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# Aerodynamic Shape Optimisation using Evolution Strategies

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## Abstract

In the recent years many successful applications of evolutionary algorithms to aerodynamic design problems have been reported. The two main problems in applying evolutionary algorithms to that field are the high computational cost of the quality evaluations and the parameterisation of the design. In order to solve these problems, it is necessary to find the best simulation methods and strategies for the optimisation in order to reduce the overall computation time. In this paper different Evolution Strategies are compared with respect to their applicability to the optimisation of gas turbine blade designs. Additionally a representation with a low number of parameters is necessary in order to reduce the time needed for the optimisation. On the other hand the representation should be as variable as possible to allow the description of a wide range of possible designs. In order to optimally solve this trade-off between compact representations and detailed descriptions an adaptive representation is proposed. This representation is compared with fixed representations with respect to the optimisation time and the quality of the final result.

## 1 Introduction

In recent years, the number of publications describing applications of evolutionary algorithms to aerodynamic design optimisation has significantly increased (see e.g. [1–7]). This is mainly due to the fact that the computational fluid dynamics calculations need considerable computing power which only recently became available at a reasonable cost. However still, one of the main problems in applying optimisation methods to aerodynamic design problems is the high computational cost of quality evaluations. This holds in particular when applying evolutionary algorithms (EA), whose demand for computational resources is usually higher than that of e.g. gradient based optimisation methods. To overcome this problem it is necessary to carefully choose both the used optimisation method and the simulation tools. The later one should need a minimum of computational resources in order to calculate the needed properties of a design. Besides the disadvantage of higher computational demands, evolutionary algorithms have several advantages compared to gradient based optimisation methods. The estimation of the gradient, which is a time consuming and, particularly in the presence of noise, difficult exercise, is not needed in evolutionary algorithms. Due to the population based approach EAs have the ability to escape from local optima, which makes them particularly suitable for design optimisation. Furthermore, they allow multiobjective optimisation in a very intuitive way.

It has been shown in [8,9] that a special variant of the Evolution Strategy, the derandomized covariance matrix adaptation, can cope with small populations and still achieve convergence speeds which are as good as or even better than other EAs with larger populations. This strategy was successfully employed for the optimisation of gas turbine blades for transonic flow conditions [6].

Another approach to increase the convergence speed and therefore, to reduce the necessary computation time is to limit the dimensionality of the search space. This can be done by using knowledge, gathered from experience with similar designs and from theoretical investigations in order to identify the most important design parameters and to restrict the optimisation to these parameters. A considerable increase of performance for a wide range of designs can be achieved by generating very compact parameterisation models with a low number of parameters.

An example for such kinds of models for stationary blades in gas turbomachineries is shown in Section 2.2. However, the determination of the correct set of design parameters is crucial and can be difficult or even impossible for problems where only a small amount of knowledge exists or where new design concepts are necessary. In aerodynamic shape design problems this is for example necessary for transonic flow conditions, which are the target of our work. By reducing the number of design parameters and the freedom for the modifications, the optimisation tends to generate solutions which are similar to already known "standard" ones. In order to find designs for significant different conditions or designs of principle novelty, the degree of freedom must be increased. In this way, we are able to represent designs, which do not necessarily comply with "engineering intuition". However besides a strong increase in the calculation time, there is a high risk of premature convergence of the algorithm to local optima in the high dimensional search space. An alternative to the fixed models can be to integrate an adaptation of the parameter space in the Evolutionary Algorithm as a variation of the parameterisation model. In this paper, we will present an approach on how such an integration of a *structure variation* in the Evolution Strategy can be achieved in order to generate an optimal set of design parameters. The optimisation starts on a simple, low dimensional model of the aerodynamic design, which leads to a fast convergence to a preliminary solution. Subsequently, the chromosome size is increased whereby the model becomes more refined and the freedom for the design increases.

