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THREE DIMENSIONAL AERODYNAMIC DESIGN OPTIMIZATION USING SINGLE AND MULTI-OBJECTIVE APPROACHES

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Abstract. *We present the application of evolutionary optimization techniques to the three dimensional aerodynamic design optimization of a gas turbine stator blade. This problem is characterized by a high dimensional search space, which results from the need to build a 3D model of the blade, and by an extremely expensive data acquisition process, namely the analysis of the 3D flow around the blade. Although aerodynamic design optimization often is treated as a single objective optimization problem by minimizing solely the aerodynamic loss, the problem is inherently multi-objective. We consider as a second objective the variation of the circumferential static outlet pressure distribution and compare several single and multi-objective methods to incorporate this second objective into the optimization. We discuss advantages and drawbacks of these methods in terms of their feasibility for our optimization task and hence, more generally, for tasks that are characterized by high costs of data acquisition, e.g. where the evaluation of the objective function for optimization is computationally demanding and time consuming.*

1 INTRODUCTION

We present the application of evolutionary optimization techniques to the 3D aerodynamic design optimization of an ultra-low-aspect-ratio (ULAR) gas turbine stator blade. This kind of stator blades is only rarely used in gas turbines because of their relatively poor stage efficiency as compared to the more conventional high-aspect-ratio (HAR) turbines. The reason for this presumably lies in the complex 3D nature of the flow in the ULAR case and the complex interaction of the secondary flow with the transonic main flow. With respect to design optimization this has two consequences: on the one hand the advanced design principles developed for HAR blades cannot be exploited to improve the efficiency of the ULAR turbine stator blades because the flow characteristics in both cases are too different. On the other hand we cannot resort to relatively fast two dimensional or quasi three dimensional methods for flow analysis but have to analyze the full three dimensional flow, a computationally expensive task.

From the optimization point of view 3D aerodynamic design optimization constitutes an interesting problem for several reasons. The need to build a 3D model of the design inevitably renders the problem high dimensional. The exploration of a high-dimensional search space, however, requires a large number of data points. In the worst case the number of necessary data points scales exponentially with the search dimension. In 3D aerodynamic design optimization this constitutes a serious problem because here acquisition of a data point means simulation of the fluid dynamic properties of the design under consideration. Computational analysis of 3D flows is still a challenging task. Even with high performance parallel codes this still may take hours. So the ultimate goals in approaching such a kind of problem must be to restrict the problem dimension and to choose methods that are able to cope with sparse data.

Another crucial question in every optimization problem is the formulation of the objective function. In aerodynamic design optimization, traditionally the minimization of the aerodynamic loss, i.e. the average pressure loss, is chosen as optimization target. However, design optimization, like most real world applications, is inherently a multi-objective optimization problem. In our case, we found it necessary to include a second objective, the minimization of the variation of the circumferential static pressure distribution. In this paper, we consider and discuss four approaches to the optimization of this multi-objective problem that span the whole range from single objective optimization via reformulating the multi-objective problem as a single objective problem to true multi-objective optimization.

In the next section, we introduce the optimization task and present our optimization framework. In Sect. 3, we discuss single objective and multi-objective approaches to the solution of this problem. The results from these approaches are presented in Sect. 4. Finally, in Sect. 5, we will discuss advantages and drawbacks of these methods in terms of their feasibility for our optimization task and hence, more generally, for tasks that are characterized by extremely high costs of data acquisition.

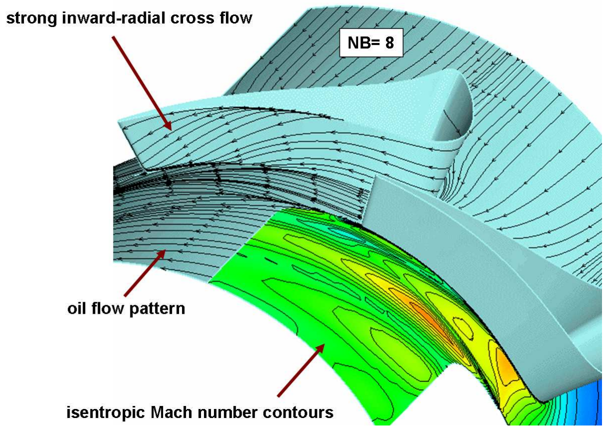

Figure 1: Ultra low aspect ratio turbine stator blades and their flow fields. The blade count is NB=8.

