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Application of Sensitivity Analysis for an Improved Representation in Evolutionary Design Optimization

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***Abstract*—The choice of the representation in evolutionary design optimization defines the flexibility and constraints of the search process. Finding an adequate representation is a pre-requisite for the success of an optimization but requires extensive knowledge about the design at hand. Based on the results of an automotive part design optimization, the authors provide evidence that an adaptation of the representation based on sensitivity information leads to new outperforming designs. For retrieving reliable sensitivity estimates a robust variant of the mutual information has been introduced. The robust sensitivity measure provides valuable information for the setup of an improved representation.**

***Keywords*-Evolutionary Design Optimization, Sensitivity Analysis, Knowledge Extraction, Automotive Design**

I. Introduction

In evolutionary design optimization, the representation of the design plays a crucial role for the success of an optimization run. It intrinsically defines the constraints and degrees of freedom for the entire search process. Especially for real world applications, extensive domain knowledge is required to find an adequate representation for a particular optimization problem. On one hand, if the representation severely limits the degrees of freedom for the search process the entire optimization might fail because the optimum cannot be represented. On the other hand, if the flexibility of the representation is too high the optimizer might fail due to the high complexity of the search space.

In situations where no or limited domain knowledge is given only a naive definition of the representation is possible, which often result in non-optimal solutions. Technologies from data mining provide a means to extract domain knowledge from existing design data which can be used to improve the representation, allowing a higher degree of freedom in design areas with high sensitivity. An investigation of the sensitivity of design parameters [2], or directly investigating the influence of surface variations on the design quality [3] provides relevant information to improve the parameterization. In this paper, we will extract information about the design of an automotive part and utilize this information to adjust the representation for computational design optimization.

II. Evolutionary Design Optimization Using Free-Form Deformation

Design optimization targets the identification of a functionally superior design configuration compared to a given baseline design. Stochastic optimization strategies imitating the course of evolution have shown to be a valuable tool for achieving this target. The setup of the design optimization process involves decisions about the representation of the design, the optimization strategy and the objective function (including constraints). In this study, we build upon the work of Menzel et. al. [6], where an evolution strategy with covariance matrix adaptation (CMA-ES) in conjunction with Free-Form Deformation (FFD) has successfully been applied, e.g. to the optimization of a 3D stator blade design of a turbofan jet engine. More details on the individual optimization components are given below.

A. Free Form Deformation Representation

Free-form deformation (FFD) has been introduced by Sederberg and Parry [10] in the field of computer graphics for deforming solid geometric objects. FFD does not build an explicit representation of the object shape, rather the object of interest is embedded into a lattice of control points in $\mathbb{R}^3$. Based on predefined transfer functions (e.g. using the tri-variate Bernstein polynomial or a linear combination of B-Spline functions) modifications of the control volume cause changes of the embedded object. Since FFD does not represent the object itself, the geometry can be arbitrarily complex. At the same time, the possible variations (deformations) that can be realized are limited by the degree of freedom of the control volume and its initial positioning.

B. Objective and Constraints

In aerodynamic design optimization the functional measure or objective function usually depends on aerodynamic forces like lift or drag. In addition, geometric constraints are required to ensure constructability or to take additional design criteria into account, e.g. cooling effects of the airflow for engine components or brakes in automotive parts design. To retrieve physical characteristics related to the motion of air, CFD simulations are carried out by means of finding a solution to the Navier stokes equations. It requires the

definition of a CFD mesh, a discretization of the space around the object where the flow is conducted. To avoid the generation of the CFD mesh for each design, FFD can be applied to deform the CFD mesh in line with the deformation of the actual object shape.

C. Optimization Strategy

The control point parameters of the FFD lattice, which define the object shape, are encoded into the chromosome of the start population of the evolution strategy. Following the concept of the CMA-ES [5], individuals of the offspring population are generated by sampling from a multi-variate normal distribution $\mathcal{N}(m, C)$, with mean $m \in \mathbb{R}^n$ and covariance matrix $C \in \mathbb{R}^{n \times n}$. The mean m is calculated according to the parameter vectors of the parent population. Base on the concept of self-adaptation in evolution strategies, the values of the covariance matrix are updated throughout the optimization, directing the search in the design space. The offspring in each generation is evaluated using CFD and the drag force is assigned as a fitness value to each individual. The μ fittest out of λ individuals are selected as parents for the subsequent generation, following a non-elitist (μ, λ) strategy.

