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Examining the Performance of Evolutionary Many-Objective Optimization Algorithms on a Real-World Application

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***Abstract*—Recently research into many-objective optimization has attracted much attention. One of the main topics of the research is to develop evolutionary many-objective optimization (EMAO) algorithms that can solve optimization problems with more than three objectives. EMAO algorithms generally differ from evolutionary multi-objective optimization (EMO) algorithms as EMO algorithms are known to work mainly for optimization problems with two or three objectives. Thus far the performance of EMO algorithms has been validated using both benchmark test problems and real-world applications. Although the performance of EMAO algorithms has also been shown using benchmark test problems, their performance on real-world applications rarely appears in the literature. In this paper we examine the performance of state-of-the-art EMAO algorithms by applying them to a real-world application, namely a hybrid car controller optimization problem with six objectives. It is demonstrated that EMAO algorithms work well for this optimization problem.**

***Keywords*-Many-objective optimization, multi-objective optimization, evolutionary algorithms, applications.**

I. Introduction

For a few decades multi-objective optimization has been one of the most active research areas in evolutionary computation [2]. In multi-objective optimization we normally consider two or three objectives that often compete against each other. In single-objective optimization a single solution or solutions that overlap in a decision variable space will be obtained as the estimated optimal solution(s) [5]. In multi-objective optimization non-dominated solutions that are not comparable with each other will be obtained as the estimated Pareto-optimal solutions. The solution in single-objective optimization is basically evaluated by its single objective value. On the other hand, the solution in multi-objective optimization is normally evaluated by its convergence to the Pareto front, diversity and uniformity in the objective space. For example, in the case of NSGA-II [3] which is one of the most well-known Pareto dominance-based (i. e., rank-based) evolutionary multi-objective optimization (EMO) algorithms, the convergence is measured by a rank value and the diversity and uniformity by a crowding distance. The rank value is assigned as follows. First non-dominated solutions in a population will be found. Then the first rank is assigned to them. Secondly non-dominated solutions in the population where solutions with the first rank are discarded will be found. Then the second rank is assigned to them. Iterating this process different rank values are assigned to each solution. In NSGA-II, selection pressure is kept via the variety of rank values. Although an assignment of the rank value plays an important role not only in NSGA-II but also in many EMO algorithms, this approach does not work anymore for optimization problems with many objectives. This is because almost all solutions get non-dominated in the higher dimensional objective space. Therefore the same rank (i. e., only the first rank) is often assigned to each solution in a population. This prevents solutions from converging to the Pareto front as there is no selection pressure based on the rank value in the population.

In this situation mainly three kinds of evolutionary many-objective optimization (EMAO) algorithms have been proposed [6]. One is an indicator-based algorithm [1], [11]. Another is a scalarizing function-based algorithm [10]. The other is a modified rank-based algorithm [8]. Although the performance of these EMAO algorithms has been shown using some benchmark test problems [9], their performance on real-world applications rarely appears in the literature. In the present paper we examine the performance of state-of-the-art EMAO algorithms (i. e., SMS-EMOA [1], MOEA/D [10], and CDAS [8]) by applying them to a real-world application. The performance of NSGA-II is also considered for comparison. As the real-world application, we have a hybrid car controller optimization problem with six objectives. It is demonstrated in computational experiments that EMAO algorithms work well not only for benchmark test problems but also for the hybrid car controller optimization problem. We also observe that an EMAO algorithm which works well for benchmark test problems does not work well for the hybrid car controller optimization problem.

This paper is organized as follows. First a general introduction to multi-objective optimization is given. We also explain EMAO algorithms in section 2. In the next section the EMAO algorithms are compared on benchmark test problems. As the benchmark test problems we use DTLZ1 and DTLZ2 [4]. In section 4 the performance of EMAO algorithms is demonstrated with the hybrid car controller optimization problem. The paper concludes in section 5.

II. Evolutionary Many-Objective Optimization Algorithms

Throughout the paper we consider minimization problems. Generally a *k*-objective minimization problem is written as follows:

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_k(\mathbf{x})), \quad (1)$$

$$\text{subject to } \mathbf{x} \in \mathbf{X}, \quad (2)$$

where $\mathbf{f}(\mathbf{x})$ is the k-dimensional objective vector, $f_i(\mathbf{x})$ is the i-th objective value to be minimized, $\mathbf{x}$ is the decision vector, $\mathbf{X}$ is the feasible region. When we have two feasible solutions **a** and **b** of (2) that satisfy the following conditions, it can be said that **a** dominates **b**.

$$\forall i : f_i(\mathbf{a}) \leq f_i(\mathbf{b}) \text{ and } \exists j : f_j(\mathbf{a}) < f_j(\mathbf{b}). \quad (3)$$

If **a** is not dominated by any other feasible solutions, **a** is referred to as a Pareto-optimal solution. The entire set of Pareto-optimal solutions forms the Pareto front. The task in multi-objective optimization is to find a set of non-dominated solutions that approximates the whole Pareto front very well [3].