The remainder of this paper is organised as follows. In the first part of section 2, two simulation methods are compared and examined with respect to their usability for the targeted flow conditions. In order to address the dimensionality of the search space, in the second part three different compressor stator air foil models used in turbomachineries are described and compared with respect to their effects on optimisations. In these models the dimensionality of the search space is fixed by the number of design variables used in the parametric description of the design. In contrast to the existing models, which are based on a fixed parameterisation, an adaptive model, which has the property that the parametrisation itself is adapted in the optimisation (additionally to the parameters of the model), is presented in Section 2.3.

In Section 3, different methods for strategy parameter adaptation are compared according to the quality of the final solution and to their convergence speed. Additionally, optimisations based on an adaptive model are carried out. Since computational fluid dynamics is very time consuming, the simulation results in Section 3 are not generated from an aerodynamic optimisation problem, but from fitting a spline to a target structure using a distance measure for closed two dimensional curves.

## 2 Aerodynamic Design Optimisation

In aerodynamic design optimisations, where complex simulations are necessary in order to determine the quality of a design, the overall calculation time very often limits the applicability of optimisation methods. Besides the overall number of iterations during the optimisation, which is influenced for example by the dimensionality of the parameterisation model and the used optimisation algorithm, the time which is needed for one single quality evaluation of course determines the computation time.

### 2.1 Simulation Methods

Often different simulation methods can be used to determine the quality of a design. This also holds for aerodynamic optimisations, where different CFD simulations can be used which vary in the degree of simplifications. Generally, there is a trade-off between calculation time and the accuracy of the results.

For two dimensional optimisations, which are the focus of this work, the Navier-Stokes method can be used together with some kind of turbulence model. These simulations provide results of high precision, however, at a high computational cost. A simplification of the calculation is given by the Euler method, where terms that consider viscosity of flowing gases are neglected. This allows a reduction of the simulation time by a factor of 5 to

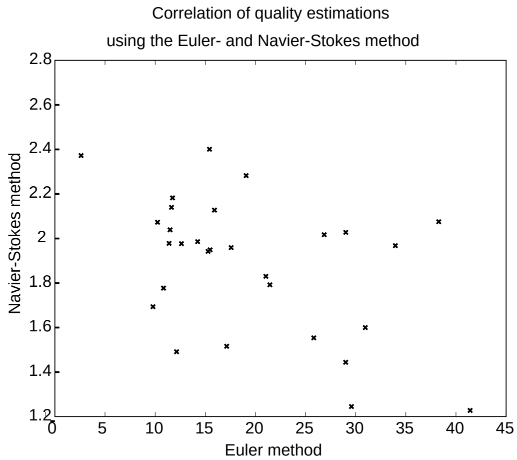


**Fig. 1.** Correlation of the estimated performance of solutions based on simulations applying the Navier-Stokes method and the Euler method.

10, which is the reason why this method is frequently used in aerodynamic optimisations. Even though the accuracy of the simulation method is low compared to other methods, it can be used for evolutionary optimisations as long as the relation between the estimated qualities of designs is preserved. The reason is that the absolute values of the criteria, e.g. the absolute value of the pressure loss, is of little interest as long as it can be decided which solution is better with respect to the observed criteria. In order to decide whether simplified methods can be used a correlation analysis was carried out. A comparison of the estimated performance of solutions based on simulations using the Navier-Stokes method and the Euler method is shown in Figure 1 for transonic flow conditions. The figure shows that there is no correlation between the two methods.

Another reason for using robust and precise simulations, which cover a wide range of designs, are the random modifications of design parameters during the optimisation. Theoretically, all possible parameter values can emerge. Simplifications of simulation algorithms reduce the number of physical effects which are taken into account and therefore, the accuracy of simulations. As a result the range of the simulation space is often restricted. The risk of incorrect results is high especially in design parameter regions which are significantly different from regions where simulation tools are usually used. This especially leads to problems when superior optima are generated by incorrect simulations, so that the algorithm converges to parameter regions which seem to be "high quality areas" due to simulation errors. In particular in preliminary design optimisations, where the quality of significantly different designs has to be evaluated, this can lead to false results. Therefore, the simplifications in simulation models can only be used after careful considerations of possible simulation errors and with error measures during the optimisation process [10].

