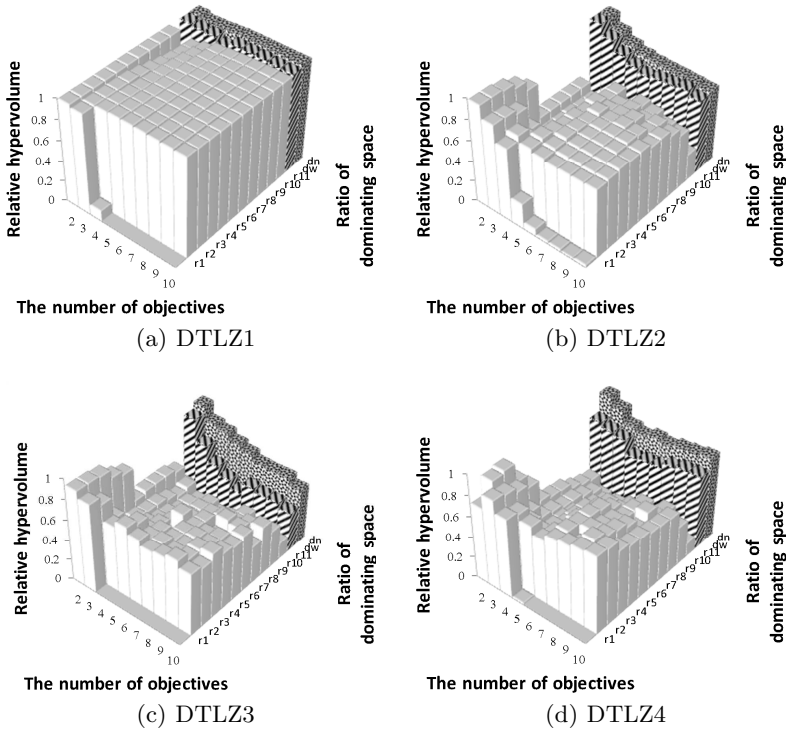
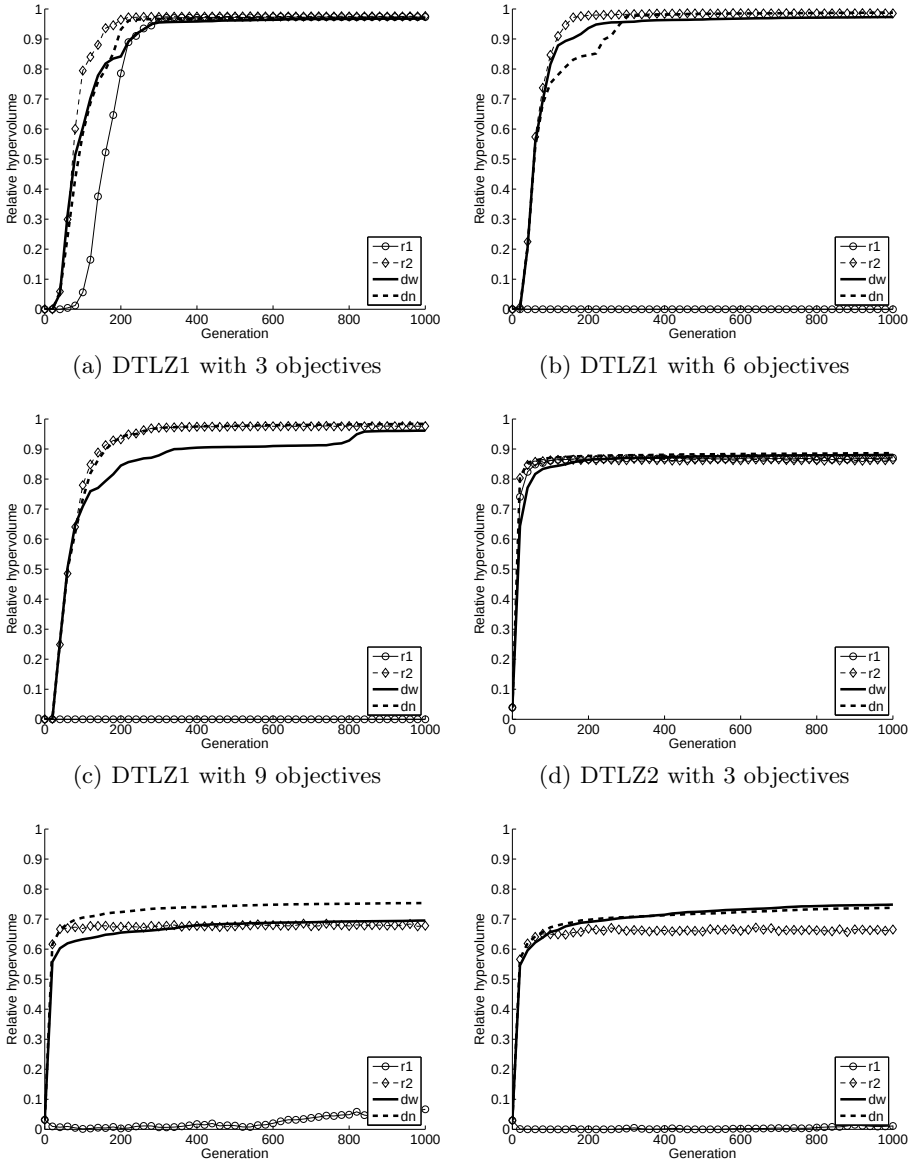