In many-objective optimization, mainly three types of EMAO algorithms have been proposed. One is an indicator-based algorithm. Another is a scalarizing function-based algorithm. The other is a modified rank-based algorithm. We use SMS-EMOA as the indicator-based algorithm, MOEA/D-TBC and MOEA/D-PBI as the scalarizing function-based algorithm, and CDAS as the modified rank-based algorithm. The following describes each algorithm in more detail.

A. SMS-EMOA

The $\mathcal{S}$ metric selection evolutionary multi-objective optimization algorithm (SMS-EMOA) is an indicator-based EMAO algorithm proposed by Beume et al. [1]. As the indicator, SMS-EMOA uses a contribution of each solution to the hypervolume for a population. This algorithm is based on a $(\mu + 1)$ strategy. In every generation, a new solution is generated and added to a population in the same manner as NSGA-II, then the solution with the lowest contribution to the hypervolume is removed from the worst-ranked solutions in the population. The maximization of the hypervolume contribution as a result makes sense because the maximal hypervolume is reached by the Pareto front even in the higher dimensional objective space [9]. The only drawback of methods that use the hypervolume is its runtime. Even for the best algorithms to calculate the hypervolume known so far, the runtime is polynomial in the number of solutions and grows exponentially with the number of objectives [1].

B. MOEA/D

The multi-objective evolutionary algorithm based on decomposition (MOEA/D) developed by Zhang et al. uses a scalarizing function for fitness evaluation [10]. This algorithm decomposes an optimization problem with many objectives into a number of scalarizing function optimization problems. Some types of scalarizing functions can be considered in MOEA/D. In this paper, we consider two types of scalarizing functions. One is based on the Tchebycheff distance. The other is based on the penalty-based boundary intersection. We call MOEA/D with the Tchebycheff distance as MOEA/D-TBC, MOEA/D with penalty-based boundary intersection as MOEA/D-PBI in this paper. In the Tchebycheff distance approach, the scalarizing function problem is described as follows:

$$\text{Minimize } g(\mathbf{x}|\lambda, \mathbf{z}^*) = \min_{1 \leq i \leq k} \{\lambda_i | f_i(\mathbf{x}) - z_i^*|\}, \quad (4)$$

$$\text{subject to } \mathbf{x} \in \mathbf{X}, \quad (5)$$

where λ is a weight vector, $\mathbf{z}^*$ is a reference vector. For the penalty-based boundary intersection approach, we consider the following problem.

$$\text{Minimize } g(\mathbf{x}|\lambda, \mathbf{z}^*) = d_1 + \theta d_2, \text{ subject to } \mathbf{x} \in \mathbf{X}, \quad (6)$$

$$d_1 = \frac{||(\mathbf{f}(\mathbf{x}) - \mathbf{z}^*)^T \lambda||}{||\lambda||}, \; d_2 = ||\mathbf{f}(\mathbf{x}) - (\mathbf{z}^* + d_1 \lambda)||, \quad (7)$$

where $\theta > 0$ is a user-specified parameter. The detailed description of MOEA/D-TBC and MOEA/D-PBI is given in [10].

C. CDAS

As discussed in section 1 and 2, there is a curse of dimensionality when the number of objectives is increased, i. e., almost all solutions in a population get non-dominated in the higher dimensional objective space, thereby EMO algorithms often lose the selection pressure based on the dominance relation in the population. In order to break the curse, an algorithm for controlling dominance area of solutions (CDAS) has been proposed by Sato et al. [8]. This algorithm relaxes the Pareto dominance relation by modifying the i-th objective value as follows:

$$f_i'(\mathbf{x}) = \frac{r \sin(\varphi_i + \omega_i)}{\sin(\varphi_i)}, \quad (8)$$

where φ_i is a user-specified parameter, r is the norm of $\mathbf{f}(\mathbf{x})$, and ω_i is the declination angle between $f_i(\mathbf{x})$ and $\mathbf{f}(\mathbf{x})$. Based on the modified objective values, CDAS assigns the rank value to each solution in a population. Then more different rank values can exist in the population than before. With the variety of rank values, CDAS can keep the selection pressure even in the higer dimensional objective space.