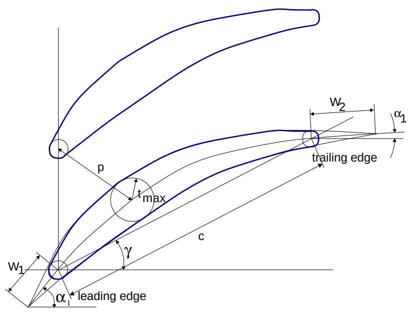


**Fig. 2.** Encoding of a stator blade which is strongly based on a priori knowledge.

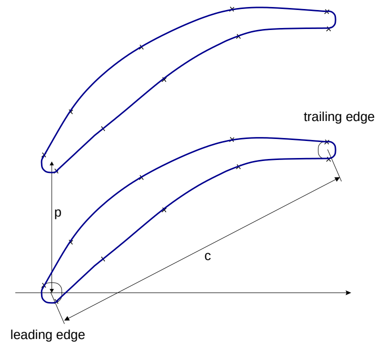


**Fig. 3.** Extended encoding for the suction and pressure side of the air foil.

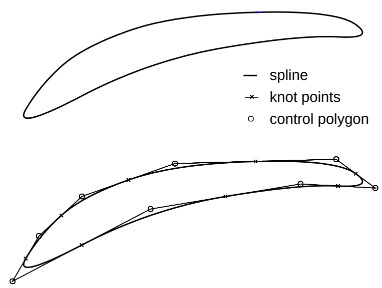


**Fig. 4.** Spline encoding of the stator blade with increased degree of freedom at the cost of a higher dimensional search space.

## 2.2 Parameterisation of the blade geometry

In order to apply evolutionary algorithms to design optimisation problems, a parameterisation model is needed. The number of parameters defines the dimension of the search space and therefore, highly influences the number of iterations which is needed for an optimisation. Therefore, a compact representation with a small number of independent parameters allows to reduce the necessary time for an optimisation.

In order to achieve a compact parameterisation, which corresponds to a low dimensional search space, a representation[1] of the turbine blade can be based on expert knowledge about the problem and about the possible designs of the blade. An example for such kind of model defined for sub-sonic turbine blades is given in Figure 2. Important parameters for the flow condition and also for the stability of the design are included in the model. The curves

[1] In this paper "representation" and "parameterisation model" will be used synonymously as describing the mapping from the genotype to the phenotype (the 2D-blade cross section).

describing the suction and pressure side of a blade are defined by $3^{rd}$ order Bézier curves, which are fixed by angles and radii at the leading and trailing edge and by a defined radius in the middle section of the blade.

Even if the model allows a highly compact representation of the design, the freedom for modifications of the shape is highly restricted which in turn also restricts the possible outcome of the optimisation.

A more detailed description of the curve can be achieved by allowing a larger degree of freedom at the middle part of the blade. Such an extended model is shown in Figure 3 where the top and the bottom curve of the blade are defined by higher order polynomials which are tangential to the circles at the leading and trailing edge. Here a more detailed structure of the blade can be represented at the cost of a higher dimensional search space. Such a representation was successfully employed for the optimisation of turbine blades with Evolution Strategies in [6].

An even more flexible model describing the cross section of a blade is shown in Figure 4. The description is based on a cyclic spline curve. Besides elementary constraints like closeness, continuity and differentiability of the curve only a minimum of a priori knowledge is used in this description, like the number of control points, which influences the smoothness of the spline and at the same time the degree of details which can be represented. Further a priori knowledge can be integrated by defining the initial position of the control points of the spline to reduce the time for the optimisation.

### 2.3 Adaptive representations

All of the described models have been used for the optimisation of turbine blades with Evolution Strategies with the expected results of increasing the quality of the final result and the likeliness of new, unsuspected designs at the cost of a substantial increase in computational demand.