2 THREE DIMENSIONAL TURBINE BLADE DESIGN OPTIMIZATION

2.1 The turbine stator blade

The aerodynamic design that we optimize is part of a gas turbine that is used in small business jets. In particular, our goal is to optimize the design of the turbine stator blades. The stator in this case is of a special type. It is a so-called ultra-low-aspect-ratio (ULAR) stator that is made up of only 8 stator blades. This is a very small number compared to 20 - 60 blades that are used in more conventional turbine designs. For details on the design specifications of the ULAR stator refer to [1].

ULAR stator blades have rarely been adopted as turbine components because of their relatively poor performance. From the flow field based on CFD calculations that is shown in Fig. 1 it can be seen that there is a very strong inward-radial cross flow on the blade suction side and – due to the interaction of the secondary flow with the transonic main flow – the flow field near the hub-end-wall is very complicated. Consequently, the loss near the hub region is significantly increased as compared to conventional HAR blades. Nevertheless, there are considerable benefits when adopting low aspect ratio blades. For example, for a low number of stator blades rotor blade resonance, and hence material fatigue, is reduced.

2.2 Evolutionary algorithms in design optimization

Evolutionary algorithms [2] are a class of stochastic optimization algorithms whose use in design optimization problems is well established by now [3]. In our approach to 3D turbine blade optimization we use a special variant of evolutionary algorithms, namely an evolution strategy (ES) with covariance matrix adaptation (CMA) [4]. The basic idea of CMA-ES is to make maximum use of the information contained in the search history for self-adaptation of the search direction which is defined in terms of the covariance matrix of a normal distribution from which new tentative solutions or, in the language of evolutionary algorithms, new individuals are drawn. Thereby the population size in the evolutionary strategy is decoupled from the dimension of the search space. This means that a drawback of stochastic search, the need for evaluation of a large number of possible solutions, is alleviated.

Especially the latter feature is indispensable in 3D blade optimization which is characterized by a fundamental conflict: on the one hand we need a 3D parametric model of the design which entails a high-dimensional design space. As a consequence a large number of different designs has to be evaluated during optimization. On the other hand each evaluation of the blade performance is a computationally extremely demanding task so that only a limited number of evaluations can be afforded.

2.3 Blade model

A crucial point in design optimization is the parametric model of the geometry that will be optimized since this determines the design space. There are a number of requirements on the design of a proper blade model. Among these are (*i*) flexibility – the model must be flexible enough to allow for a wide variety of different designs, (*ii*) compactness – the number of parameters describing the model must be low enough to allow for reasonable convergence times of the optimization algorithm, and (*iii*) locality – variations of a single model parameter should result in only local variations of the model and should not affect the global model shape.

A good choice to fulfill these requirements is to use non-uniform rational B-spline (NURBS) surfaces [5] to represent the blade. A B-spline surface is a tensor product of two B-spline curves and hence is defined by two parameters, a set of control points and two knot vectors, one for each parameter. Usually not all of these parameters are subject to optimization. Often the variables in the design optimization problem are given by the coordinates of the control points only. However, the suitable number of control points must be chosen with care: the use of too few control points may unnecessarily restrict the design space and exclude potentially interesting designs while the use of too many control points complicates the optimization problem and additionally may have unwanted side-effects like the creation of cusps or even self-intersections of the resulting surfaces.

Due to manufacturing reasons, our blade geometry is defined by two sections, the hub section and the tip section. The remaining blade geometry is defined by linear interpolation between these two sections. Thus our blade model consists of a B-spline surface defined by a periodically closed cubic B-spline in one parameter direction and a second order open B-spline in the other direction. The hub section and the tip section of the blade are each modeled using 25 control points so that we have all in all 50 control points. Using the coordinates of the 50 three-dimensional control points directly as design variables would result in a 150-dimensional search space. Fortunately, we can exploit two facts to reduce the search space dimension to just 88. We only need to include the 44 independent, non-periodic control points as design variables since they determine the remaining 6 control points that are only needed to periodically close the B-Splines in the first parameter direction. Furthermore the hub and the tip section of the blade are defined to lie on cylindrical surfaces. This means the z-coordinates of the control points are implicitly determined by the blade geometry. Hence we only need to consider the x- and y-coordinates of the non-periodic control points as design variables.