Fig. 7. The relative hypervolume of solutions obtained by CDAS-D for each benchmark test problem is shown for the ratio of the dominating space and the number of objectives. Results of CDAS-D are shown for the wide range (dw) setting by bars with diagonals, for the narrow range (dn) setting by bars with confetti, respectively. The results of CDAS (i. e., Fig. 5) are also included by bars in white.

CDAS with the optimal ratio of the dominating space. It can be also said that CDAS-D with the narrow range shows better performance than that with the wide range. It should be noted that we do not have to specify the ratio of the dominating space in CDAS-D. In order to see a process of optimization, we show the relative hypervolume during optimization in the case of three, six, and nine objectives for CDAS with $r = r1$ (i. e., NSGA-II), $r = r2$, CDAS-D with the wide range (dw), and CDAS-D with the narrow range (dn) on DTLZ1 and DTLZ2 in Fig. 8, on DTLZ3 and DTLZ4 in Fig. 9, respectively. From Figs. 8(a)



(a) DTLZ1 with 3 objectives

(b) DTLZ1 with 6 objectives

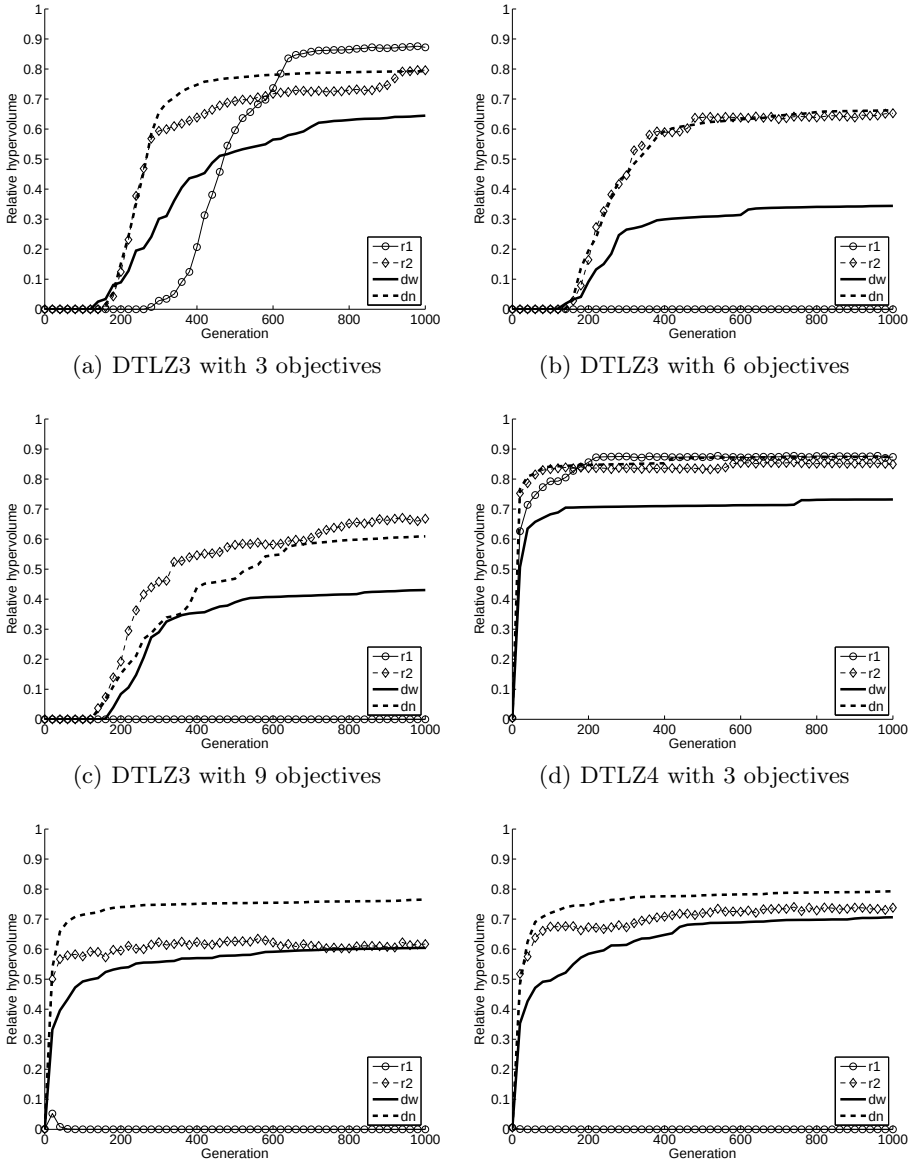
(c) DTLZ1 with 9 objectives

(d) DTLZ2 with 3 objectives

(e) DTLZ2 with 6 objectives

(f) DTLZ2 with 9 objectives

Fig. 8. CDAS-D for DTLZ1 and DTLZ2



(a) DTLZ3 with 3 objectives

(b) DTLZ3 with 6 objectives

(c) DTLZ3 with 9 objectives

(d) DTLZ4 with 3 objectives

(e) DTLZ4 with 6 objectives

(f) DTLZ4 with 9 objectives

Fig. 9. CDAS-D for DTLZ3 and DTLZ4

to 8(c), we can see that the convergence speed of CDAS with $r = r2$ is very fast for all DTLZ1 problems. On the other hand, CDAS with $r = r1$, CDAS-D with the narrow range, and CDAS-D with the wide range is very slow in the three-, six-, and nine-objective space, respectively. The relative hypervolume at the last generation (i. e., 1000th generation) is almost one for every CDAS and CDAS-D. On DTLZ2 in Figs. 8(d) to 8(f), there is no remarkable difference on the performance among CDAS and CDAS-D in the case of three objectives. However, CDAS-D with the narrow range outperforms CDAS on DTLZ2 with six and nine objectives. Regarding DTLZ3 from Figs. 9(a) to 9(c), CDAS with $r = r2$ performs quite well. We can also say that the performance of CDAS-D with the narrow range is good although CDAS-D with the wide range gives the worst results. From Figs. 9(d) to 9(f) on DTLZ4, CDAS-D with the narrow range performs the best. It is also interesting to point out that CDAS-D with the wide range performs the worst. From Figs. 8 and 9 we can see that CDAS with $r = r1$ (i. e., NSGA-II) does not work anymore in six- and nine-objective space. We can also see that the relative hypervolume of CDAS-D (i. e., dw and dn in Figs. 8 and 9) monotonically increases in all cases.

5 Summary