However, increasing the number of design parameters in order to increase the quality of the final result by adding new parameters, for example by adding spline control points in a spline model (see Figure 4), do not necessarily leads to solutions which are significantly better than existing designs. This is due to the risk of premature convergence of the algorithm to a local optimum in a high dimensional search space.

In order to combine fast and efficient search with sufficient variability, an adaptive representation is proposed. By starting the optimisation with a minimal number of parameters, a large part of the search space can be covered in a small number of iterations. If the optimal solution in the limited search space is found, the representation is extended in order to allow a more refined design based on the already found solution. However the parameter, which should be added in order to allow a further increase of quality, is not known. Therefore the model adaptation is realised by an additional evolutionary mutation operator [11].

Thus, two evolutionary processes are combined. One is the adaptation

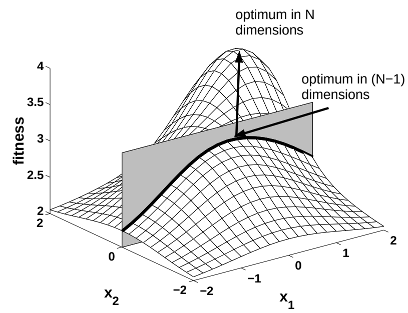


**Fig. 5.** Schematic description of possible changes in the fitness landscape when the dimensionality of the search space is increased.

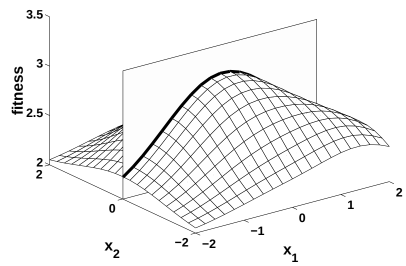


**Fig. 6.** Example for an extension where no further increase of the fitness is possible after the extension of the dimensionality.

of the parameters of the underlying model or representation, the other the adaptation of the model itself. The changes of the design variables, i.e. the parameters of the model, directly result in changes of the quality. Therefore, it is straightforward to evaluate the changes in parameters. Unfortunately, this is not the case for modifications of the representation itself. Indeed, modifications of the model are made in order to increase the degree of freedom and only subsequently, through variations of the new parameters, to increase the quality. Whether a change of the model is beneficial can be seen only after a sufficiently long time when favourable modifications of the parameter values have been reached. Therefore, the two adaptations work on different time scales. In Figure 5 an explanation of the effect of an extension of the search space by an extension of the parameterisation model is given. In an early stage of the optimisation, the search space is low dimensional, which is visualised by the one dimensional fitness function which is lying in the two dimensional gray plane. When the optimum is found after a sufficient number of iterations no further progress is possible in the low dimensional search space. By extending the search space new optima can appear in the higher dimensional fitness function. If the found position in the low dimensional search space is a good starting point for the search in the higher dimensional space, the extension of the representation has been successful. If the extension does not lead to a higher dimensional space, where the fitness can be further increased, see Figure 6, the extension of the parameters should not be selected and inherited to following generations.

The adaptation of the representation therefore describes a way in which the different approaches towards "preliminary design" and "design optimisation" can be smoothly integrated into one method. In preliminary design, a large part of a limited, i.e. not including all necessary design details, search space is covered and a wide variety of unconventional designs are tried. On the other hand, in design optimisation the search space must include a detailed description in order to be precise enough for experimental tests, which usually follow a successful design optimisation.

## 3 Comparison of algorithms and encoding methods

In order to compare different Evolution Strategies and different parameterisation models according to their usability in aerodynamic design optimisations, it is necessary to compare the averaged performance for different parameter settings. Due to the high computational cost, it is not possible to use aerodynamic optimisations for this comparison. Therefore, a test function is used, which has similar attributes and is based on the same representation as the quality function which is used in aerodynamic design optimisation.
In the experiments, which are described in this section, the control polygon of a closed spline curve, represented by spline control points in Cartesian coordinates, is adapted in such a way, that the spline is as close as possible to a given target curve. The distance measure is defined by a modified Hausdorff measure between the target curve and the spline represented by the parameters encoded in the chromosome of an individual.