2.4 Flow analysis

For the evaluation of a blade design, we use CFD simulations of the flow from which the necessary fluid dynamic quantities are calculated. For these simulations we use the parallelized 3D Navier-Stokes flow solver HSTAR3D [6], with Wilcox's k-ω two equations model [7]. In order to obtain a high resolution of the boundary layer development, CFD calculations for the baseline blade, that was used to initialize the optimization process, were performed prior to optimization to determine the grid size. The computational grid for the solution of the Navier Stokes equation consisted of $175 \times 52 \times 64 = 582,400$ cells. For each evaluated blade design a new grid was generated. This is a relatively inexpensive operation that takes on average about 40 seconds on an AMD Opteron 2 GHz double processor. The flow analysis, however, is an extremely time consuming task that is parallelized into 4 processes and takes between 2 hours and 3.5 hours on AMD Opteron 2 GHz double processor machines depending on the number of flow solver iterations needed for convergence. The calculation of about 300 generations comprised of 10 individuals of the evolutionary optimization, took about 6 weeks time using a cluster of 40 computers!

3 SINGLE AND MULTI-OBJECTIVE APPROACHES

A complex optimization problem like aerodynamic design optimization is inherently multi-objective even if often only a single objective is taken into account for optimization. But also if multiple objectives are employed, the problem often is rendered as a single objective one and solved with familiar and efficient single objective optimization methods. An example for this approach is to linearly aggregate multiple objectives weighted by an appropriate choice of parameters and to solve the resulting single objective problem. Another approach that is often used to avoid multi-objective optimization is to use all but one objectives as soft constraints, i.e. to tolerate small deviations from a target value.

Multi-objective optimization has been the subject of intense research in the recent years in the field of evolutionary computation [8]. In contrast to single objective optimization, which aims at finding a single optimal solution, multi-objective optimization strives for searching a set of optimal solutions in problems with possibly conflicting objectives. This Pareto set is characterized by the fact that no solution from this set is better than another Pareto solution in all objectives involved. Since evolutionary algorithms inherently operate on sets, i.e. a population of solutions, they seem to be particularly suitable for multi-objective optimization problems. The drawback here is that most multi-objective optimization algorithms seem to require more function evaluations than efficient single-objective methods like the derandomized evolution strategies with cumulative step-size adaptation. As already mentioned, the larger number of required function evaluations poses a problem when each function evaluation is computationally demanding, like for aerodynamic design optimization problems [9, 10], other fluid-dynamic problems [11] or even multi-disciplinary problems [12]. In these cases, effort has been undertaken to minimize the needed population size [10] or to employ meta-models [13, 14]. Since we deal with a particularly computationally demanding aerodynamic optimization problem, we have to employ algorithms which minimize the required number of individual evaluations. We use both "standard" static aggregated methods and the dynamic weight aggregation method [15, 16] which employs an evolution strategy to represent the set of Pareto optimal solutions in an archive.

3.1 Fitness functions and constraints

The main performance index in aerodynamic blade design is the aerodynamic loss of the blade which is measured by the mass averaged pressure loss ω. In order to reduce the stator-rotor-interaction in the turbine, we include the variation of the pitch-wise static outlet pressure $\mathrm{PST}_{\mathrm{VAR}}$ as a second criterion into the optimization.

In addition to these two objectives, we use a number of geometrical and manufacturing constraints that are included in the objective function as penalty terms that will only contribute if the constraints are violated. The constraints are (*i*) the outflow angle and (*ii*) the mass flow rate which are determined as a result of the flow analysis. The other constraints concern the blade geometry and basically can be considered as manufacturing constraints. These are (*ii*) the minimum blade thickness, (*iii*) the minimum trailing edge thickness, and (*iv*) the blade solidity.