III. Exploiting Design Data

Data mining technologies have shown to provide valuable insights into the design domain by discovering potentially unknown information from an existing set of designs, see e.g. [3], [2]. We assume that information about the influence of design variations on the design quality can be used for the setup of a suitable representation for the search process. Fig. 1 provides an executive summary of the basic concept. The initial representation is configured based on the engineer's preference that mostly results in a uniform lattice centered around the area of the design which is subject to changes. Optimization is carried out until a certain stopping criterion is met (e.g. time). If the overall design goal is reached the whole process stops. If the result of the optimization does not meet the final design goal the representation of the design is adapted based on the extracted information from previous optimization runs. In this study, a robust method for global sensitivity analysis is introduced to rate the importance of design features and to provide the information to support the tuning of the representation.

A. Sensitivity Analysis

Following [8], we analyze the sensitivities represented by the relation between design features and quality measures. The quantification of the sensitivity allows an ordering of the design features according to their functional importance. A wide range of methods have already been studied, see e.g. Ascough et al. [1] for an overview. Here, a variant of the mutual information, termed *Robust Mutual Information* is applied which is less sensitive to quantification errors.

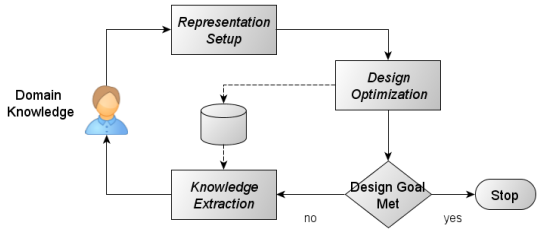


Figure 1. Flow chart sketching the underlying strategy for the adaptation of the design representation based on augmented domain knowledge.

Mutual information quantifies the (nonlinear) correlation between random variables by comparing the underlying distributions. Given the marginal and joint probability distributions of two discrete random variables X_1 and X_2, the mutual information is defined as,

$$MI(X_1; X_2) = \sum_{x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2} p(x_1, x_2) log \frac{p(x_1, x_2)}{p(x_1)p(x_2)}, \quad (1)$$

with $p(x_1, x_2)$, $p(x_1)$ and $p(x_2)$ denoting the marginal and joint probability masses. $MI(X_1; X_2) = 0$ if X_1 and X_2 are independent. In all other cases, $MI(X_1; X_2) > 0$ bounded by the maximum of the marginal Shannon entropies $H(X_1)$ and $H(X_2)$. In real world applications the generation of

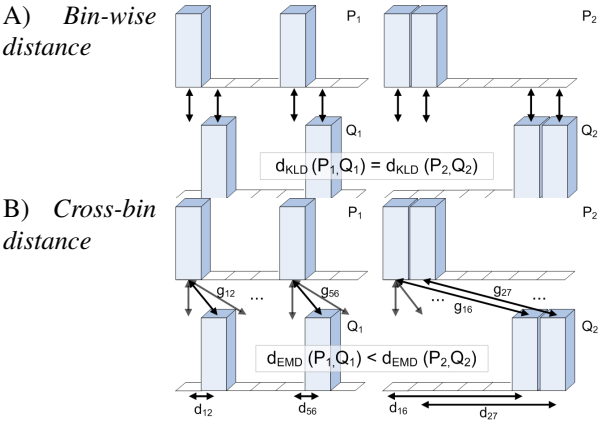


Figure 2. Visualization of the difference between the bin-wise A) and cross-bin B) distance. The EMD cross-bin distance is defined by a weighted superposition of the grounding distances d_{ij} and the mass differences g_{ij}.

data is often computational expensive, what hinders a reliable estimation of the probability masses and a correct estimation of the mutual information. In order to estimate the mutual information more reliably, we firstly note that the formulation of the mutual information equals the Kullback Leibler divergence (KLD), which is a bin-wise difference measure, between the observed $P = p(x_1, x_2)$ and the approximated joint probabilities $Q = p(x_1)p(x_2)$ (Kirkwood superposition approximation) assuming independence, $MI(X_1, X_2) = D_{KLD}(P||Q)$. In a second step, we replace the Kullback Leibler divergence with an alternative distance measure between distributions: the Earth Mover's distance