### 3.1 Distance measure

Assuming two sets of data points $\mathcal{A}$ and $\mathcal{B}$ the used fitness measure is defined by

$$
\begin{aligned}
f_{dist}(\mathcal{A},\mathcal{B}) = \tfrac{1}{2}\Bigg( & \sum_{i=1}^{|\mathcal{A}|} \min\{\mid \boldsymbol{a}_i - \boldsymbol{b} \mid^2; \boldsymbol{b} \in \mathcal{B}\} && (1)\\
+ & \sum_{i=1}^{|\mathcal{B}|} \min\{\mid \boldsymbol{b}_i - \boldsymbol{a} \mid^2; \boldsymbol{a} \in \mathcal{A}\}\Bigg). && (2)
\end{aligned}
$$

The data sets $\mathcal{A}$ and $\mathcal{B}$ consist of sample points from both curves, the size of the sets is given by $\mid \mathcal{A} \mid$ and $\mid \mathcal{B} \mid$. In case of a perfect match the distance measure should return a value $f_{dist} = 0$. In the presented results this cannot be observed due to the limited number of sample points, which are used to evaluate the fitness. Furthermore, it has to be noted, that the defined quality measure is not invariant against translation of the curves, in contrast to the aerodynamic quality measure.

### 3.2 Comparison of Evolutionary Algorithms

In a first set of experiments different adaptation methods for strategy parameters in Evolution Strategies are compared for a fixed number of spline control points which implies a fixed chromosome length. The following methods are employed:

GSA global mutative step size adaptation [12]
ISA individual step size adaptation[12]
IDA derandomised individual step size adaptation and the estimation of one mutation direction [8]
CMA covariance matrix adaptation [13]

The four strategies are chosen to give an overview over the main variants of Evolution Strategies, beginning with the simplest form of step size adaptation with only one global step size (GSA) to the estimation of individual step sizes including correlations between parameters (CMA).

The overall number of fitness evaluations is recorded here, which is proportional to the overall computation time in case of fitness evaluations with high computational costs compared to other calculations necessary in the evolutionary algorithm.

All results which are shown in the following are averaged over 20 runs of identical optimisations, which are started with different sets of random numbers. The initial parameter set for the optimisation defines a spline in the shape of a circle around the target curve as shown in Figure 7. In all experiments an Evolution Strategy with two parent individuals and 10 offspring individuals per generation (ES(2,10)) was used.

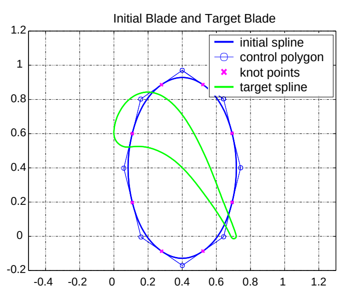


**Fig. 7.** Initial spline used in optimisations with a fixed chromosome length and target curve for all optimisations.

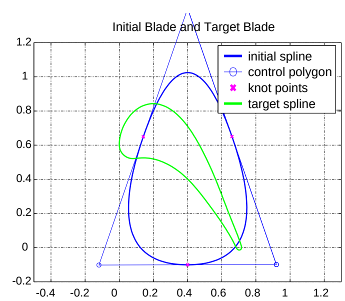


**Fig. 8.** Initial spline used in optimisations with an adaptive chromosome length and target curve for all optimisations.

In the case of a fixed number of parameters the final quality of the solution largly depends on the number of design parameters. If the number is too low, the variability and the possible degree of details, which can be described is often not sufficient. One could argue, that the final quality of an optimisation can be increased by simply increasing the number of free variables if there is only sufficient time to adapt all parameters. In a different set of experiments, see [11], it has been shown that an optimisation tends to converge to local optima if the number of parameters is too high. In the used test function the main reasons for this are loops in the spline, which describe local optima in the fitness function. In the experiments in [11] it has been shown, that a number of eight spline control points, corresponding to 16 design parameters, is the optimal number for the used test function and starting conditions. This number of control points was used for the comparison.