We conducted a series of both single and multi-objective optimizations using the following objective functions and optimization approaches, respectively:

A. We only minimized the pressure loss ω and observed $\mathrm{PST}_{\mathrm{VAR}}$. This is a single objective approach with the objective function

$$f_1 = \omega \rightarrow \min \ . \tag{1}$$

B. We used static linear aggregation of the two objectives according to

$$f_3 = w_1 \, \omega + w_2 \, \mathrm{PST}_{\mathrm{VAR}} \rightarrow \min \ . \tag{2}$$

Here the weights w_1 and w_2 of both objectives were constant throughout the optimization and were initially chosen such that both objectives contributed equally to the objective function. This approach constitutes a simple, naïve approach to multi-objective optimization.

C. We minimized the pressure loss and included the static outlet pressure as a constraint thus turning the multi-objective problem into a single objective one. Thus the objective is

$$f_2 = \omega + w_1 \left[\min(0, \mathrm{PST}_{\mathrm{VAR,init}} - \mathrm{PST}_{\mathrm{VAR}})\right]^2 \rightarrow \min\,, \tag{3}$$

where $\mathrm{PST}_{\mathrm{VAR,init}}$ is the initial blade's value of $\mathrm{PST}_{\mathrm{VAR}}$ and w_1 is the corresponding weight.

D. We used the multi-objective approach of dynamic weight aggregation (DWA) [15, 16]. In this algorithm the two objectives are linearly aggregated according to

$$f_3(t) = w_1(t)\ \ \omega + w_2(t)\ \mathrm{PST}_{\mathrm{VAR}} \rightarrow \min\ . \tag{4}$$

Here the weights of the two objectives are varied dynamically, gradually, and periodically during the optimization according to $w_1(t) = \left|\sin\left(\frac{2\pi}{p}t\right)\right|$ and $w_2(t) = 1 - w_1(t)$, where p defines the period of the variation and t is given by the generation of the population. This approach can easily be combined with arbitrary evolutionary strategies which has the advantage that we can retain using CMA-ES and the associated small populations.

4 RESULTS

In our experiments, we used a (μ, λ) CMA-ES with parameter settings as described in [4]. We used $\mu = 1$ parent individual and $\lambda = 10$ offspring individuals. The strategy parameters σ were initialized with $\sigma = 0.1$. This value gave good results in preliminary tests with a wider range of possible initialization values. We did not use recombination. The simulations were run on a cluster of AMD Opteron 2GHz double processors.

We ran four single- and multi-objective variants of the optimization as detailed in Sect. 3.1 above, cf. Eq. (1) - Eq. (4). In the multi-objective optimization using DWA according to Eq. (4) we used a period $p = 100$ generations, i.e. the weights w_1 and w_2 were periodically varied from 0 to 1 within 50 generations. The results reported for this run were taken after 426 generations.

A detailed analysis of the blade resulting from optimization with objective function Eq. (1) from the aerodynamic point of view is given in [17]. In this paper we will restrict our discussion of the results to a comparison of the various optimization approaches.

In Fig. 2, we show the evolution of the pressure loss and the variation of the static outlet pressure, respectively, as a function of the number of generations for the three single objective optimization approaches described in Sect. 3.1.

The curves denoted with "A." give the optimization results according to Eq. (1). Here only the pressure loss was included in the objective function. From the optimization point of view this simulation was quite successful: a reduction in the pressure loss of about 10% was achieved. However, as shown in Fig. 2 (right), this was reached at the expense of a considerable increase in the variation of the static pressure which is a very undesirable result from the engineering point of view.