III. Benchmark Test Problems

In this section, first we show the performance of EMAO algorithms described in the previous section on benchmark test problems with six objectives. We do not include the results of SMS-EMOA simply because its calculation cost

Table I
MEAN, STANDARD DEVIATION (S. D.), AND THE BEST VALUE OVER 20 RUNS ON THE CONVERGENCE (CONV.) AND RELATIVE HYPERVOLUME (RHV) MEASURES FOR DTLZ1 WITH SIX OBJECTIVES

Measure	Statistic	NSGA-II	M/D-TBC	M/D-PBI	CDAS
Conv.	Mean	376.4	28.91	1.883	0.014
	S. D.	29.36	27.72	4.587	0.008
	Best	328.7	0.155	0.001	0.002
RHV	Mean	0.000	0.092	0.756	0.994
	S. D.	0.000	0.249	0.326	0.001
	Best	0.000	0.824	0.919	0.995

Table II
MEAN, STANDARD DEVIATION (S. D.), AND THE BEST VALUE OVER 20 RUNS ON THE CONVERGENCE (CONV.) AND RELATIVE HYPERVOLUME (RHV) MEASURES FOR DTLZ2 WITH SIX OBJECTIVES

Measure	Statistic	NSGA-II	M/D-TBC	M/D-PBI	CDAS
Conv.	Mean	1.785	0.005	0.000	0.082
	S. D.	0.097	0.002	0.000	0.014
	Best	1.603	0.001	0.000	0.061
RHV	Mean	0.004	0.422	0.894	0.671
	S. D.	0.009	0.036	0.000	0.019
	Best	0.034	0.501	0.894	0.709

is too huge to get statistics. However it should perform as the best if the same amount of evaluations is possible. The performance of NSGA-II is also considered for comparison. As benchmark test problems, we use DTLZ1 and DTLZ2 [4]. The number of decision variables is specified as 10 in DTLZ1 and 15 in DTLZ2, respectively, as recommended in [4]. The population size in MOEA/D is determined by the combination of $C(H+k-1,\ k-1)$ where H is a user-specified parameter, k is the number of objectives. Here we choose $H=4$ and $k=6$ yielding $C(9,5)=126$ as the population size. We set the number of solutions as 126 for all algorithms in section 3 and 4. In MOEA/D with the penalty-based boundary intersection approach, we choose $\theta=5.0$. The parameter φ_i in CDAS is specified as $\varphi_i=0.4\pi$ for all i. For benchmark test problems, the stopping condition is $126\times 101=12726$ evaluations. The other parameters are the same as in Wagner et al. [9].

We use two performance measures. One is the convergence measure. The other is the relative hypervolume. The convergence measure is used to calculate the convergence of solutions in a population to the Pareto front in the objective space [9]. The convergence measures for DTLZ1 and DTLZ2 are written as follows:

$$d_{\mathrm{DTLZ1}} = |f_1+f_2+...+f_k-0.5|, \quad (9)$$

$$d_{\mathrm{DTLZ2}} = |\mathbf{f}(\mathbf{x})|-1. \quad (10)$$

In order to calculate the hypervolume, the reference point $\mathbf{r}$ should be specified. In this section we use $\mathbf{r}=0.7^k$ for DTLZ1 and $\mathbf{r}=1.1^k$ for DTLZ2. Solutions that did not dominate the reference point $\mathbf{r}$ are removed in the calculation. The relative hypervolume is calculated using the analytical optimal value [9].

First of all, we show mean values, standard deviations, and the best values over 20 runs on the convergence and relative hypervolume measures in Table I for DTLZ1 and Table II for DTLZ2, respectively, where MOEA/D is denoted by M/D due to space limitation. From Table I, we can see that CDAS outperforms the other algorithms both on the convergence and relative hypervolume for DTLZ1. Although MOEA/D-PBI performs very well, the standard deviation is relatively high. This is because failure cases giving no solutions that dominate the reference point in the hypervolume calculation are found in 3 out of 20 trials. This characteristic feature of MOEA/D-PBI could be a problem in real-world applications as the number of evaluations is restricted, thereby any failures are not allowed. We can also see that the MOEA/D-TBC sometimes works well. However its standard deviation is also high. It should be noted that NSGA-II does not perform anymore in the six-dimensional objective space. In the result for DTLZ2 in Table II, we can see that the performance of MOEA/D-PBI is the best. Furthermore both MOEA/D-TBC and CDAS perform well among the tradeoff between the convergence and relative hypervolume measures. It should be noted that all tested EMAO algorithms do not have large standard deviations for DTLZ2. This means EMAO algorithm can stably find good non-dominated solutions that approximate the Pareto-front very well for this case.