The optimisations using an adaptive encoding started with three control points located as shown in Figure 8 and every $g_{mut} = 4$ generations the model mutation was applied. The population size was the same as in the experiments using a fixed encoding. The details on how the operators for changing the representation are integrated in the Evolution Strategy can be found in [11].

The convergence properties of the different experiments are summarised in Figure 9 where the logarithm of the distance between both curves according to equation 1 is plotted as the fitness value against the number of evaluations. It can be seen, that the Evolution Strategy with individual step size adaptation shows the worst convergence properties concerning convergence speed as well as the quality of the final result. This is mainly an effect of the small population size which does not allow a stable adaptation of the strategy parameter. Note that an increase of the population size leads to a more stable adaptation of the strategy parameters, however at the expense of an increase in the number of evaluations, [14]. The global step size adaptation shows a very high convergence speed in the first about 50 generations (500 evaluations). In the beginning of the optimisation, where a rough adaptation of the control points is necessary, one single step size is sufficient and it can be adapted very fast. Later on, the step sizes for different parameters should be different and the convergence speed is low compared to the strategies called IDA and CMA, where an individual step size is adapted with a derandomized method. In case of a fixed encoding the best convergence speed can be observed for the covariance matrix adaptation method.

The convergence speed of the adaptive encoding can be compared in the beginning of the optimisation with the covariance matrix adaptation. Due to the lower number of parameters which has to be adapted in order to increase the quality of the match the convergence speed is high. In order to simplify the step size adaptation for encodings with a variable number of parameters, a global step size adaptation method was used.

Due to the step wise growth of the representation, the disadvantages of the global step size method GSA, namely the premature convergence can be overcome. Additionally, the structure mutations and there evaluations make it necessary to increase the number of evaluations. However, as can be seen from Figure 9, even this increase is more than compensated by the more efficient search realised by the adaptive model. Later on during the optimisation, a higher quality can be observed in the solutions due to the higher degree of freedom, which allows a better match of the target structure.

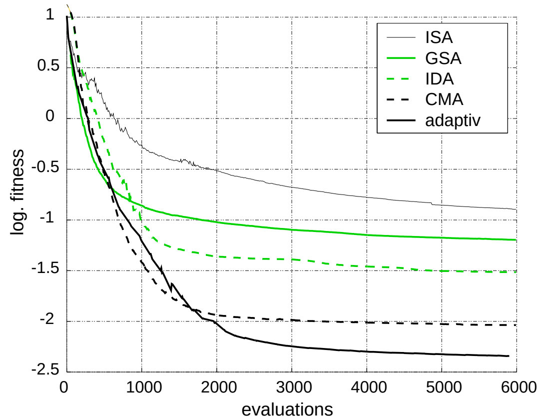


**Fig. 9.** Comparison of the convergence speed for different strategies.

## 4 Discussion and Outlook

In order to apply evolutionary algorithms to aerodynamic design optimisations the parameterisation of the design, the simulation of aerodynamic properties and the used optimisation method have a high influence on the optimisation process and on the final result. The used parameterisation model is always a compromise between a low dimensional model which allows to achieve a high convergence speed combined with a reduced probability of a convergence to local optima and a high dimensional model which allows a detailed description. In order to overcome this trade-off an adaptive parameterisation model was introduced which allows an adaptation of the degree of freedom for the design. Using an adaptive model principles from a preliminary design optimisation and a design optimisation can be combined. This model was compared to static models with the outcome, that the convergence speed even when using simple forms of strategy parameter adaptations, is similar to the high convergence speed of derandomized strategies. Furthermore, it has been shown that the quality of the final result of an optimisation is higher than that of fixed representations due to the ability to refine the model.

All comparisons have been done using a test function, which was chosen in such a way that it is as similar as possible to the quality function in aerodynamic designs of turbine blades, which is the target application. However, a comparison of adaptive representations and fixed representations for aerodynamic optimisation has to be done in the future.

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