In this paper we examined the effect of dominance balance on the performance of CDAS for DTLZ1, DTLZ2, DTLZ3, and DTLZ4 benchmark test problems. The dominance balance is defined by the ratio of the dominating space to the objective space. In CDAS, the ratio of the dominating space can be changed by specifying the parameter S which is used to transform objective values. However, it is very difficult for the parameter S to control the dominance balance properly in the higher-dimensional objective space. In order to show this difficulty, we examined the performance of CDAS by changing the value of $a = \pi/2 - S \cdot \pi$ ($0 \leq a \leq \pi/4$) in even steps (see Fig. 4). Then we can see that the optimal value of a that gives the best performance of CDAS increases with an increase in the number of objectives except for DTLZ1. In DTLZ2 with more than five objectives, for example, we can also see that the optimal value of a is large enough but not the maximum in its range (i. e., $a = 0.225\pi$) from Fig. 4(b). This is because the performance of CDAS will be affected by the ratio of the dominating space directly and it is considered to be proportional to the value of a to the $(d-1)$-th power in the d-dimensional objective space. Therefore, next we showed the performance of CDAS by changing the ratio of the dominating space from the minimum to the maximum in even steps directly. The corresponding parameter S is calculated from an equation which we propose in the paper. From our computational experiments in Fig. 4 and Fig. 5, we can see that CDAS with the ratio of the dominating space $r = r2, r3, \ldots, r10$ frequently shows more stable and better performance than that with the value of a in the higher-dimensional objective space. In this paper, we also proposed a new version of CDAS with the dynamic ratio of the dominating space, which we call CDAS-D. In CDAS-D, the hypervolume of solutions is calculated at each generation. As long as the

hypervolume is increased, we do not change the ratio of the dominating space. Once the hypervolume is decreased, the ratio of the dominating space is replaced with a random value from a uniform distribution. The idea behind this is that we do not want the ratio to be fixed during optimization. As we do not know the optimal ratio, the random value is tried for the ratio when the hypervolume is decreased. There are two settings for the range of the random value; the wide range and the narrow range. From computational experiments, we can see that our CDAS-D is comparable to CDAS with the optimal ratio of the dominating space. It can be also said that CDAS-D with the narrow range shows better performance than that with the wide range. It should be noted that the ratio of the dominating space in our CDAS-D does not need to be specified by the user in advance and during optimization although in the original CDAS it needs to be fixed properly before optimization. In our CDAS-D, when the hypervolume of solutions is decreased, the ratio of the dominating space is updated with the random value. However, there may be much smarter ways like adaptive approaches to specify the ratio. This issue will be left for one of further works. We should also consider the performance of CDAS-D on real-world applications in the future.

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