(EMD), see [7]. Contrary to the bin-wise KLD measure, the cross-bin EMD measure allows a more robust estimation of the distance between two distributions for a low number of observations. This is depicted in Fig. 2. The standard KLD, Fig. 2 A), would derive the same distance between distributions (P1,Q1) and (P2,Q2). However, we could easily imagine that the difference between (P1,Q1) could result from measurement noise or from a limited number of data samples while the difference between (P2,Q2) seems to be more distinct. Therefore, an intuitive measure should rate the distributions (P1,Q1) being closer together than (P2,Q2). The EMD measure, Fig. 2 B), does this by taking neighboring bins into account. As a result small perturbations in the data do not have a drastic effect on the cross-bin distance measure (EMD), as indicated in Fig. 2.

The calculation of the EMD is in itself the solution to an optimization problem and computational expensive. Therefore, Shirdonkar and Jacobs [9] proposed a fast algorithm for the calculation of the EMD. The authors found a closed solution of the EMD in the wavelet domain and termed this technique wavelet EMD, or in short WEMD. By applying the WEMD as a cross-bin distance measure, $MI(X_1, X_2) \approx D_{WEMD}(P, Q)$, we derive a robust measure for the quantification of the mutual dependency between two random variables as:

$$RMI(X_1, X_2) = \sum_{\lambda} 2^{-j(1+\frac{n}{2})} |p_\lambda|, \qquad (2)$$

where p_λ are the wavelet coefficients of the transformed n dimensional difference histogram $P - Q$. The WEMD integrates over the shift as well as scale coefficients j indexed with λ. The computation time of the WEMD increases only linearly with the number of bins [9].

B. Utilize Sensitivity Information

The analysis of the sensitivities can be directly carried out based on the parameter and objective values from the optimization run as suggested by [2]. The investigation of the parameter sensitivities allows the ranking of optimization parameters according to their relevance. Thus insensitive parameters can be hidden from a subsequent optimization run, while sensitive parameters should be kept.

However, the effect of the design parameters (in particular for FFD) on the actual design is often not clear to the user. Thus, when applying FFD as design representation neighboring control points have to be considered together in order to identify their effect on the actual design. Therefore, a direct analysis of the design surface representation can provide additional insight into the data. In particular, the direct analysis of the surface representation allows to decide whether additional control points should be inserted into the lattice (increasing the degree of freedom selectively) or whether some control points can be grouped and changed together, reducing the effective degree of freedom.

IV. Results

A. Air Dam Optimization with a Basic Representation

An air dam is a small spoiler under the front bumper of a passenger car with the purpose to reduce drag. As described in Section III, an initial optimization with a standard FFD representation to allow changes to the air dam geometry has been carried out. As shown in Fig. 4 A), control points have been equally distributed along the y-direction of the air dam. Since the air dam is symmetric, half of the control point lattice has been parameterized, with overall $n = 25$ parameters allowing deformations in x- and z-direction ($n_x = 18$, $n_z = 7$).

Mutations applied to the parameters during the evolutionary process result in different air dam geometries. For each new air dam geometry the aerodynamic properties of the complete car are calculated using CFD simulations. The drag value is calculated and assigned as fitness to the respective parameter vector. For the optimization a $(2, 15)$ CMA-ES has been applied with an initial global step size of $\sigma = 0.01$. Two optimization runs using the standard representation with different random seeds have been carried out resulting in 127 different air dam designs. Each optimization was run for 12 generations only due to the high computational effort required for the fitness evaluation. Some of the designs could not be evaluated due to distortions of the CFD mesh. The drag value of the best air dam shape decreased by 5.6%.