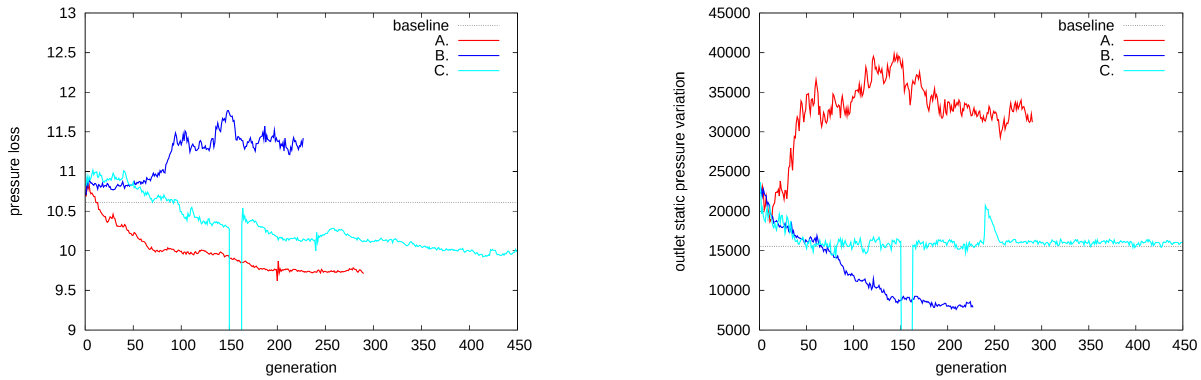

Figure 2: The developing of the pressure loss (left) and the variation of the static outlet pressure (right). Here "baseline" denotes the performance of the baseline blade. The performance of the fitness-best individuals of each generation are shown for the following fitness functions: A - minimization of the pressure loss, cf. Eq. (1), B - static linear aggregation of the two objectives, cf. Eq. (2), C - minimization of the pressure loss subject to an upper bound on the static outlet pressure, cf. Eq. (3).

This increase can be inhibited by including $\mathrm{PST}_{\mathrm{VAR}}$ explicitly in the objective function. The results of doing so by static linear weight aggregation according to Eq. (2) are shown as curves "B." in Fig. 2. As a first approach in this case, we chose the weights of the two objectives such that initially both objectives received the same weight. The results show that this choice is not the best one. The variation of the static outlet pressure received too much weight, so that the optimizer was able to reduce only $\mathrm{PST}_{\mathrm{VAR}}$ while allowing an increase of the pressure loss of about 9%.

In a second approach, we included the variation of the static outlet pressure as a soft constraint according to Eq. (3). We bounded $\mathrm{PST}_{\mathrm{VAR}}$ from above using approximately the value of the initial blade. The results are labeled by "C." in Fig. 2. The constraint is effective in controlling $\mathrm{PST}_{\mathrm{VAR}}$ while at the same time the pressure loss is reduced. Compared with curve "A.", this reduction is achieved at a slightly slower pace. The vertical lines in Fig. 2 indicate that from generation 150 to generation 160 the process of grid generation and flow analysis failed for the whole population. In principle, this is critical because it may lead to a failure of the whole optimization process. But here the optimizer was sufficiently robust to tolerate the missing results from the flow analysis and to drive the population back to a region in which flow analysis was possible with relatively small loss in performance.

The results from the multi-objective optimization using dynamic weight aggregation according the Eq. (4) are shown labeled as "D."in Fig. 3. Here we plotted for each individual that was produced in each of the four the optimization processes discussed in this paper the values of $\mathrm{PST}_{\mathrm{VAR}}$ against the value of the pressure loss ω. The Pareto optimal individuals are marked by filled circles. In Fig. 3 also the performance of the initial blade design is plotted as a dotted line for each of the two objectives. This means all blade designs in the upper right quadrant are inferior to the initial blade and all blade

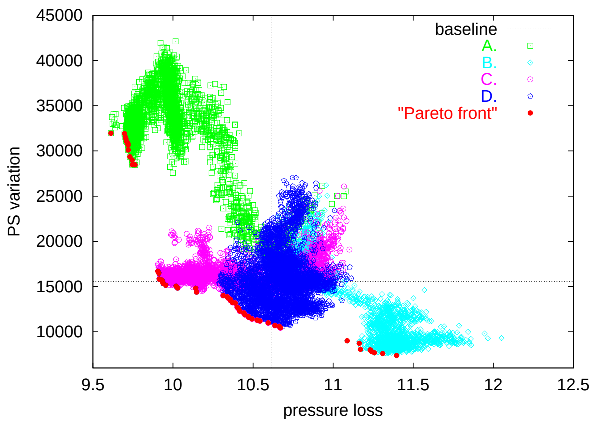

Figure 3: The "Pareto front" aggregated from all four optimization runs. Here "baseline" denotes the performance of the baseline blade. A - minimization of the pressure loss, cf. Eq. (1), B - static linear aggregation of the two objectives, cf. Eq. (2), C - minimization of the pressure loss subject to an upper bound on the static outlet pressure, cf. Eq. (3), D - multi-objective optimization using dynamic weight aggregation, cf. Eq. (4). The individuals that build the Pareto front are marked by filled circles.