IV. REAL-WORLD APPLICATION

In this section we apply four EMAO algorithms to a real-world application. The result from NSGA-II is also included for comparison. As the application we use a hybrid car controller optimization problem with six objectives. This problem has 11 inputs (decision variables). First a rule of the controller in a simulator is determined by 11 decision variables. Then a driving cycle is simulated based on the rule-based controller. Finally six outputs (objective values) to be minimized are obtained from the simulator. We handle the inside of the simulator as a black box. With the same parameter settings as in the previous section except that the stopping condition is $126\times 21=2646$ evaluations, we examine the performance of EMAO algorithms. Regarding the performance measure, we use the hypervolume measure with a reference point consisting of the worst (maximal) objective values of non-dominated solutions that are calculated from all solutions found by all EMAO algorithms and NSGA-II. The result is shown in Fig. 1. From Fig. 1, we can see that SMS-EMOA apparently outperforms the other algorithms. However its calculation time including the simulator running is about 50 hours. The MOEA/D-TBC and CDAS algorithms are found as the second best EMAO algorithms. Furthermore their calculation time including the simulator running is around 9 hours. Although MOEA/D-PBI does work quite

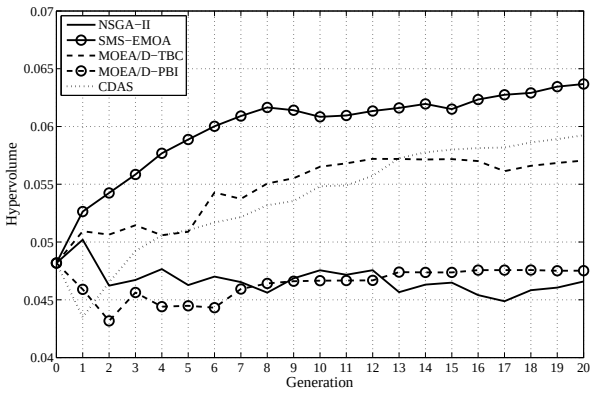

Figure 1. Performance of NSGA-II and EMAO algorithms on the hybrid car controller optimization problem with six objectives

well for DTLZ2, it does not perform well for this problem. We can also see that NSGA-II does not perform very well. It should be noted that the hypervolume of SMS-EMOA sometimes gets worse in the middle of generations although it should maximize the hypervolume contribution. This is because the exact nadir point is not used as the reference point in the hypervolume measure.

V. Summary

In this paper we examined the performance of state-of-the art EMAO algorithms on DTLZ1, DTLZ2, and a hybrid car controller optimization problem. We also included the result of NSGA-II for comparison. It is clearly shown in computational experiments that SMS-EMOA performed as the best on the hybrid car controller optimization problem. If designers are only interested in non-dominated solutions that approximate the Pareto front very well, the choice of SMS-EMOA may be the best to work with real-world applications. However the calculation time of SMS-EMOA including the simulator running was more than five times as much as each of the other algorithms. Therefore approximation methods for the hypervolume contribution in SMS-EMOA should be considered (e. g., [7]). On the other hand, CDAS might be also suitable for real-world applications as shown in the previous section. This is because CDAS stably obtained a good solutions set on benchmark test problems. The feature is quite important especially when failures are not allowed. Although MOEA/D-PBI worked quite well on DTLZ2, its performance on the hybrid car controller optimization problem was poor. The MOEA/D-PBI algorithm may work only for well-defined optimization problems. Contrary to MOEA/D-PBI, MOEA/D-TBC performed very well on the hybrid car controller optimization problem. This suggests that the Tchebycheff distance used in MOEA/D-TBC is useful in the higher dimensional objective space. It should be noted that NSGA-II does not work anymore in the higher dimensional objective space as there are only solutions with the first rank giving no selection pressure after a few generations. Although we examined the performance of EMAO algorithms on the hybrid car controller optimization problem in this paper, their performance should be demonstrated with a number of applications in order to statistically evaluate them in the near future.

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