1) Analyzing The Design Surface: As we mentioned in Section III-B, for FFD representations it is more informative to calculate sensitivities directly for the design surface and not just for the parameter vector used during optimization. Therefore, for each of the 127 air dam designs a STL (Standard Tessellation Language) surface mesh of vertices has been generated. Firstly, the initial design has been selected as a pivot or baseline design. Secondly, the local difference between all designs and the pivot design has been calculated by estimating all displacements of corresponding vertices between two STL meshes, as suggested in [3]. The sensi-

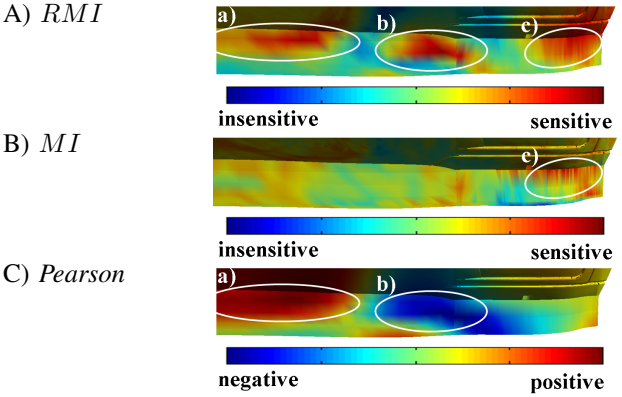


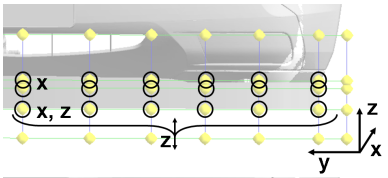
Figure 3. Surface sensitivity analysis: results of the robust mutual information are compared against the standard mutual information and the linear correlation analysis. The respective quantities are color coded.

tivity analysis is applied to the surface mesh representation to identify those regions which have a strong impact on the performance value. The results of the calculation are shown in Fig. 3. Fig. 3 A): three main sensitive areas, marked from a) to c) have been identified based on the RMI (equation 2). For comparison, the sensitivities using the standard mutual information, Fig. 3 B), and the linear Pearson correlation coefficient, Fig. 3 C), have been calculated as well. From the results of the mutual information, the sensitive regions a) and b) can hardly be identified. The linear sensitivity analysis provides additional information about the direction of expected quality improvement for regions a) and b). But it fails to identify the sensitive region depicted by c).

B. Air Dam Optimization with the Adapted Representation

Using the information provided by the sensitivity analysis, we are able to adapt the initial position of the control points of the FFD representation. New control points have been inserted in the vicinity of sensitive design regions. Furthermore, constraints on the direction of modification are introduced to guide the optimization. The updated control point lattice is shown in Fig. 4 B). A reduced set of 12 control point parameters, which focus on highly sensitive design regions, has been used for a third optimization run. The optimization curve of the third run, carried out for another 12 generations, shows a consistently better performance for all generations than the optimization runs using the standard representation. The resulting drag decrease of 9.9% is almost twice as high as the performance gain in the first two runs.

A) *Initial FFD grid*



B) *Adapted FFD grid*

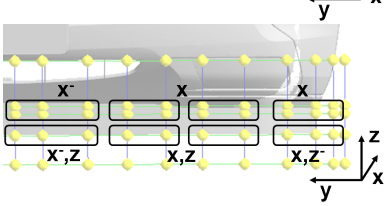


Figure 4. Illustration of the initial and adapted FFD grid and their parameterization for the distinct evolutionary runs. 25 parameters for the initial and 12 for the adapted representation have been used.

V. Conclusions

In this study, the effect of the adaptation of the representation for evolutionary design optimization based on sensitivity information has been investigated. An improved sensitivity measure, the robust mutual information, has been applied, which is less sensitive to errors from small data sets. The sensitivity information has been extracted from previous optimization runs using a standard FFD representation. The adapted representation allows to restrict degrees of freedom where they are not necessary and to increase degrees of freedom where they can be exploited during the optimization process. Using a computationally demanding real world application from the area of automotive parts optimization, we were able to show that the adaptation of the representation can almost double the performance gain during the optimization. Due to the high computational costs of the CFD simulations a proper statistical analysis of the results was not feasible and remains future work. The mutual information and its robust alternative quantify the direct influence of the design variations on the objective function only. Higher order interactions between design features and objective function, as described in [4] have not been considered. The interaction analysis can provide additional insights for further improving the representation used during optimization.

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