designs in the lower left quadrant are superior to the initial blade with respect to both objectives. From the results of using dynamic weight aggregation – blue open pentangles – we see that using multi-objective optimization techniques it is possible to reduce PST_{VAR} and the pressure loss at the same time. However, the progress in this case is much slower than in the single objective optimization.

The representation in Fig. 3 allows us to directly compare the results from all optimizations with respect to the two involved optimization criteria and at the same time we gain a more complete impression of the Pareto front in this problem. The designs found by solely minimizing the pressure loss – denoted by A. in Fig. 3 – clearly achieve the lowest pressure loss of all designs but are also characterized by the highest static pressure variation. The other extreme, a high pressure loss and a low variation in the outlet pressure, is represented by the solutions from the static linear aggregation of both objectives – denoted by B. in Fig. 3. This is due to the minor influence of the pressure loss in this case that was caused by the choice of the weights in the linear aggregation Eq. (2). Best at simultaneously minimizing both objectives is the multi-objective approach using dynamic weight aggregation – denoted by D. in Fig. 3, while using PST_{VAR} as a soft constraint – denoted by C. in Fig. 3 – yields solutions with low pressure loss and slightly decreased static pressure variation.

5 CONCLUSION

Three dimensional aerodynamic design optimization constitutes a challenging problem because the need to work with a 3D model of the design entails a high dimensional search space and in general requires large amounts of data. These data, however, are obtained by analyzing the 3D flow around the blade, a process that consumes huge amounts of computation time and makes data acquisition expensive. This is the main limiting factor in the optimization problem. Hence decisions on the choice of models and algorithms must be governed by the goals of using a compact, yet flexible computational model and optimization algorithms that are able to cope with sparse data in order to allow for only a small number of objective function evaluations in the optimization. Our proposal here is to use a B-spline model of the blade and to optimize this using a derandomized evolution strategy with cumulative step-size adaptation, namely the covariance matrix adaptation [4].

The target in this study was to find aerodynamic designs that achieve low aerodynamic losses and at the same time low static outlet pressure variations. The trade-off relation between both objectives has been previously pointed out in the literature [18]. In this paper, we employed and compared several methods to combine the two objectives. The simplest way to do so is static linear aggregation. Its main advantage is that the problem remains single objective and more efficient algorithms can be used than are available for multi-objective optimization. The drawbacks, besides the linearity of the combination, are the ad-hoc choice and the inflexibility of the relative weights. A successful application of this method requires careful exploration of the weight space, a possibly time-consuming task that may well outweigh the advantage of being able to use efficient single objective optimization algorithms. To make matters worse, the appropriate weighting of the constraints may change during optimization. Indeed as our results show, the ad-hoc choice of the weights drew too much attention towards the static pressure variation.

Alternatively, we regarded the static pressure variation as a soft constraint – slight overshooting was penalized only slightly – and only optimized the pressure loss. This again was a single objective optimization approach with the aforementioned advantages that will yield good results if, as in our case, it is sufficient to impose a bound on all but one of the objectives.

Finally, we used a multi-objective approach, dynamic weight aggregation [15, 16], which searches for the Pareto front by dynamically, gradually, and periodically changing the weighting factors of a linear aggregation of the objectives during optimization. The advantage of this approach is that it can easily be combined with arbitrary evolutionary strategies, so that we can retain using CMA-ES and the associated small populations. This is especially important in cases like ours where a large number of individuals cannot be afforded because of hardware and time restrictions. Small populations would not be possible with all multi-objective evolutionary algorithms. NSGA-II [19], for example, a successful and popular method that is based on genetic algorithms or real coded genetic algorithms requires substantially larger populations than CMA-ES.

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